April 27, 2023

Advanced Mechanical Elements (Lecture 3)

Dynamic analysis of planar/spatial link mechanisms

- Driving force and joint force analysis using the systematic kinematic analysis -

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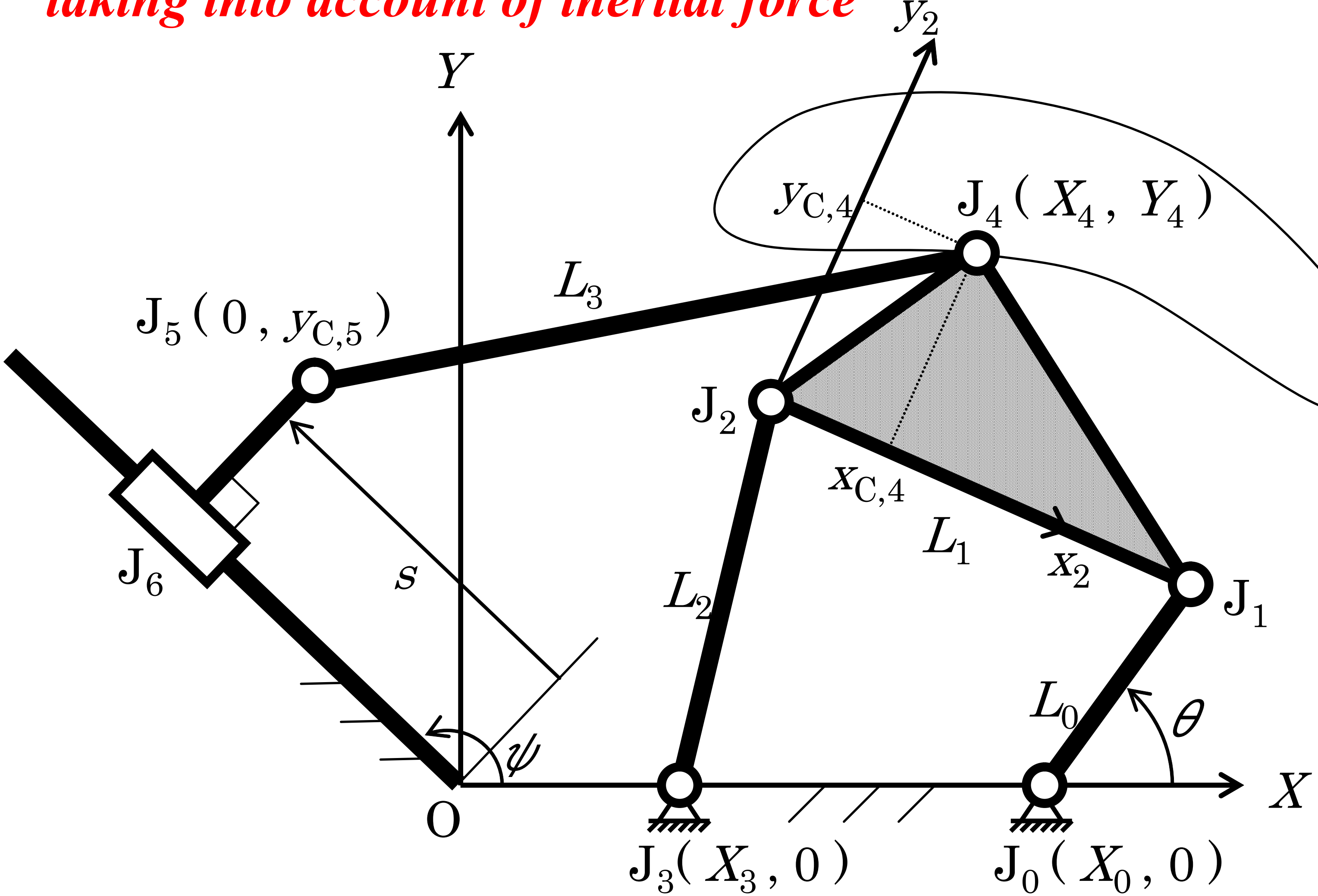
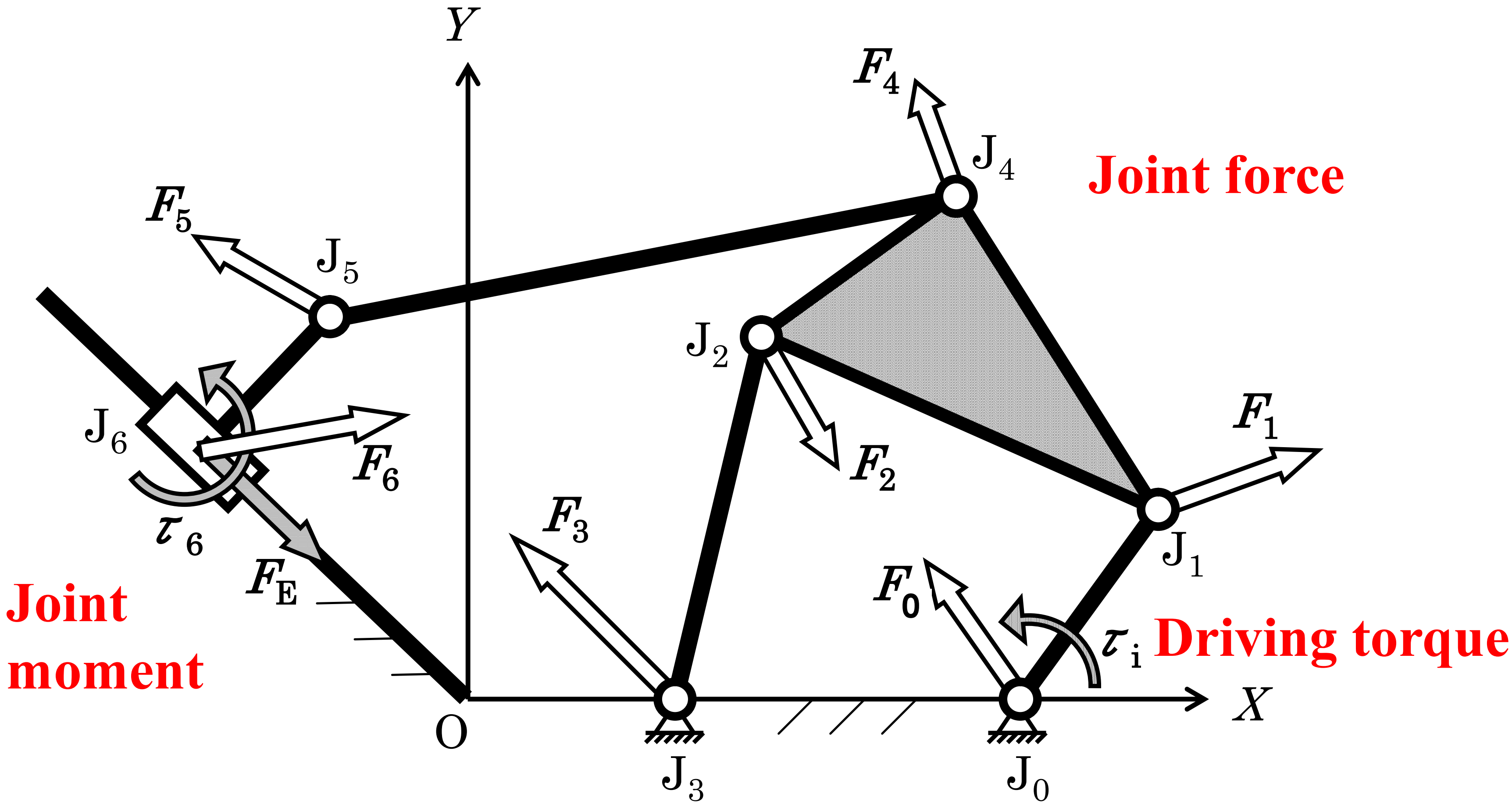
1. Dynamics Analysis of Planar Link Mechanisms

"Dynamics Analysis : Motion and Applying Force Analysis taking into account of inertial force"



a. Forces and moments applying on a mechanism



Forces and moments applying on pairs in a crank-input slider-output planar 6-bar link mechanism

A driving torque is transferred via joint force and moments and drives all of moving links.

a. Forces and moments applying on a mechanism

We can achieve the following two kinds of analyses:

(1) Forward dynamics analysis:

By specifying driving force/torque, motion of mechanism should then be calculated.

(2) Inverse dynamics analysis:

By specifying the desired motion of mechanism, the driving force/torque and joint forces and moments should then be calculated.

Question:

Which is easy to calculate?

— Forward dynamics or Inverse dynamics? —

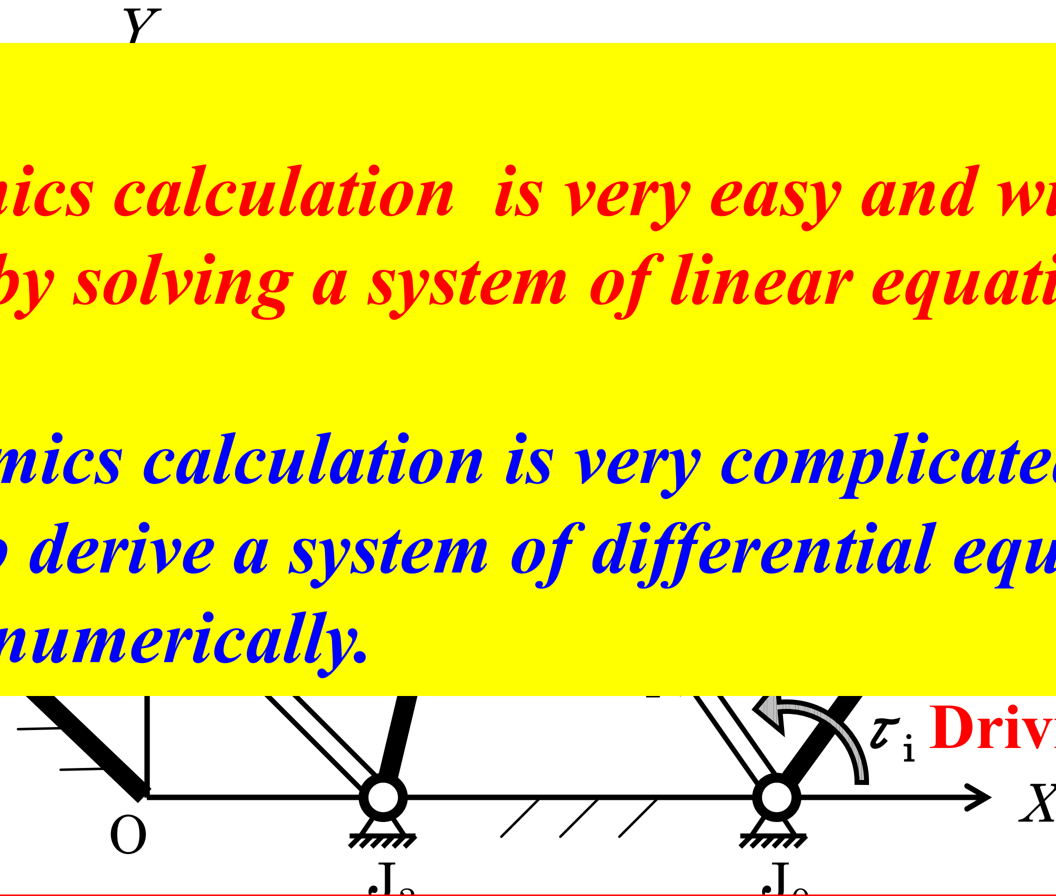
and drives all of moving links.

a. Forces and moments applying on a mechanism

Answer:

Inverse dynamics calculation is very easy and will be achieved only by solving a system of linear equations.

Forward dynamics calculation is very complicated and requires to derive a system of differential equations and to solve it numerically.



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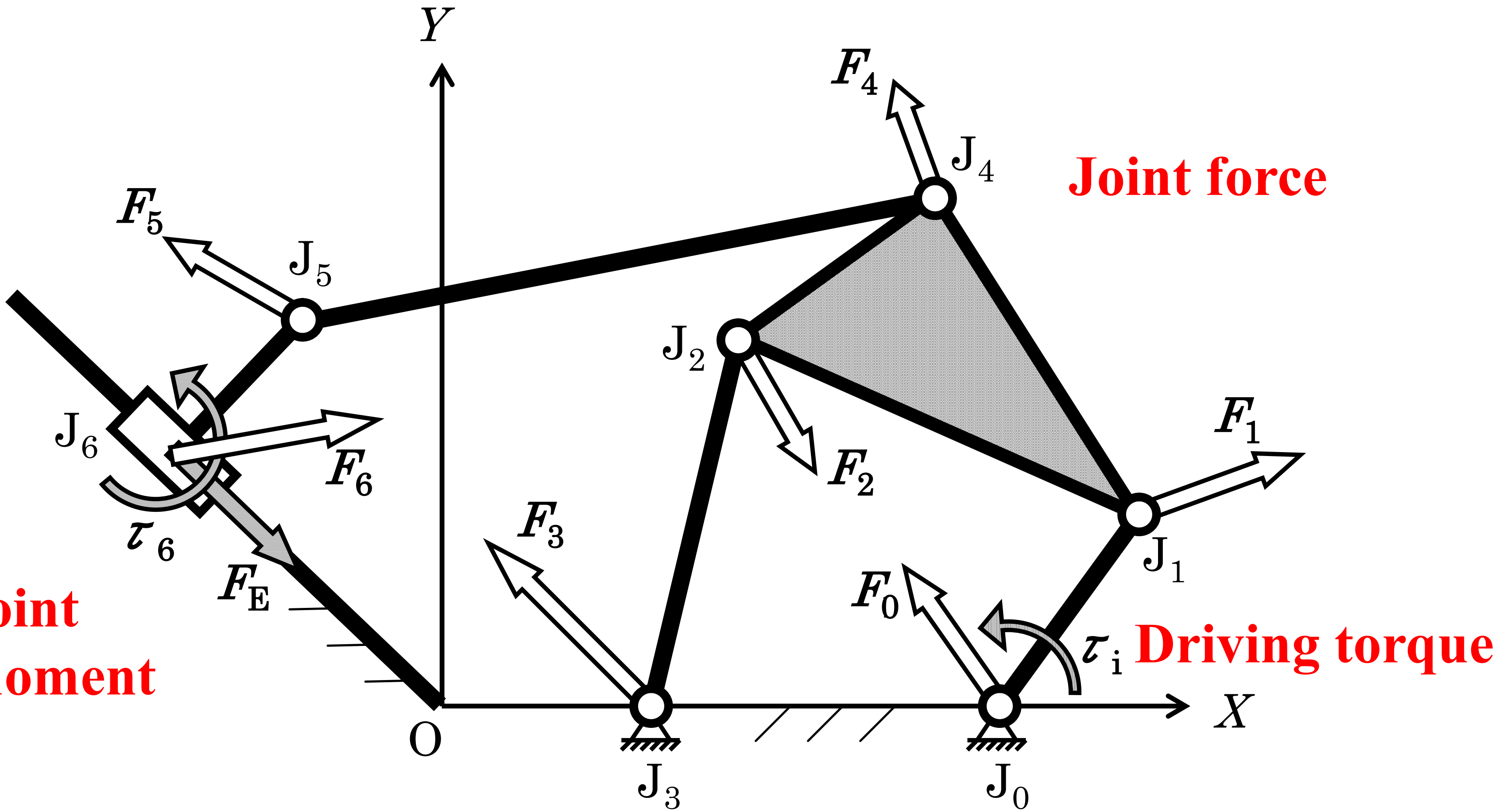
So, firstly let's try inverse dynamics calculation !

A

and drives all of moving links.

a. Forces and moments applying on Mechanism

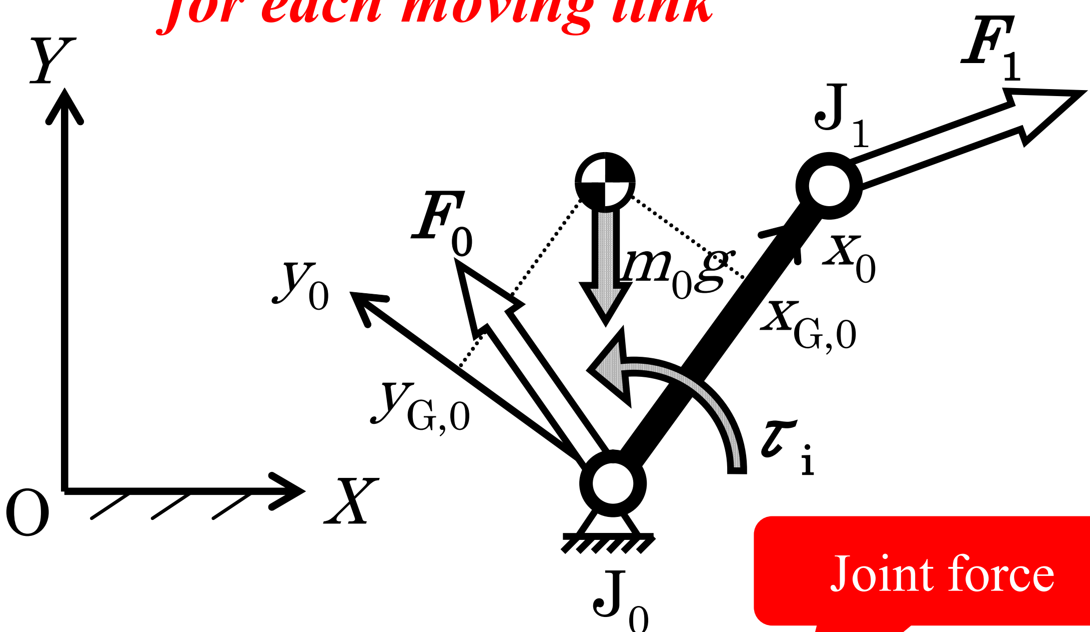


Forces and moments applied on pairs in a crank-input slider-output planar 6-bar link mechanism

Let's derive an equation of motion for each moving link by taking account of joint forces and moments.

b. Equations of motion of planar link mechanism

"Equation of translational motion of COG
Equation of angular motion about COG
for each moving link"



(1)Crank link

Joint force

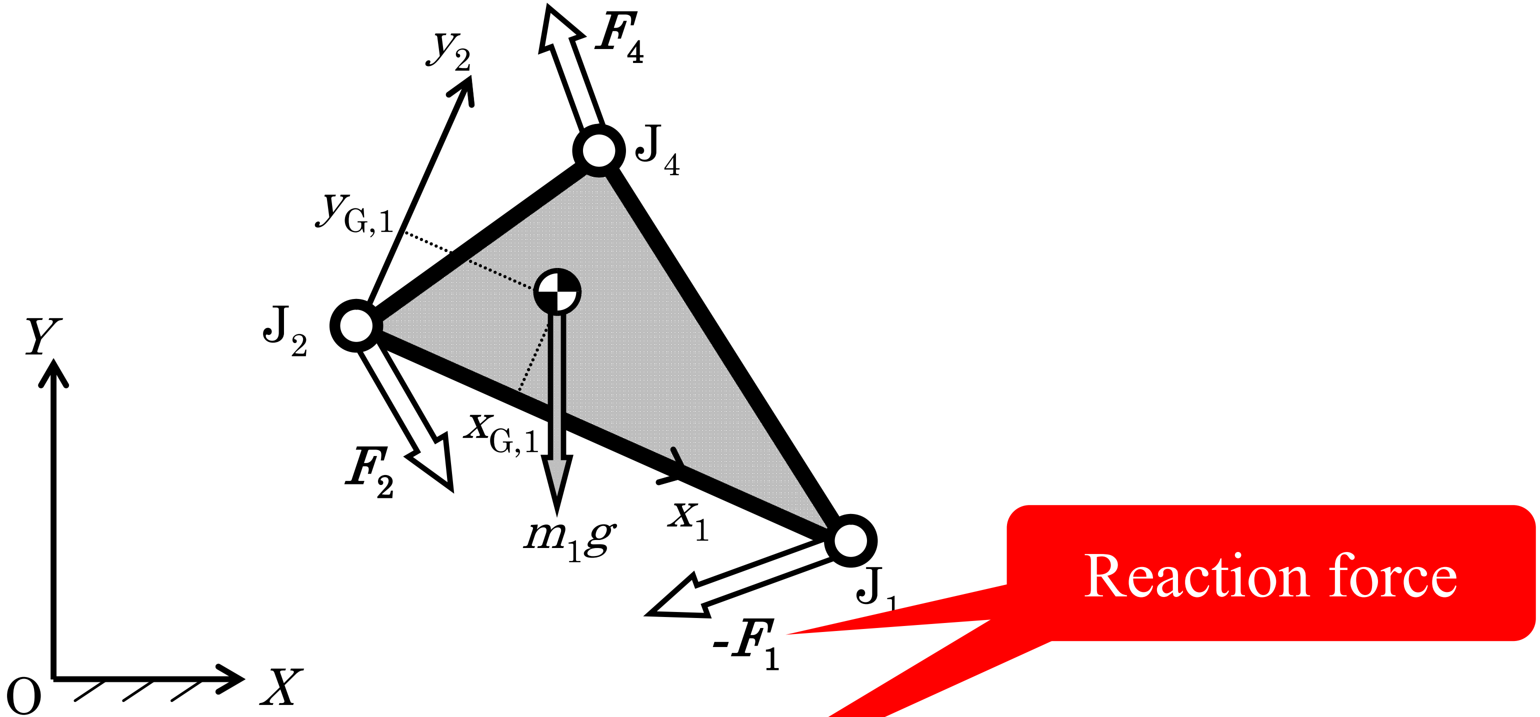
Gravitational force

Translational: $m_0\ddot{\boldsymbol{G}}_0 = \boldsymbol{F}_0 + \boldsymbol{F}_1 + m_0\boldsymbol{g}$

Angular: $I_0\ddot{\phi}_{G,0} = \tau_i + (\boldsymbol{J}_0 - \boldsymbol{G}_0)\times\boldsymbol{F}_0 + (\boldsymbol{J}_1 - \boldsymbol{G}_0)\times\boldsymbol{F}_1$

Driving torque

Moment due to joint force

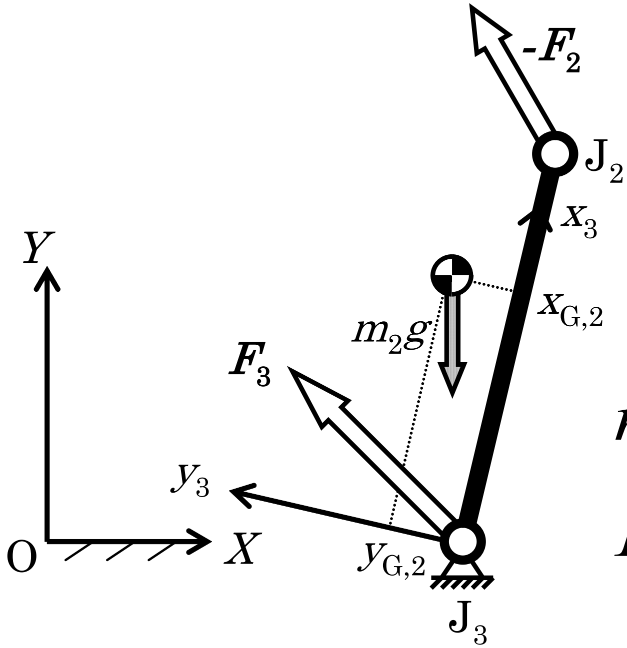


(2)Coupler link 1

Translational: $m_1\ddot{\boldsymbol{G}}_1 = -\boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_4 + m_1\boldsymbol{g}$

Angular:
$$I_1\ddot{\phi}_{G,1} = -(\boldsymbol{J}_1 - \boldsymbol{G}_1)\times\boldsymbol{F}_1 + (\boldsymbol{J}_2 - \boldsymbol{G}_1)\times\boldsymbol{F}_2 + (\boldsymbol{J}_4 - \boldsymbol{G}_1)\times\boldsymbol{F}_4$$

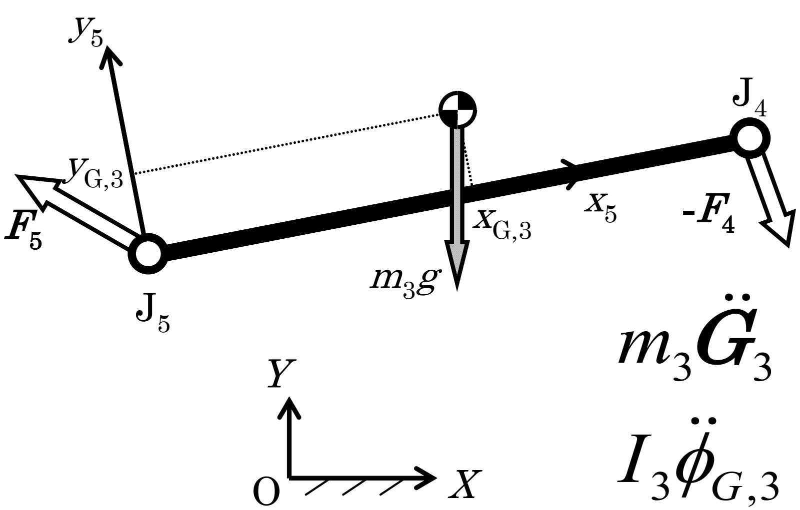
where $\boldsymbol{J}_i = \begin{Bmatrix} X_i \\ Y_i \end{Bmatrix}$ $\boldsymbol{G}_j = \begin{Bmatrix} X_{G,j} \\ Y_{G,j} \end{Bmatrix}$ $\boldsymbol{F}_k = \begin{Bmatrix} F_{k,X} \\ Y_{k,Y} \end{Bmatrix}$



$$m_2\ddot{\boldsymbol{G}}_2 = -\boldsymbol{F}_2 + \boldsymbol{F}_3 + m_2\boldsymbol{g}$$

$$I_2\ddot{\phi}_{G,2} = -(\boldsymbol{J}_2 - \boldsymbol{G}_2)\times\boldsymbol{F}_2 + (\boldsymbol{J}_3 - \boldsymbol{G}_2)\times\boldsymbol{F}_3$$

(3)Coupler link 2

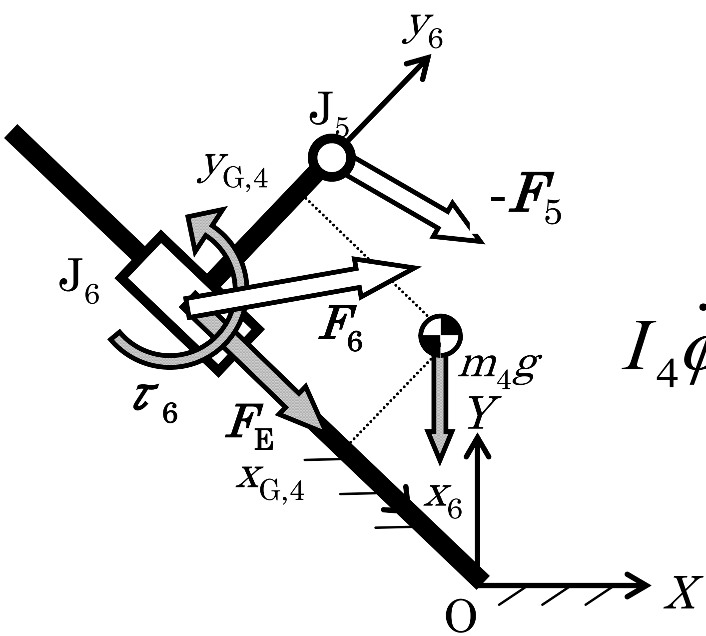


$$m_3\ddot{\boldsymbol{G}}_3 = -\boldsymbol{F}_4 + \boldsymbol{F}_5 + m_3\boldsymbol{g}$$

$$I_3\ddot{\phi}_{G,3} = -(\boldsymbol{J}_4 - \boldsymbol{G}_3)\times\boldsymbol{F}_4 + (\boldsymbol{J}_5 - \boldsymbol{G}_3)\times\boldsymbol{F}_5$$

(4)Coupler link 3

$$m_4\ddot{\boldsymbol{G}}_4 = -\boldsymbol{F}_5 + F_{6,y}\begin{Bmatrix}\sin\psi\\ -\cos\psi\end{Bmatrix} - F_E\begin{Bmatrix}\cos\psi\\ \sin\psi\end{Bmatrix} + m_4\boldsymbol{g}$$



$$I_4\ddot{\phi}_{G,4} = -(\boldsymbol{J}_5 - \boldsymbol{G}_4)\times\boldsymbol{F}_5 + (\boldsymbol{J}_6 - \boldsymbol{G}_4)\times\begin{Bmatrix}F_{6,y}\sin\psi - F_E\cos\psi\\ -F_{6,y}\cos\psi - F_E\sin\psi\end{Bmatrix} + \tau_6$$

(5)Output link

Joint force from guide rail

External force appled on output slider

You can derive equations 3 times more than moving links.

c. Inverse dynamics analysis with systematic kinematics analysis method

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Y_{G,0}-Y_0 & X_0-X_{G,0} & Y_{G,0}-Y_1 & X_1-X_{G,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Y_1-Y_{G,1} & X_{G,1}-X_1 & Y_{G,1}-Y_2 & X_2-Y_{G,1} & 0 & 0 & Y_{G,1}-Y_4 & X_4-X_{G,1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Y_2-Y_{G,2} & X_{G,2}-X_2 & Y_{G,2}-Y_3 & X_3-X_{G,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_4-Y_{G,3} & X_{G,3}-X_4 & Y_{G,3}-Y_5 & X_5-X_{G,3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \sin\psi & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\cos\psi & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_5-Y_{G,4} & X_{G,4}-X_5 & (X_{G,4}-X_6)\cos\psi+(Y_{G,4}-Y_6)\sin\psi & 1 & 0
\end{bmatrix}
\begin{Bmatrix}
F_{0,X} \\ F_{0,Y} \\ F_{1,X} \\ F_{1,Y} \\ F_{2,X} \\ F_{2,Y} \\ F_{3,X} \\ F_{3,Y} \\ F_{4,X} \\ F_{4,Y} \\ F_{5,X} \\ F_{5,Y} \\ F_{6,y} \\ \tau_6 \\ \tau_i
\end{Bmatrix}
$$

$$
= \begin{Bmatrix}
m_0\ddot{X}_{G,0} \\
m_0(\ddot{Y}_{G,0}+g) \\
I_0\ddot{\phi}_{G,0} \\
m_1\ddot{X}_{G,1} \\
m_1(\ddot{Y}_{G,1}+g) \\
I_1\ddot{\phi}_{G,1} \\
m_2\ddot{X}_{G,2} \\
m_2(\ddot{Y}_{G,2}+g) \\
I_2\ddot{\phi}_{G,2} \\
m_3\ddot{X}_{G,3} \\
m_3(\ddot{Y}_{G,3}+g) \\
I_3\ddot{\phi}_{G,3} \\
m_4\ddot{X}_{G,4}+F_E\cos\psi \\
m_4(\ddot{Y}_{G,4}+g)+F_E\sin\psi \\
I_4\ddot{\phi}_{G,4}+F_E[(X_6-X_{G,4})\sin\psi+(Y_{G,4}-Y_6)\cos\psi]
\end{Bmatrix}
$$

They can be calculated with the systematic kinematics analysis.

Position of COG, posture of link can also be calculated with programs:
coupler_point, link_angle

c. Inverse dynamics analysis with systematic kinematics analysis method

1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	$F_{0,X}$
0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	$F_{0,Y}$
$Y_{G,0}-Y_0$	$X_0-X_{G,0}$	$Y_{G,0}-Y_1$	$X_1-X_{G,0}$	0	0	0	0	0	0	0	0	0	0	1	$F_{1,X}$
0	0	-1										0	0	0	$F_{1,Y}$
0	0	0										0	0	0	$F_{2,X}$
0	0	$Y_1-Y_{G,1}$										0	0	0	$F_{2,Y}$
0	0	0										0	0	0	$F_{3,X}$
0	0	0											0	0	$F_{3,Y}$
0	0	0										0	0	0	$F_{4,X}$
0	0	0										0	0	0	$F_{4,Y}$
0	0	0										0	0	0	$F_{5,X}$
0	0	0										0	0	0	$F_{5,Y}$
0	0	0										$\sin\psi$	0	0	$F_{6,Y}$
0	0	0										$\cos\psi$	0	0	τ_6
0	0	0										$s\psi+(Y_{G,4}-Y_6)\sin\psi$	1	0	τ_i

Unknown variable vector of Joint force and moment and driving torque can easily be solved with Gauss-Jordan method

$$F=[\mathrm{A}]^{-1}b$$

$$
=\begin{Bmatrix}
m_0\ddot{X}_{G,0}\\
m_0(\ddot{Y}_{G,0}+g)\\
I_0\ddot{\phi}_{G,0}\\
m_1\ddot{X}_{G,1}\\
m_1(\ddot{Y}_{G,1}+g)\\
I_1\ddot{\phi}_{G,1}\\
m_2\ddot{X}_{G,2}\\
m_2(\ddot{Y}_{G,2}+g)\\
I_2\ddot{\phi}_{G,2}\\
m_3\ddot{X}_{G,3}\\
m_3(\ddot{Y}_{G,3}+g)\\
I_3\ddot{\phi}_{G,3}\\
m_4\ddot{X}_{G,4}+F_E\cos\psi\\
m_4(\ddot{Y}_{G,4}+g)+F_E\sin\psi\\
I_4\ddot{\phi}_{G,4}+F_E[(X_6-X_{G,4})\sin\psi+(Y_{G,4}-Y_6)\cos\psi]
\end{Bmatrix}
$$

A system of linear equations on joint force and moment and driving torque:

$$[\mathrm{A}]F=b$$

d. Example of analysis

Example of program:

Pidp6barpo.cpp----Main program

```
//------ Main loop ------
        for( ith=0; ith<=360; ith++ ){
                th = (double)ith;
                theta.the = th*DR;

                ichk = MP6BARPO( L, gsi, eta, PG, psi, theta, J, &s );

                if( ichk != SUCCESS ) {
                        printf( "¥n ++ Error in MP6BARPO! ier= %d ¥n", ichk );
                        exit ( 0 );
                }

                GRVP6BARPO( gsig, etag, J, CG, phig );

                ichk = IDP6BARPO( M, I, J, psi, CG, phig, fe, fjx, fjy, &tauj6, &taui );
                if( ichk != SUCCESS ) {
                        printf( "¥n ++ Error in IDP6BARPO ! ier = %d ¥n", ichk );
                        exit ( 0 );
                }
                fprintf(fpDAT, "%7.1f %13.5f %13.5f %13.5f %13.5f %13.5f %13.5f",
              " %13.5f %13.5f %13.5f %13.5f %13.5f %13.5f",
              " %13.5f %13.5f %13.5f %13.5f %13.5f %13.5f¥n",
                        th, s.v,  J[4].P.x, J[4].P.y,
                                fjx[0], fjy[0], fjx[1], fjy[1], fjx[2], fjy[2],
               fjx[3], fjy[3], fjx[4], fjy[4], fjx[5], fjy[5],

        }
```

Kinematics calculation (MP6BARPO)

Motion of COG and link posture (GRVP6BARPO)

Inverse dunamics calculation (IDP6BARPO)

mp6barpo.cpp----kinematics calculation

```
int MP6BARPO( double L[], double gsi[], double eta[], POINT PG,
              ANGLE psi, ANGLE theta, POINT J[], VARIABLE *s )
{
        ANGLE dumma;
        POINT dummp;
        int ichk;
        int minv[2];

        dumma.the = dumma.dthe = dumma.ddthe = 0.0;
        dummp.P.x = dummp.DP.x = dummp.DDP.x = 0.0;
        dummp.P.y = dummp.DP.y = dummp.DDP.y = 0.0;
//------ Displacement , velocity and acceleration of pairs
        minv[0] = 1;
        minv[1] = 1;

        crank_input( J[0], L[0], theta, dumma, &J[1] );

        ichk = RRR_links( J[3], J[1], L[2], L[1], minv[0],  &J[2] );
        if( ichk !=SUCCESS ) return ( 991 );

        coupler_point( J[2], J[1], gsi[0], eta[0],  &J[4], &dumma );

        ichk = PRR_links( J[4], L[3], PG, psi, gsi[1], eta[1], minv[1], s, &J[6], &J[5] );
        if( ichk !=SUCCESS ) return ( 992 );

        return ( SUCCESS );
}
```

Systematic kinematics subprograms

grvp6barpo.cpp----position of COG and link posture

```
void GRVP6BARPO( double gsig[], double etag[], POINT J[], POINT CG[], ANGLE phig[] )
{

//-- Displacement, velocity, acceleration of COG and link posture, angular velocity, angular acceleration--

      coupler_point( J[0], J[1], gsig[0], etag[0], &CG[0] , &phig[0] );
      coupler_point( J[2], J[1], gsig[1], etag[1], &CG[1] , &phig[1] );
      coupler_point( J[3], J[2], gsig[2], etag[2], &CG[2] , &phig[2] );
      coupler_point( J[5], J[4], gsig[3], etag[3], &CG[3] , &phig[3] );
      coupler_point( J[6], J[5], gsig[4], etag[4], &CG[4] , &phig[4] );

}
```

Kinematics calculation can be used

id6barpo.cpp----Inverse dynamics calculation

```
int IDP6BARPO( double M[], double I[],
   POINT J[], ANGLE psi, POINT CG[], ANGLE phig[], double fe,
   double fjx[], double fjy[], double *tauj6, double *taui )
{
    double  wa[NV][NV], wb[NV], wk[NV];
    int     ipw[NV];
    int     ichk,  i, j;
    double  cp, sp,  double  Grav;
    Grav = -9.805;

    for(j=0;j<NV;j++)  {
       for(i=0;i<NV;i++) {
            wa[i][j] = 0.0;
       }
     }
//----- Coefficient matrix -----
    wa[ 0][ 0] =  1.0;
    wa[ 0][ 2] =  1.0;
    wa[ 1][ 1] =  1.0;
    wa[ 1][ 3] =  1.0;
    wa[ 2][ 0] =  CG[0].P.y - J[0].P.y;
    wa[ 2][ 1] =  J[0].P.x - CG[0].P.x;
    wa[ 2][ 2] =  CG[0].P.y - J[1].P.y;
    wa[ 2][ 3] =  J[1].P.x - CG[0].P.x;
    wa[ 2][14] =  1.0;
```

Setting coefficient matrix

id6barpo.cpp----Inverse dynamics calculation

```
:
:
//----- Constatnt vector-----
   wb[ 0] = M[0]*CG[0].DDP.x;
   wb[ 1] = M[0]*( CG[0].DDP.y - Grav );
   wb[ 2] = I[0]*phig[0].ddthe;
    :
    :
//----- Gauss-Jordan method -----
   ichk = GAUSS( wa, wb, ipw, wk );
   if( ichk != 1 )  return ( ichk );

//----- Joint forces and driving torque -----
   fjx[0] = wb[ 0];
   fjy[0] = wb[ 1];
   fjx[1] = wb[ 2];
    :
    :
   *tauj6 = wb[13];
   *taui =  wb[14];
    return ( SUCCESS );
}
```

Setting constant vector

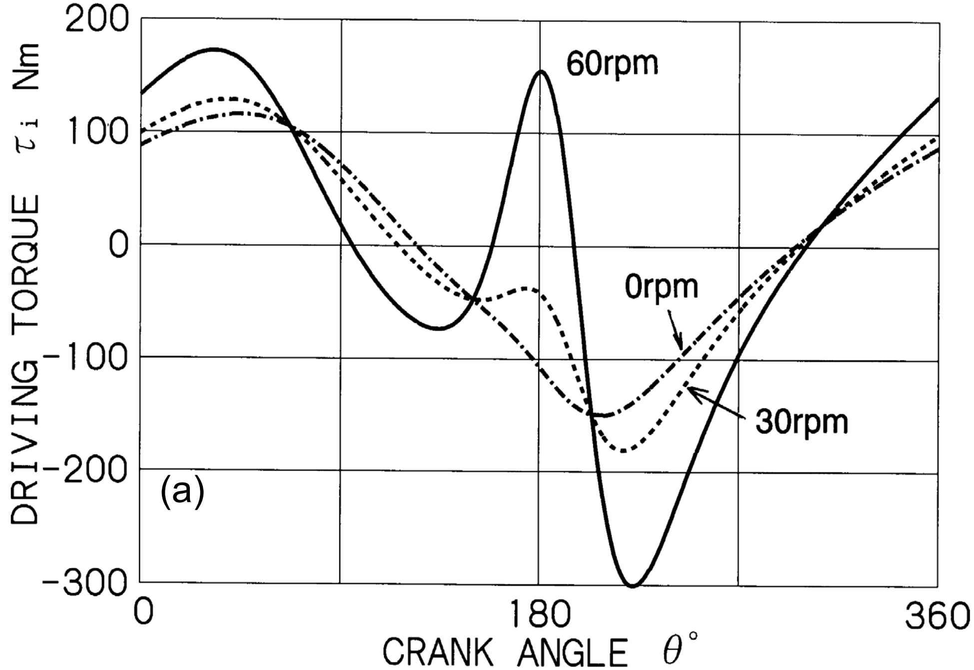
Solving a system of linear equations with Gauss-Jordan method

Example of analysis:

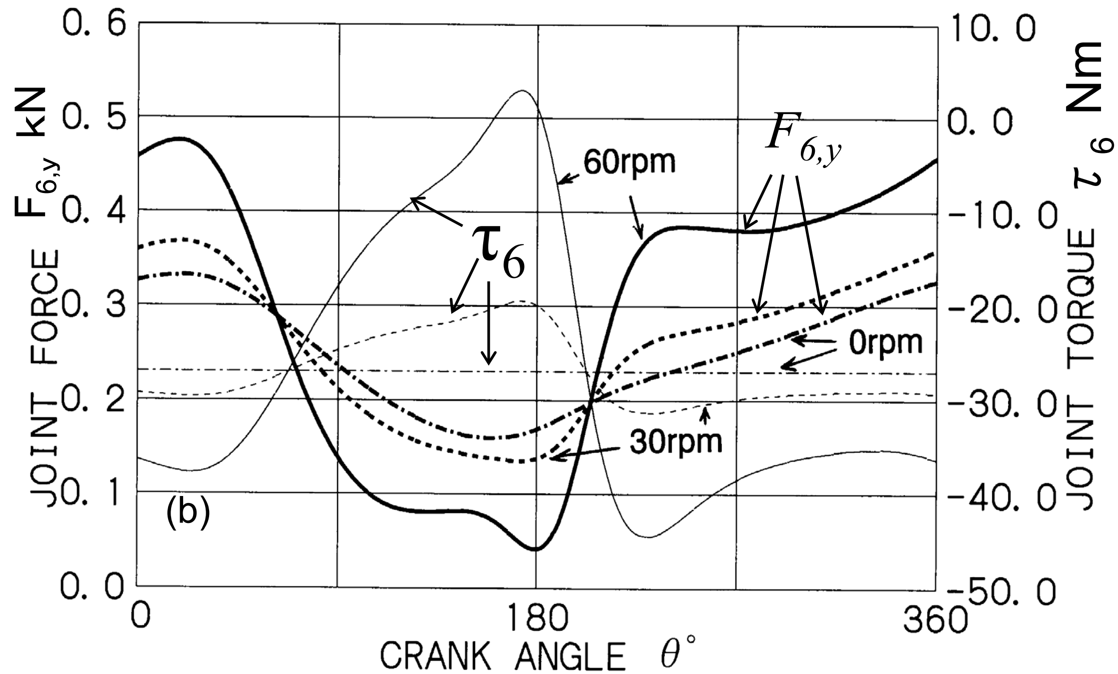
Kinematic and mass-inertia parameters of the 6-bar mechanism

j	J_1	J_2	m_j kg	$\xi_{G,j}$ m	$\eta_{G,j}$ m	I_j kgm^2
0	0	1	4.72	0.15	0.00	0.0354
1	2	1	15.72	0.40	0.00	0.8000
2	3	2	7.86	0.25	0.05	0.1638
3	5	4	12.58	0.40	0.00	0.6709
4	6	5	5.00	0.00	0.00	0.0200

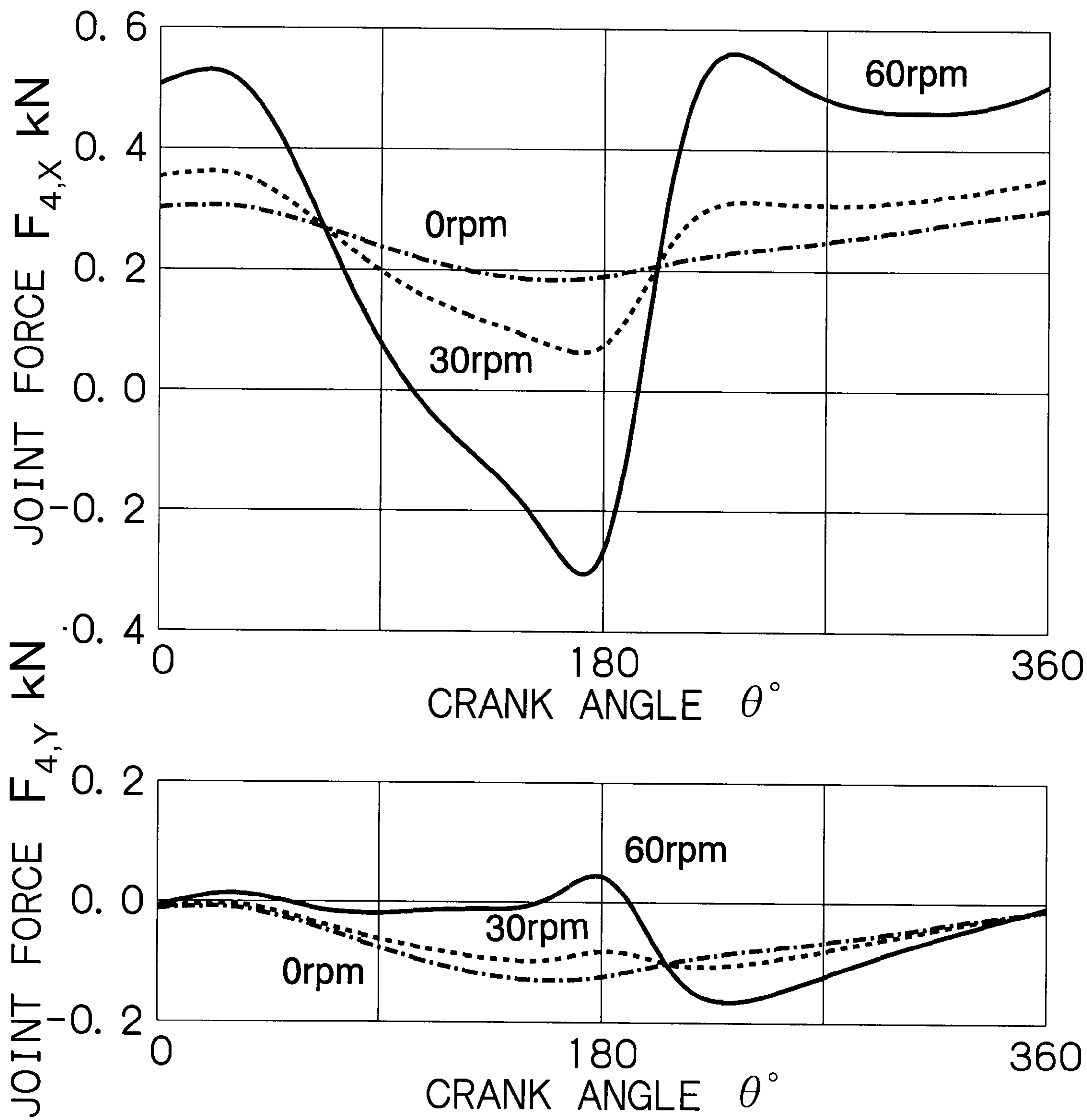
Pair number to specify a moving coordinate system to represent position of COG (i)Driving torque



(ii)Force and moment applying on output slider



(iii)Joint force applying on pair J_4



Results of analysis of inverse kinematics

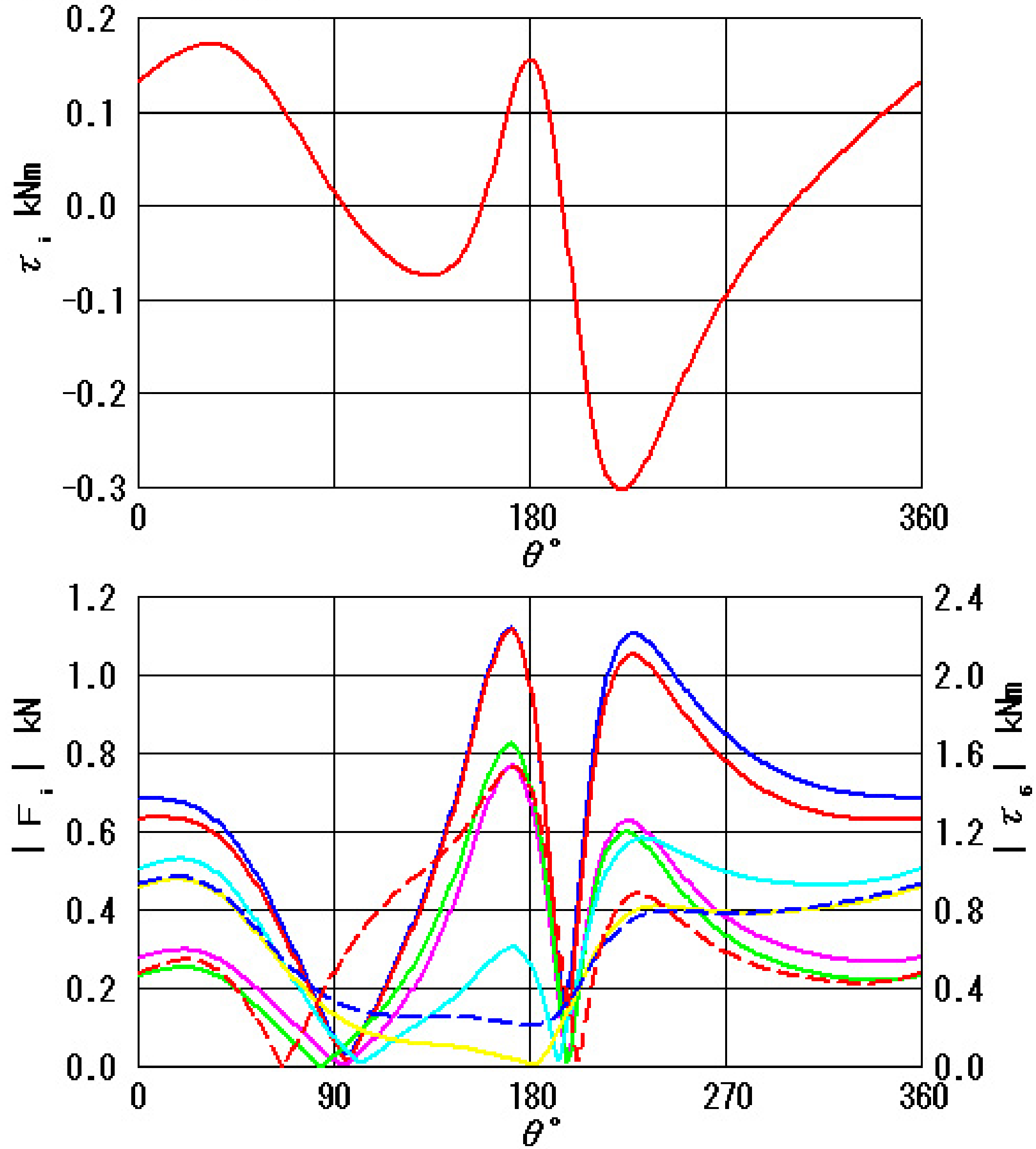
Joint forces and moments can easily calculated!

by nob

直進対偶出力平面６節リンク機構の逆動力学解析 by RPIDP6BARPO.BAS

$\dot{\theta}$ = 60 RPM　　F_x = 100N

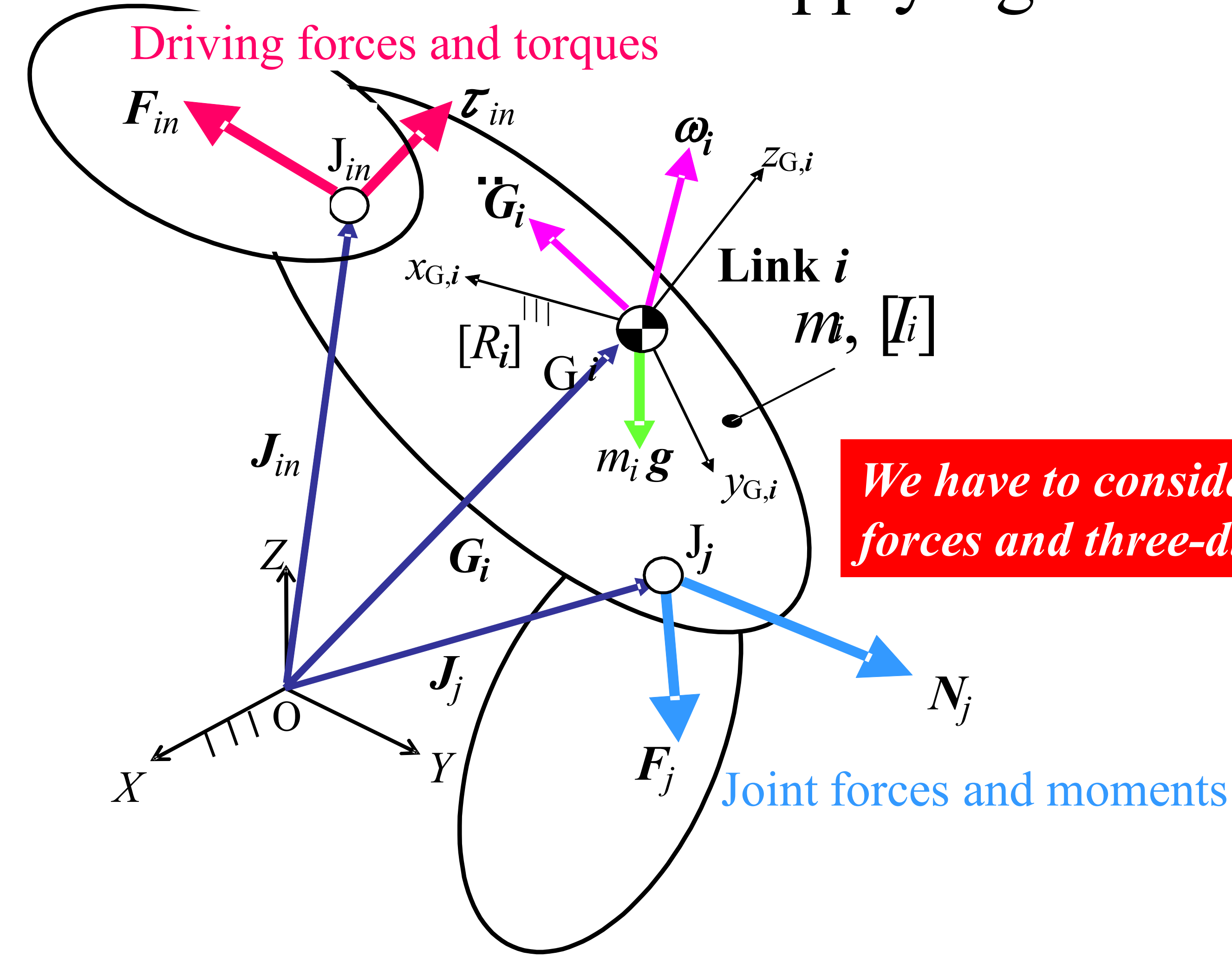
駆動トルクと対偶作用力



Summarized driving torque and Joint forces and moments

2. Dynamics Analysis of Spatial Link Mechanisms

a. Forces and moments applying on a mechanism



We have to consider three-dimensional forces and three-dimensional moments!

Forces and moments applying on spatial link mechanisms

b. Equations of motion of spatial link mechanism

"Equation of motion of spatially moving link"

$$m_j\ddot{\boldsymbol{G}}_j = \sum_k \boldsymbol{F}_k + \sum \boldsymbol{F}_{in} + m_j\boldsymbol{g}$$

Three-dimensional translational motion of COG

$$[T_j]^T([I_j]\dot{\boldsymbol{\omega}}_j + \boldsymbol{\omega}_j \times [I_j]\boldsymbol{\omega}_j) = \sum_k \boldsymbol{\tau}_k + \sum_k (\boldsymbol{J}_k - \boldsymbol{G}_j)\times \boldsymbol{F}_k + \sum \boldsymbol{\tau}_{in} + \sum (\boldsymbol{J}_{in} - \boldsymbol{G}_j)\times \boldsymbol{F}_{in}$$

Coordinate transformation from moving coordinate system

Three-dimensional angular motion about COG

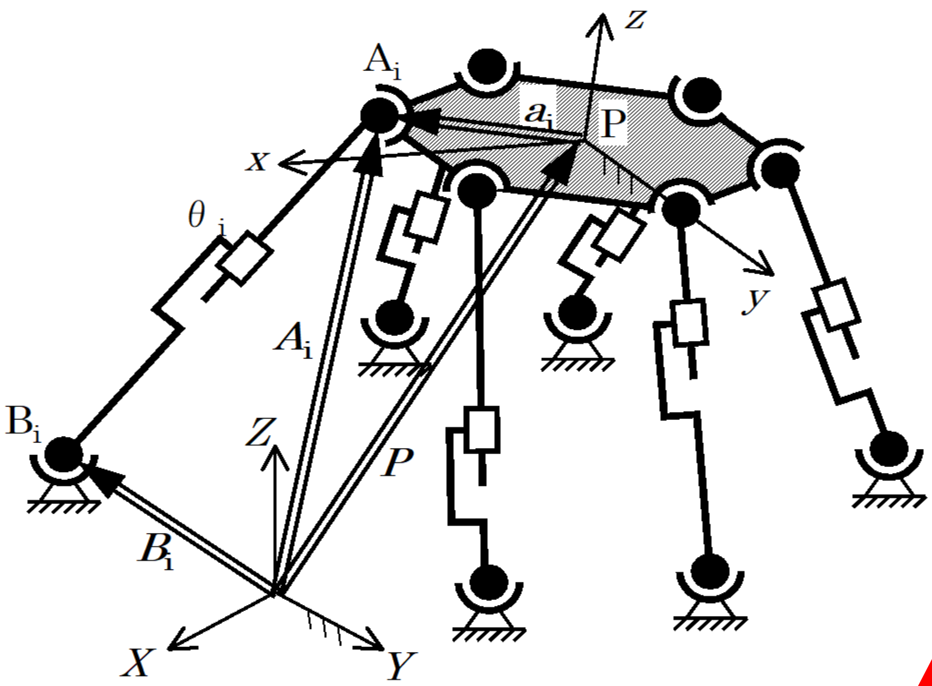
where angular velocity and acceleration are described on moving coordinate system as

$$\omega_j = \begin{Bmatrix} \dot{\alpha}_j \cos\beta_j \cos\gamma_j + \dot{\beta}_j \sin\gamma_j \\ -\dot{\alpha}_j \cos\beta_j \sin\gamma_j + \dot{\beta}_j \cos\gamma_j \\ \dot{\alpha}_j \sin\beta_j + \dot{\gamma}_j \end{Bmatrix}$$

Note that they are given as the function of derivatives of roll-, pitch- and yaw-angles of moving link

$$\dot{\omega}_j = \begin{Bmatrix} \ddot{\alpha}_j \cos\beta_j \cos\gamma_j + \ddot{\beta}_j \sin\gamma_j - \dot{\alpha}_j\dot{\beta}_j \sin\beta_j \cos\gamma_j + \dot{\beta}_j\dot{\gamma}_j \cos\gamma_j - \dot{\gamma}_j\dot{\alpha}_j \cos\beta_j \sin\gamma_j \\ -\ddot{\alpha}_j \cos\beta_j \sin\gamma_j + \ddot{\beta}_j \cos\gamma_j + \dot{\alpha}_j\dot{\beta}_j \sin\beta_j \sin\gamma_j - \dot{\beta}_j\dot{\gamma}_j \sin\gamma_j - \dot{\gamma}_j\dot{\alpha}_j \cos\beta_j \cos\gamma_j \\ \ddot{\alpha}_j \sin\beta_j + \ddot{\gamma}_j + \dot{\alpha}_j\dot{\beta}_j \cos\beta_j \end{Bmatrix}$$

c. Inverse dynamics analysis with systematic kinematics analysis method



$$\left[A_{78\times78}\right]\begin{bmatrix} F_{A,0,X} \\ F_{A,0,Y} \\ F_{A,0,Z} \\ \vdots \\ \vdots \\ F_{in3} \\ F_{in4} \\ F_{in5} \end{bmatrix}_{78\times1} = \left[b_{78\times1}\right]$$

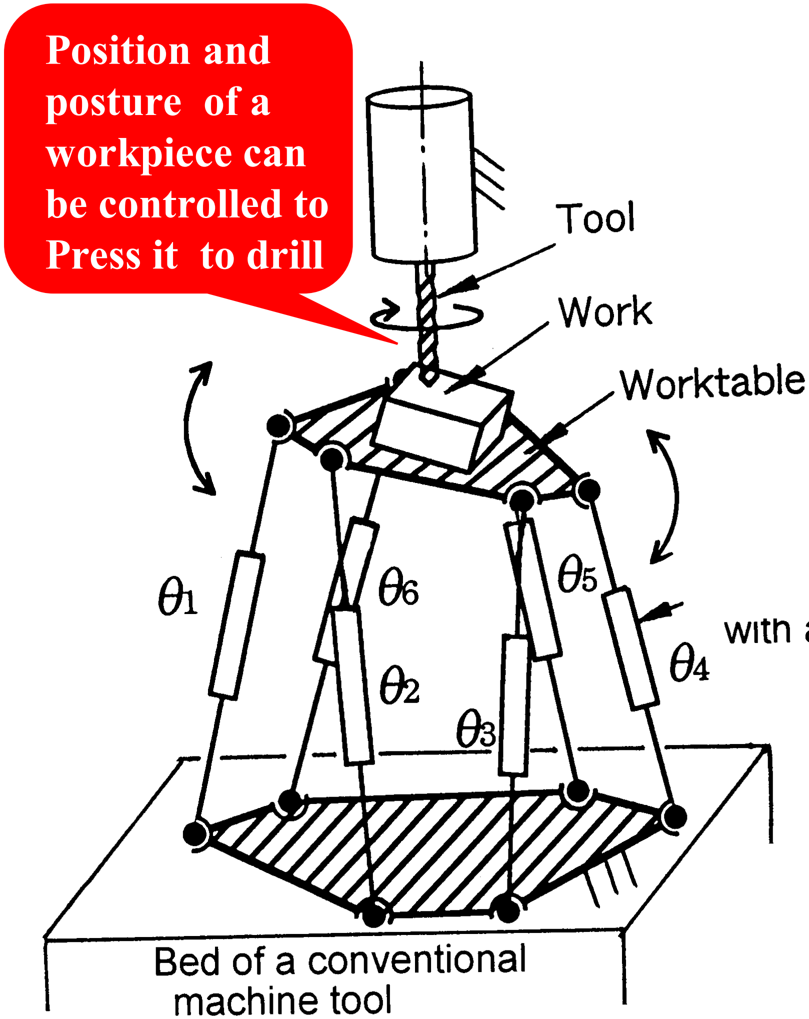
They can be calculated with the systematic kinematics analysis method

A system of 78 linear equations for Stewart platform manipulator with 6 DOF

Joint forces applying on spherical pairs and driving forces of linearactuator can be calculated as

$$\begin{bmatrix} F_{A,0,X} \\ F_{A,0,Y} \\ F_{A,0,Z} \\ \vdots \\ \vdots \\ F_{in3} \\ F_{in4} \\ F_{in5} \end{bmatrix}_{78\times 1} = \left[A_{78\times 78}\right]^{-1}\left[b_{78\times 1}\right]$$

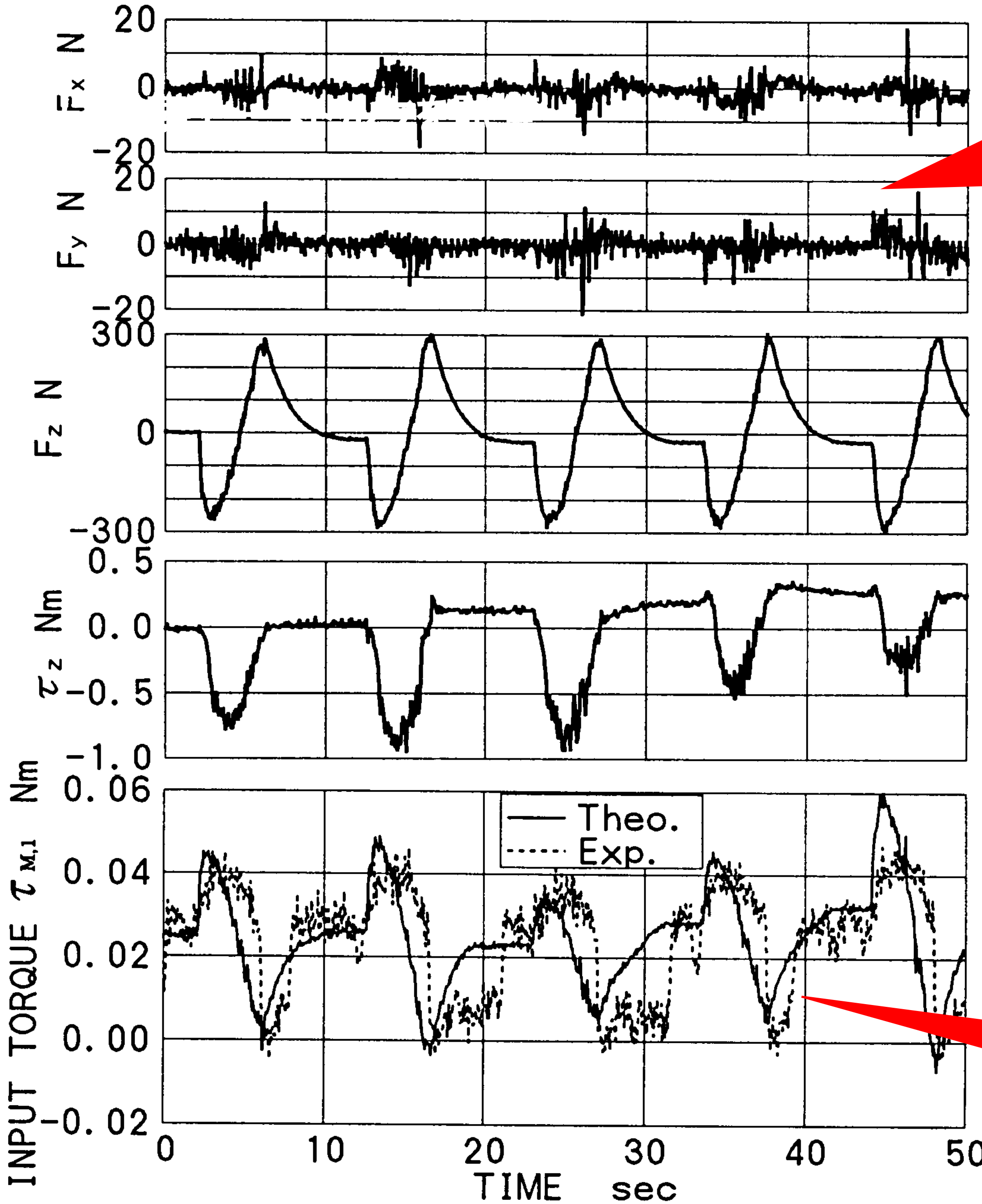
Calculation result and experimental validation

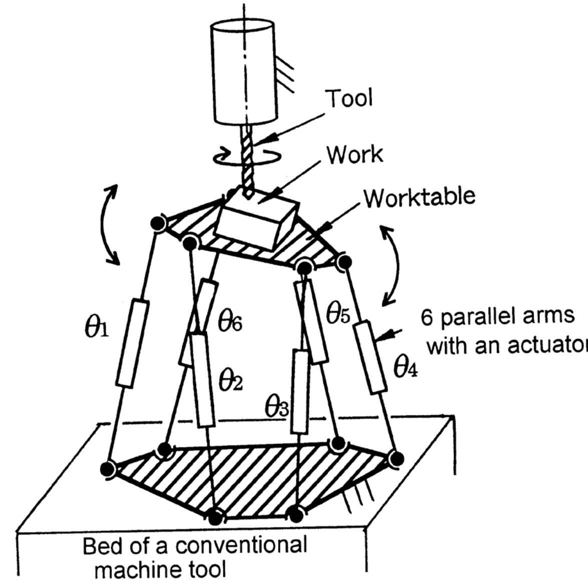


Active worktable of spatial parallel manipulator for NC machine tool

Photograph of machining and workpiece

External force and moments to be input in inverse dynamics analysis





Analyzed value agreed well with experimental value

Machining forces and actuator torques

Inverse dynamics analysis can easily be achieved by using the systematic kinematics analysis method!

4. Forward Dynamics Analysis of Planar/Spatial Link Mechanisms

Next let's consider forward dynamics analysis to calculate motion of link mechanisms after specifying driving forces/torques.

Let's consider equation of motion as

$$[\mathrm{A}]\boldsymbol{F}=\boldsymbol{b}$$

An example for planar 6-bar link mechanism:

Motion of mechanism to be calculated in forward dynamics

$$
\underbrace{\begin{bmatrix}
1 & 0 & 1 & \cdots & & & & & & & & & & 0 & 0 \\
0 & 1 & 0 & \cdots & & & & & & & & & & 0 & 0 \\
Y_{G,0}-Y_0 & X_0-X_{G,0} & Y_{G,0}-Y_1 & X\cdots & & & & & & & & & & 0 & 1 \\
0 & 0 & -1 & \cdots & & & & & & & & & & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Y_1-Y_{G,1} & X_{G,1}-X_1 & Y_{G,1}-Y_2 & X_2-Y_{G,1} & 0 & 0 & Y_{G,1}-Y_4 & X_4-X_{G,1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Y_2-Y_{G,2} & X_{G,2}-X_2 & Y_{G,2}-Y_3 & X_3-X_{G,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_4-Y_{G,3} & X_{G,3}-X_4 & Y_{G,3}-Y_5 & X_5-X_{G,3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \sin\psi & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\cos\psi & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_5-Y_{G,4} & X_{G,4}-X_5 & (X_{G,4}-X_6)\cos\psi+(Y_{G,4}-Y_6)\sin\psi & 1 & 0
\end{bmatrix}}_{[A]}
\underbrace{\begin{Bmatrix}
F_{0,X} \\ F_{0,Y} \\ F_{1,X} \\ F_{1,Y} \\ F_{2,X} \\ F_{2,Y} \\ F_{3,X} \\ F_{3,Y} \\ F_{4,X} \\ F_{4,Y} \\ F_{5,X} \\ F_{5,Y} \\ F_{6,Y} \\ \tau_6 \\ \tau_i
\end{Bmatrix}}_{F}
$$

$$
= \underbrace{\begin{Bmatrix}
m_0\ddot{X}_{G,0} \\
m_0(\ddot{Y}_{G,0}+g) \\
I_0\ddot{\phi}_{G,0} \\
m_1\ddot{X}_{G,1} \\
m_1(\ddot{Y}_{G,1}+g) \\
I_1\ddot{\phi}_{G,1} \\
m_2\ddot{X}_{G,2} \\
m_2(\ddot{Y}_{G,2}+g) \\
I_2\ddot{\phi}_{G,2} \\
m_3\ddot{X}_{G,3} \\
m_3(\ddot{Y}_{G,3}+g) \\
I_3\ddot{\phi}_{G,3} \\
m_4\ddot{X}_{G,4}+F_E\cos\psi \\
m_4(\ddot{Y}_{G,4}+g)+F_E\sin\psi \\
I_4\ddot{\phi}_{G,4}+F_E[(X_6-X_{G,4})\sin\psi+(Y_{G,4}-Y_6)\cos\psi]
\end{Bmatrix}}_{b}
$$

Input parameters for forward dynamics analysis

An example for planar 6-bar link mechanism:

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Y_{G,0}-Y_0 & X_0-X_{G,0} & Y_{G,0}-Y_1 & X_1-X_{G,0} & & & & & & & & & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & & & & & & & & & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & & & & & & & & & 0 & 0 & 0 \\
0 & 0 & Y_1-Y_{G,1} & X_{G,1}-X_1 & & & & & & & & & & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & & & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Y_2-Y_{G,2} & X_{G,2}-X_2 & Y_{G,2}-Y_3 & X_3-X_{G,2} & & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_4-Y_{G,3} & X_{G,3}-X_4 & Y_{G,3}-Y_5 & X_5-X_{G,3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \sin\psi & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\cos\psi & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_5-Y_{G,4} & X_{G,4}-X_5 & (X_{G,4}-X_6)\cos\psi+(Y_{G,4}-Y_6)\sin\psi & 1 & 0
\end{bmatrix}
\begin{Bmatrix}
F_{0,X} \\ F_{0,Y} \\ F_{1,X} \\ F_{1,Y} \\ F_{2,X} \\ F_{2,Y} \\ F_{3,X} \\ F_{3,Y} \\ F_{4,X} \\ F_{4,Y} \\ F_{5,X} \\ F_{5,Y} \\ F_{6,Y} \\ \tau_6 \\ \tau_i
\end{Bmatrix}
$$

$$
=\begin{Bmatrix}
m_0\ddot{X}_{G,0} \\
m_0(\ddot{Y}_{G,0}+g) \\
I_0\ddot{\phi}_{G,0} \\
m_1\ddot{X}_{G,1} \\
m_1(\ddot{Y}_{G,1}+g) \\
I_1\ddot{\phi}_{G,1} \\
m_2\ddot{X}_{G,2} \\
m_2(\ddot{Y}_{G,2}+g) \\
I_2\ddot{\phi}_{G,2} \\
m_3\ddot{X}_{G,3} \\
m_3(\ddot{Y}_{G,3}+g) \\
I_3\ddot{\phi}_{G,3} \\
m_4\ddot{X}_{G,4}+F_E\cos\psi \\
m_4(\ddot{Y}_{G,4}+g)+F_E\sin\psi \\
I_4\ddot{\phi}_{G,4}+F_E[(X_6-X_{G,4})\sin\psi+(Y_{G,4}-Y_6)\cos\psi]
\end{Bmatrix}
$$

[A]

F

b

Joint forces and moments should be eliminated!

Motion of mechanism should be rewritten as function of input crank angle

An example for planar 6-bar link mechanism:

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
Y_{G,0}-Y_0 & X_0-X_{G,0} & Y_{G,0}-Y_1 & X_1-X_{G,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & Y_1-Y_{G,1} & X_{G,1}-X_1 & Y_{G,1}-Y_2 & X_2-Y_{G,1} & 0 & 0 & Y_{G,1}-Y_4 & X_4-X_{G,1} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & Y_2-Y_{G,2} & X_{G,2}-X_2 & Y_{G,2}-Y_3 & X_3-X_{G,2} & \cdots & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_4-Y_{G,3} & X_{G,3}-X_4 & Y_{G,3}-Y_5 & X_5-X_{G,3} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \sin\psi & 0 & 0\\
0 & \cdots & & & & & & & & & & & \cdots\psi & 0 & 0\\
0 & \cdots & & & & & & & & & & & \cdots(Y_{G,4}-Y_6)\sin\psi & 1 & 0
\end{bmatrix}
\begin{Bmatrix}
F_{0,X}\\ F_{0,Y}\\ F_{1,X}\\ F_{1,Y}\\ F_{2,X}\\ F_{2,Y}\\ F_{3,\cdots}\\ F_{\cdots}\\ F_{4,X}\\ F_{4,Y}\\ F_{5,X}\\ F_{5,Y}\\ F_{6,Y}\\ \tau_6\\ \tau_i
\end{Bmatrix}
$$

$$
= \begin{Bmatrix}
\cdots\\
\cdots\ddot{\phi}_{G,1}\\
\cdots\ddot{X}_{G,2}\\
m_2(\ddot{Y}_{G,2}+g)\\
I_2\ddot{\phi}_{G,2}\\
m_3\ddot{X}_{G,3}\\
m_3(\ddot{Y}_{G,3}+g)\\
I_3\ddot{\phi}_{G,3}\\
m_4\ddot{X}_{G,4}+F_E\cos\psi\\
m_4(\ddot{Y}_{G,4}+g)+F_E\sin\psi\\
I_4\ddot{\phi}_{G,4}+F_E[(X_6-X_{G,4})\sin\psi+(Y_{G,4}-Y_6)\cos\psi]
\end{Bmatrix}
$$

[A] **F** **b**

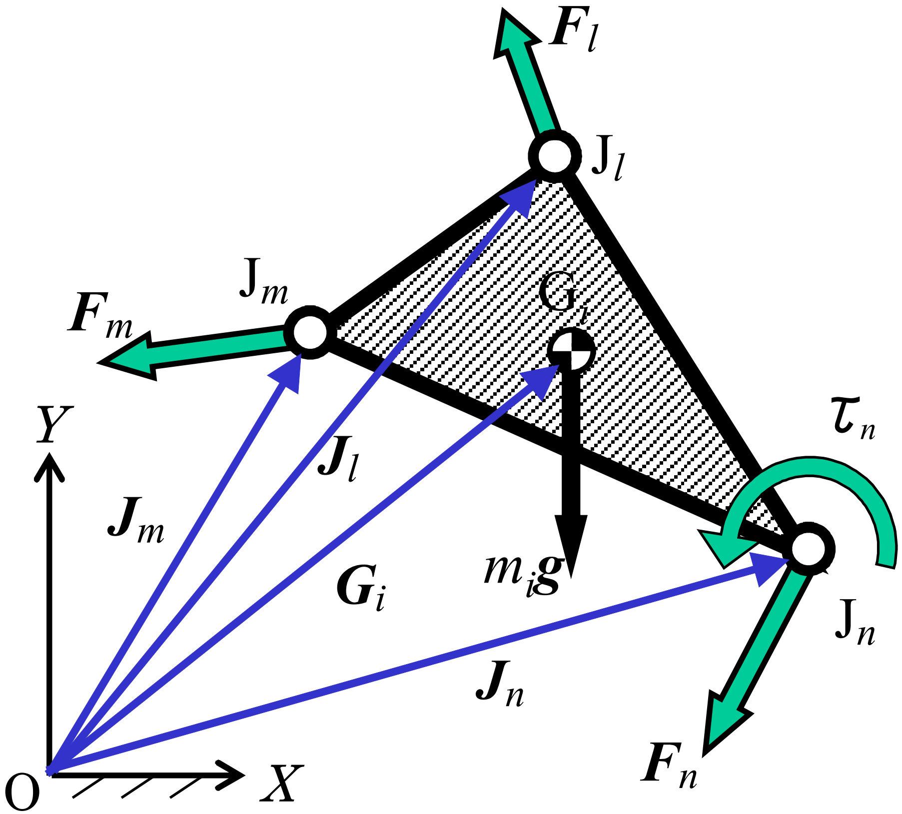
Differential equation on crank angle:

$f(\theta,\dot{\theta},\ddot{\theta})=0$ can be derived.

However this procedure is very complicated!

For general planar closed-loop link mechanisms

Equations of Motion



A link in planar motion

Equations of motion for each link:

$$m_i \ddot{\boldsymbol{G}}_i = \boldsymbol{F}_m + \boldsymbol{F}_n + \boldsymbol{F}_l + m_i \boldsymbol{g}$$

$$I_i \ddot{\varphi}_i = (\boldsymbol{J}_m - \boldsymbol{G}_i) \times \boldsymbol{F}_m + (\boldsymbol{J}_n - \boldsymbol{G}_i) \times \boldsymbol{F} + (\boldsymbol{J}_l - \boldsymbol{G}_i) \times \boldsymbol{F}_l + \tau_n$$

$\boldsymbol{G}_i$:	Position vector of COG
φ_i :	Posture angle
$\boldsymbol{J}_l, \boldsymbol{J}_m, \boldsymbol{J}_n$:	Position vector of joint
$\boldsymbol{F}_l, \boldsymbol{F}_m, \boldsymbol{F}_n$:	Joint forces
τ_n :	Driving torque

For all moving links

Joint forces **Driving Torques**

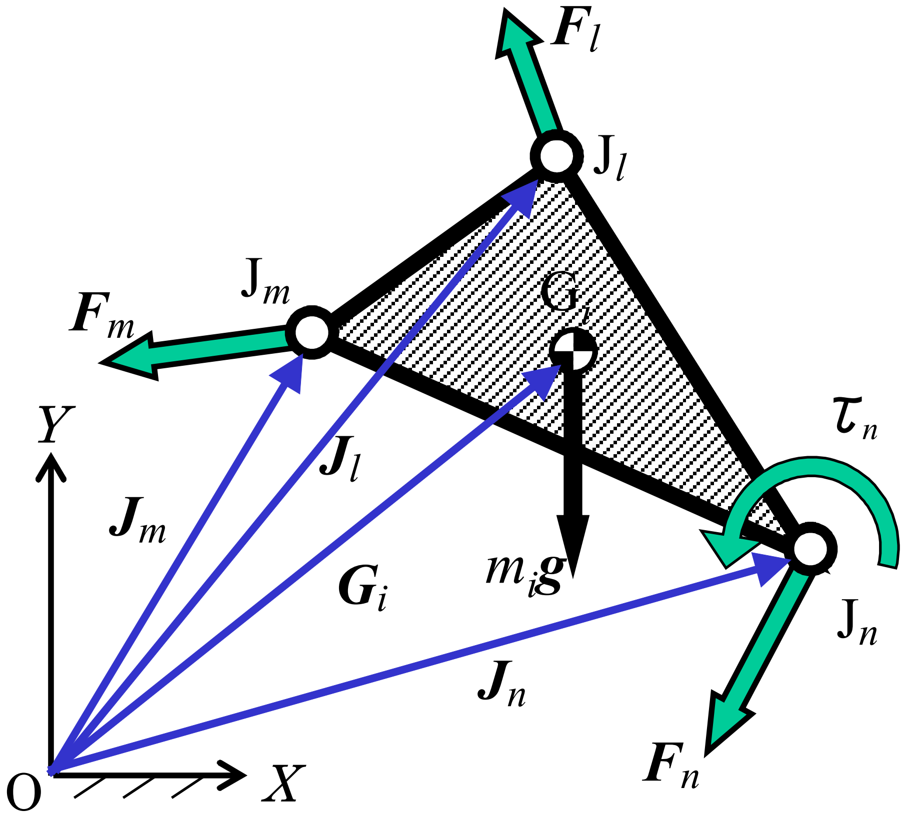
$$[P(\boldsymbol{G}, \boldsymbol{J})] \begin{bmatrix} \boldsymbol{F} \\ \boldsymbol{\tau} \end{bmatrix} = \boldsymbol{q}(\ddot{\boldsymbol{G}}, \ddot{\boldsymbol{\varphi}}, \boldsymbol{J})$$

Parameters determined by displacements and accelerations

One can carry out inverse dynamics by solving a system of linear equations with respect to joint forces and driving torques.

For general planar closed-loop link mechanisms

Equations of Motion



Forward dynamics:

$$[P(\boldsymbol{G},\boldsymbol{J})]\begin{bmatrix}\boldsymbol{F}\\ \boldsymbol{\tau}\end{bmatrix}=\boldsymbol{q}(\ddot{\boldsymbol{G}},\ddot{\boldsymbol{\varphi}},\boldsymbol{J})$$

Substituting motions of mechanism

$$\boldsymbol{G}=f_G(\boldsymbol{\theta}),\qquad \ddot{\boldsymbol{G}}=g_G(\ddot{\boldsymbol{\theta}},\dot{\boldsymbol{\theta}},\boldsymbol{\theta})$$

$$\ddot{\varphi}=g_\varphi(\ddot{\boldsymbol{\theta}},\dot{\boldsymbol{\theta}},\boldsymbol{\theta})$$

$$\boldsymbol{J}=f_J(\boldsymbol{\theta})$$

$\boldsymbol{\theta}$: **Crank inputs**

Eliminating joint forces $\boldsymbol{F}$ by adding and subtracting Eqs.

$$\boldsymbol{f}_0(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}},\boldsymbol{\tau})=\boldsymbol{0}$$

"A system of Nonlinear differential equations with respect to crank inputs"

By modifying the system of Eqs. as

$$\ddot{\boldsymbol{\theta}}=f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},t)$$

, then it can be solved with numerical calculation such as Runge—Kutta method.

Numerical solution to calculate both of motion and joint forces from the equation of motion

Consequently we have to use numerical method!

$$[P(\boldsymbol{G},\boldsymbol{J})]\begin{bmatrix}\boldsymbol{F}\\ \boldsymbol{\tau}\end{bmatrix}=\boldsymbol{q}(\ddot{\boldsymbol{G}},\ddot{\boldsymbol{\varphi}},\boldsymbol{J})$$

The acceleration of COG and angular acceleration of each link should be represented as linear combination of angular acceleration of crank.

$$\ddot{\boldsymbol{G}}=[r(\boldsymbol{\theta})]\ddot{\boldsymbol{\theta}}+\boldsymbol{s}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$$

$$\ddot{\boldsymbol{\varphi}}=[u(\boldsymbol{\theta})]\ddot{\boldsymbol{\theta}}+\boldsymbol{v}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$$

Coefficients of accelerations should be systematically derived.

$$[T(\boldsymbol{\theta})]\begin{bmatrix}\ddot{\boldsymbol{\theta}}\\ \boldsymbol{F}\end{bmatrix}=\boldsymbol{w}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{\tau})$$

"A system of linear equations with respect to angular acceleration of crank and joint forces."

Calculate $\ddot{\boldsymbol{\theta}},\boldsymbol{F}$ by a numerical method such as Gauss's method

$$\ddot{\boldsymbol{\theta}}_k=f(\boldsymbol{\theta}_k,\dot{\boldsymbol{\theta}}_k,t_k)$$

Solve $\boldsymbol{\theta}$ by a numerical method such as Runge-Kutta method

$$\boldsymbol{\theta}_{k+1}$$

Numerical solution to calculate both of motion and joint forces from the equation of motion

Consequently we have to use numerical method!

$$[P(\boldsymbol{G},\boldsymbol{J})]\begin{bmatrix}\boldsymbol{F}\\ \boldsymbol{\tau}\end{bmatrix}=\boldsymbol{q}(\ddot{\boldsymbol{G}},\ddot{\boldsymbol{\varphi}},\boldsymbol{J})$$

The acceleration of COG and angular acceleration of each link should be represented as linear combination of angular acceleration of crank.

$$\ddot{\boldsymbol{G}}=[r(\boldsymbol{\theta})]\ddot{\boldsymbol{\theta}}+\boldsymbol{s}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$$

$$\ddot{\boldsymbol{\varphi}}=[u(\boldsymbol{\theta})]\ddot{\boldsymbol{\theta}}+\boldsymbol{v}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$$

Coefficients of accelerations should be systematically derived.

$$[T(\boldsymbol{\theta})]\begin{bmatrix}\ddot{\boldsymbol{\theta}}\\ \boldsymbol{F}\end{bmatrix}=\boldsymbol{w}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{\tau})$$

"A system of linear equations with respect to angular acceleration of crank and joint forces."

Calculate $\ddot{\boldsymbol{\theta}},\boldsymbol{F}$ by a numerical method such as Gauss's method

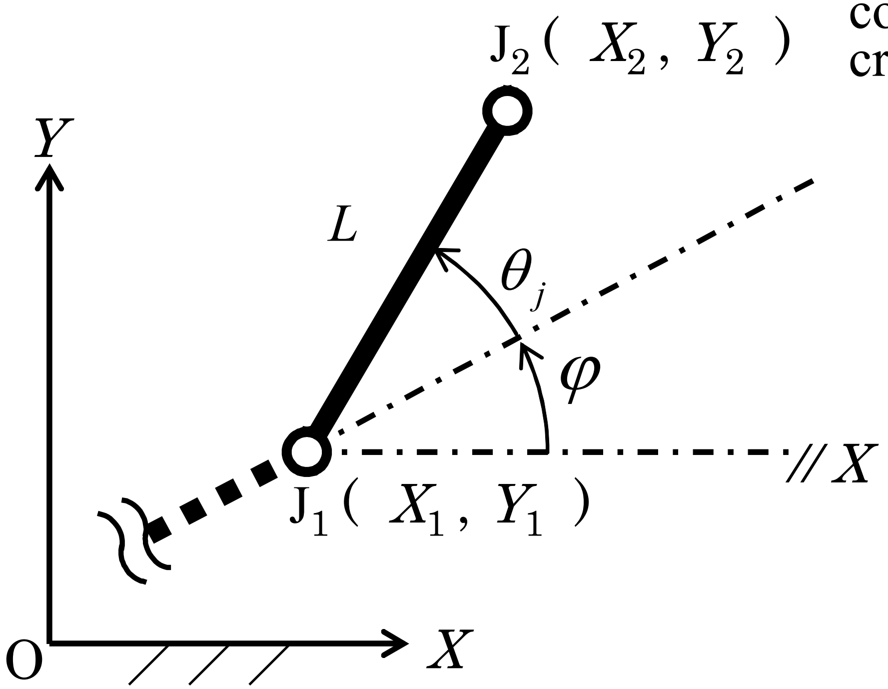
$$\ddot{\boldsymbol{\theta}}_k = f$$

$$\boldsymbol{\theta}_{k+1}$$

- ○ ***It is not necessary to derive equations by eliminating joint forces.***
- ○ ***Both of crank motion and joint forces can be calculated.***
- △ ***The acceleration can be obtained from the values in the last step.***
  (There is no problem since time step is sufficiently short.)

Equations to Derive Accelerations

Crank input



A crank

Acceleration of joint J_1 and angular acceleration of the previous link are given as the linear combination of angular accelerations of other cranks as

$$\ddot{X}_1 = \sum_k A_{1,k}\ddot{\theta}_k + B_1$$

$$\ddot{Y}_1 = \sum_k C_{1,k}\ddot{\theta}_k + D_1$$

$$\ddot{\varphi} = \sum_k E_k\ddot{\theta}_k + F$$

A new crank input : θ_j

$$X_2 = L\cos(\theta_j + \varphi) + X_1$$

$$Y_2 = L\sin(\theta_j + \varphi) + Y_1$$

By differentiating

$$\ddot{X}_2 = -L[(\ddot{\theta}_j + \ddot{\varphi})\sin(\theta_j + \varphi) + (\dot{\theta}_j + \dot{\varphi})^2\cos(\theta_j + \varphi) + \ddot{X}_1$$

$$\ddot{Y}_2 = L[(\ddot{\theta}_j + \ddot{\varphi})\cos(\theta_j + \varphi) - (\dot{\theta}_j + \dot{\varphi})^2\sin(\theta_j + \varphi) + \ddot{Y}_1$$

Equations to Derive Accelerations

Crank input

Acceleration of joint J_1 and angular acceleration of the previous link are given as the linear

They can be written as linear combination of the angular acceleration of crank as

$$\ddot{X}_2 = \sum_k A_{2,k}\ddot{\theta}_k + B_2, \qquad \ddot{Y}_2 = \sum_k C_{2,k}\ddot{\theta}_k + D_2$$

where

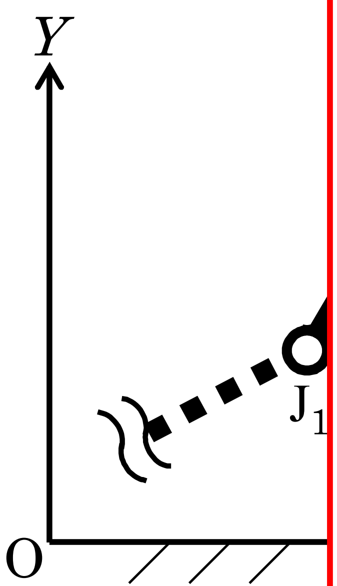
$$A_{2,k} = \begin{cases} -LE_k \sin(\theta_j + \varphi) + A_{1,k} & (k \neq j) \\ -L(E_k + 1)\sin(\theta_j + \varphi) + A_{1,k} & (k = j) \end{cases}$$

$$B_2 = -L[F\sin(\theta_j + \varphi) + (\dot{\theta}_j + \dot{\varphi})^2 \cos(\theta_j + \varphi)] + B_1$$

$$C_{2,k} = \begin{cases} LE_k \cos(\theta_j + \varphi) + C_{1,k} & (k \neq j) \\ L(E_k + 1)\cos(\theta_j + \varphi) + C_{1,k} & (k = j) \end{cases}$$

$$D_2 = L[F\cos(\theta_j + \varphi) - (\dot{\theta}_j + \dot{\varphi})^2 \sin(\theta_j + \varphi)] + D_1$$

Dynamics calculation program: FD_CRANK_INPUT

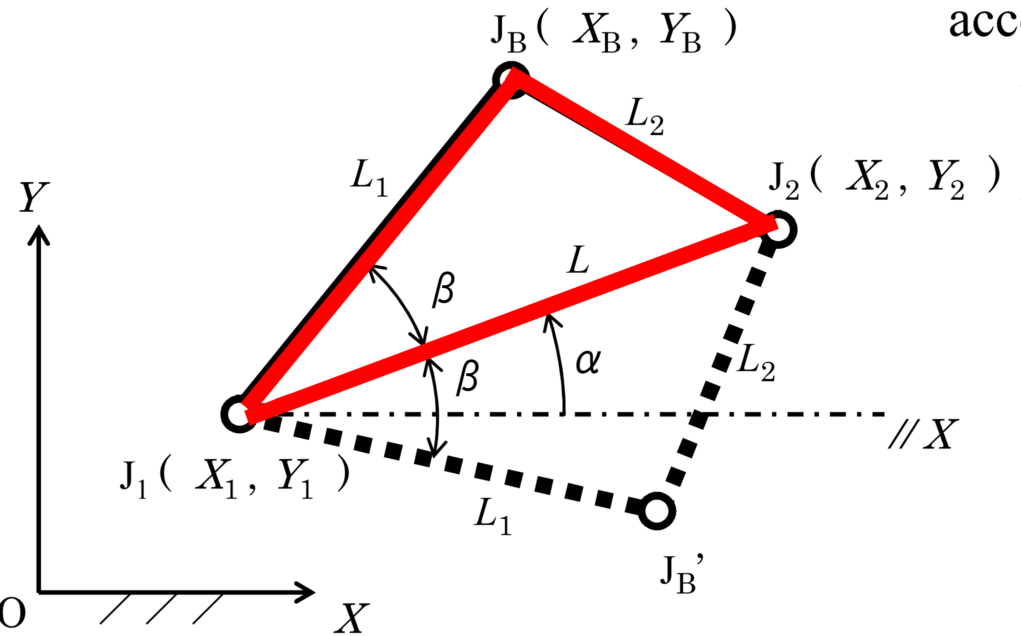


A c

$$\ddot{X}_2 = -L[(\ddot{\theta}_j + \ddot{\varphi})\sin(\theta_j + \varphi) + (\dot{\theta}_j + \dot{\varphi})^2 \cos(\theta_j + \varphi) + \ddot{X}_1$$

$$\ddot{Y}_2 = L[(\ddot{\theta}_j + \ddot{\varphi})\cos(\theta_j + \varphi) - (\dot{\theta}_j + \dot{\varphi})^2 \sin(\theta_j + \varphi) + \ddot{Y}_1$$

Two adjacent links with three revolute joints



R-R-R two adjacent links

Accelerations of joints, J_1, J_2, at both ends are given as the linear combination of angular accelerations of cranks as

$$\ddot{X}_1 = \sum_k A_{1,k}\ddot{\theta}_k + B_1,\quad \ddot{Y}_1 = \sum_k C_{1,k}\ddot{\theta}_k + D_1$$

$$\ddot{X}_2 = \sum_k A_{2,k}\ddot{\theta}_k + B_2,\quad \ddot{Y}_2 = \sum_k C_{2,k}\ddot{\theta}_k + D_2$$

Displacement analysis of the chain

$$X_B = L_1\cos(\alpha \pm \beta) + X_1$$

$$Y_B = L_1\sin(\alpha \pm \beta) + Y_1$$

$$\alpha = \tan^{-1}\frac{\Delta Y}{\Delta X},\quad \beta = \cos^{-1}\frac{L_1^2 + L^2 - L_2^2}{2L_1 L},$$

$$L = \sqrt{\Delta X^2 + \Delta Y^2},$$

Cosine theorem

$$\Delta X = X_2 - X_1, \Delta Y = Y_2 - Y_1$$

By differentiating

$$\ddot{X}_B = -L_1[(\ddot{\alpha} \pm \ddot{\beta})\sin(\alpha \pm \beta) + (\dot{\alpha} \pm \dot{\beta})\cos(\alpha \pm \beta)] + \ddot{X}_1$$

$$\ddot{Y}_B = L_1[(\ddot{\alpha} \pm \ddot{\beta})\cos(\alpha \pm \beta) - (\dot{\alpha} \pm \dot{\beta})\sin(\alpha \pm \beta)] + \ddot{Y}_1$$

$$\ddot{\alpha} = \cdots\cdots,\quad \ddot{\beta} = \cdots\cdots,\quad \ddot{L} = \cdots\cdots$$

Two adjacent links with three revolute joints

They can be written as linear combination of angular acceleration of crank as

$$\ddot{X}_B = \sum_k A_{B,k}\ddot{\theta}_k + B_B, \qquad \ddot{Y}_B = \sum_k C_{B,k}\ddot{\theta}_k + D_B$$

where

$$A_{B,k} = -L_1(P_k \pm R_k)\sin(\alpha \pm \beta) + A_{1,k}$$

$$B_B = -L_1(Q \pm S)\sin(\alpha \pm \beta) - L_1(\dot{\alpha} \pm \dot{\beta})^2\cos(\alpha \pm \beta) + B_1$$

$$C_{B,k} = L_1(P_k \pm R_k)\cos(\alpha \pm \beta) + C_{1,k}$$

$$D_B = L_1(Q \pm S)\cos(\alpha \pm \beta) - L_1(\dot{\alpha} \pm \dot{\beta})^2\sin(\alpha \pm \beta) + D_1$$

Dynamics calculation program: FD_RRR_LINKS

$$H_k = [\Delta X(A_{2,k} - A_{1,k}) + \Delta Y(C_{2,k} - C_{1,k})]/L$$

$$K = \{L[\Delta X(B_2 - B_1) + \Delta Y(D_2 - D_1)] + L(\Delta\dot{X}^2 + \Delta\dot{Y}^2) - \dot{L}(\Delta\dot{X}\Delta X + \Delta\dot{Y}\Delta Y)\}/L^2$$

$$P_k = \cos^2\alpha[\Delta X(C_{2,k} - C_{1,k}) + \Delta Y(A_{2,k} - A_{1,k})]/\Delta X^2$$

$$Q = \cos^2\alpha[\Delta X(D_2 - D_1) + \Delta Y(B_2 - B_1)]/\Delta X^2 - 2\cos\alpha(\Delta\dot{Y}\Delta X - \Delta Y\Delta\dot{X})(\Delta X\dot{\alpha}\sin\alpha + \Delta\dot{X}\cos\alpha)$$

$$R_k = (L_1^2 - L_2^2 - L^2)H_k/(2L_1L^2\sin\beta)$$

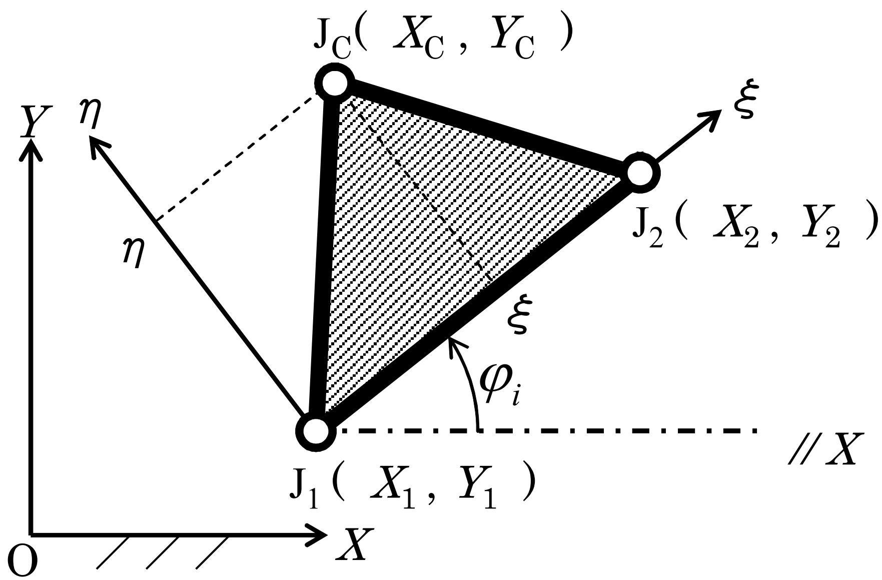
$$S = (L_1^2 - L_2^2 - L^2)K/(2L_1L^2\sin\beta) + \dot{L}[(L^2 - L_1^2 + L_2^2)L\dot{\beta}\cos\beta + 2(L_2^2 - L_1^2)\dot{L}\sin\theta]/(2L_1L^3\sin^2\beta)$$

$$\ddot{X}_B = -L_1[(\ddot{\alpha} \pm \ddot{\beta})\sin(\alpha \pm \beta) + (\dot{\alpha} \pm \dot{\beta})\cos(\alpha \pm \beta)] + \ddot{X}_1$$

$$\ddot{Y}_B = L_1[(\ddot{\alpha} \pm \ddot{\beta})\cos(\alpha \pm \beta) - (\dot{\alpha} \pm \dot{\beta})\sin(\alpha \pm \beta)] + \ddot{Y}_1$$

$$\ddot{\alpha} = \cdots\cdots, \quad \ddot{\beta} = \cdots\cdots, \quad \ddot{L} = \cdots\cdots$$

Point on a coupler and posture of coupler



A coupler

The accelerations of two joints, J_1, J_2, are given as the linear combination of angular accelerations of cranks as

$$\ddot{X}_1 = \sum_k A_{1,k}\ddot{\theta}_k + B_1, \quad \ddot{Y}_1 = \sum_k C_{1,k}\ddot{\theta}_k + D_1$$

$$\ddot{X}_2 = \sum_k A_{2,k}\ddot{\theta}_k + B_2, \quad \ddot{Y}_2 = \sum_k C_{2,k}\ddot{\theta}_k + D_2$$

Displacement analysis (Coordinate transform)

$$X_C = \xi\cos\varphi_i - \eta\sin\varphi_i + X_1$$

$$Y_C = \xi\sin\varphi_i + \eta\cos\varphi_i + Y_1$$

$$\varphi_i = \tan^{-1}\frac{\Delta Y}{\Delta X}$$

$$\Delta X = X_2 - X_1, \quad \Delta Y = Y_2 - Y_1$$

By differentiating

$$\ddot{X}_C = -\xi(\ddot{\varphi}_i\sin\varphi_i + \dot{\varphi}_i^2\cos\varphi_i) - \eta(\ddot{\varphi}_i\cos\varphi_i - \dot{\varphi}_i^2\sin\varphi_i) + \ddot{X}_1$$

$$\ddot{Y}_C = \xi(\ddot{\varphi}_i\cos\varphi_i - \dot{\varphi}_i^2\sin\varphi_i) - \eta(\ddot{\varphi}_i\sin\varphi_i + \dot{\varphi}_i^2\cos\varphi_i) + \ddot{Y}_1$$

$$\ddot{\varphi}_i = \cdots\cdots$$

Point on a coupler and posture of coupler

The accelerations of two joints, J_1, J_2, are
of angular

$_k + D_1$

$\ddot{\theta}_k + D_2$

lysis
form)

They can be written as the linear combination of angular acceleration of cranks as

$$\ddot{X}_C = \sum_k A_{C,k}\ddot{\theta}_k + B_C, \quad \ddot{Y}_C = \sum_k C_{C,k}\ddot{\theta}_k + D_C, \quad \ddot{\varphi}_i = \sum_k E_{i,k}\ddot{\theta}_k + F_i$$

$$A_{C,k} = -(\xi \sin\varphi_i + \eta\cos\varphi_i)E_{i,k} + A_{1,k}$$

$$B_C = -(\xi \sin\varphi_i + \eta\cos\varphi_i)F - \xi\dot{\varphi}_i^{\,2}\cos\varphi_i + \eta\dot{\varphi}_i^{\,2}\sin\varphi_i + B_1$$

$$C_{C,k} = (\xi\cos\varphi_i - \eta\sin\varphi_i)E_{i,k} + C_{1,k}$$

$$D_B = (\xi\cos\varphi_i - \eta\sin\varphi_i)F - \xi\dot{\varphi}_i^{\,2}\sin\varphi_i - \eta\dot{\varphi}_i^{\,2}\cos\varphi_i + D_1$$

$$E_{i,k} = [\Delta X(C_{2,k} - C_{1,k}) - \Delta Y(A_{2,k} - A_{1,k})]/L^2$$

$$F_i = [\Delta X(D_2 - D_1) - \Delta Y(B_2 - B_1)]/L^2{}_1$$

Dynamics calculation module: FD_COUPLER_POINT

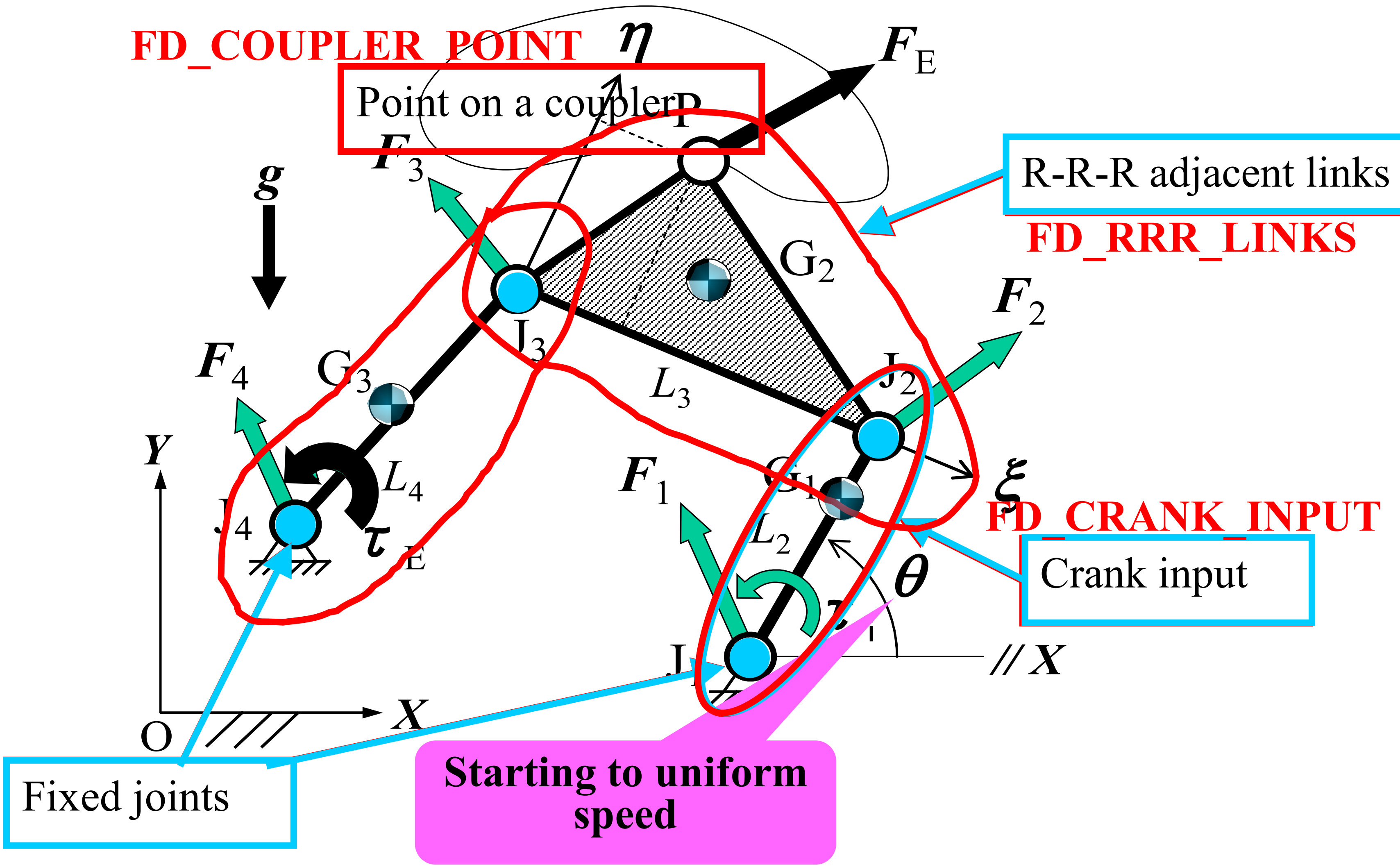
$$\ddot{X}_C = -\xi(\ddot{\varphi}_i\sin\varphi_i + \dot{\varphi}_i^2\cos\varphi_i) - \eta(\ddot{\varphi}_i\cos\varphi_i - \dot{\varphi}_i^2\sin\varphi_i) + \ddot{X}_1$$

$$\ddot{Y}_C = \xi(\ddot{\varphi}_i\cos\varphi_i - \dot{\varphi}_i^2\sin\varphi_i) - \eta(\ddot{\varphi}_i\sin\varphi_i + \dot{\varphi}_i^2\cos\varphi_i) + \ddot{Y}_1$$

$$\ddot{\varphi}_i = \cdots\cdots$$

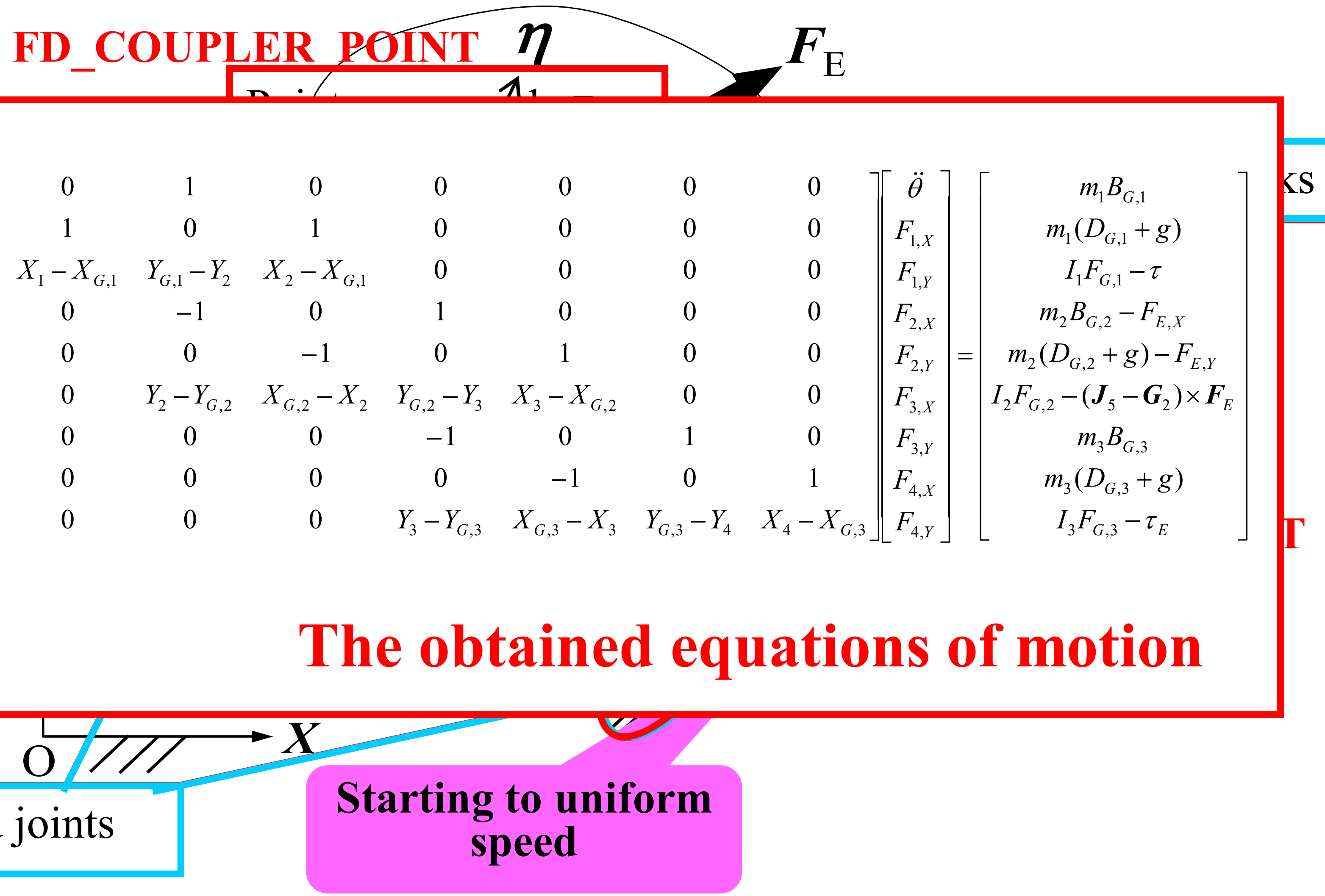
Examples of Forward Dynamics of Planar Closed-loop Mechanisms

Example 1 : Starting process of a 4-bar mechanism



Examples of Forward Dynamics of Planar Closed-loop Mechanisms

Example 1 : Starting process of a 4-bar mechanism



$$
\begin{bmatrix}
-m_1A_{G,1} & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-m_1C_{G,1} & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
-I_1E_{G,1} & Y_{G,1}-Y_1 & X_1-X_{G,1} & Y_{G,1}-Y_2 & X_2-X_{G,1} & 0 & 0 & 0 & 0 \\
-m_2A_{G,2} & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
-m_2C_{G,2} & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
-I_2E_{G,2} & 0 & 0 & Y_2-Y_{G,2} & X_{G,2}-X_2 & Y_{G,2}-Y_3 & X_3-X_{G,2} & 0 & 0 \\
-m_3A_{G,3} & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
-m_3C_{G,3} & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
-I_3E_{G,3} & 0 & 0 & 0 & 0 & Y_3-Y_{G,3} & X_{G,3}-X_3 & Y_{G,3}-Y_4 & X_4-X_{G,3}
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta} \\ F_{1,X} \\ F_{1,Y} \\ F_{2,X} \\ F_{2,Y} \\ F_{3,X} \\ F_{3,Y} \\ F_{4,X} \\ F_{4,Y}
\end{bmatrix}
=
\begin{bmatrix}
m_1B_{G,1} \\
m_1(D_{G,1}+g) \\
I_1F_{G,1}-\tau \\
m_2B_{G,2}-F_{E,X} \\
m_2(D_{G,2}+g)-F_{E,Y} \\
I_2F_{G,2}-(\boldsymbol{J}_5-\boldsymbol{G}_2)\times\boldsymbol{F}_E \\
m_3B_{G,3} \\
m_3(D_{G,3}+g) \\
I_3F_{G,3}-\tau_E
\end{bmatrix}
$$

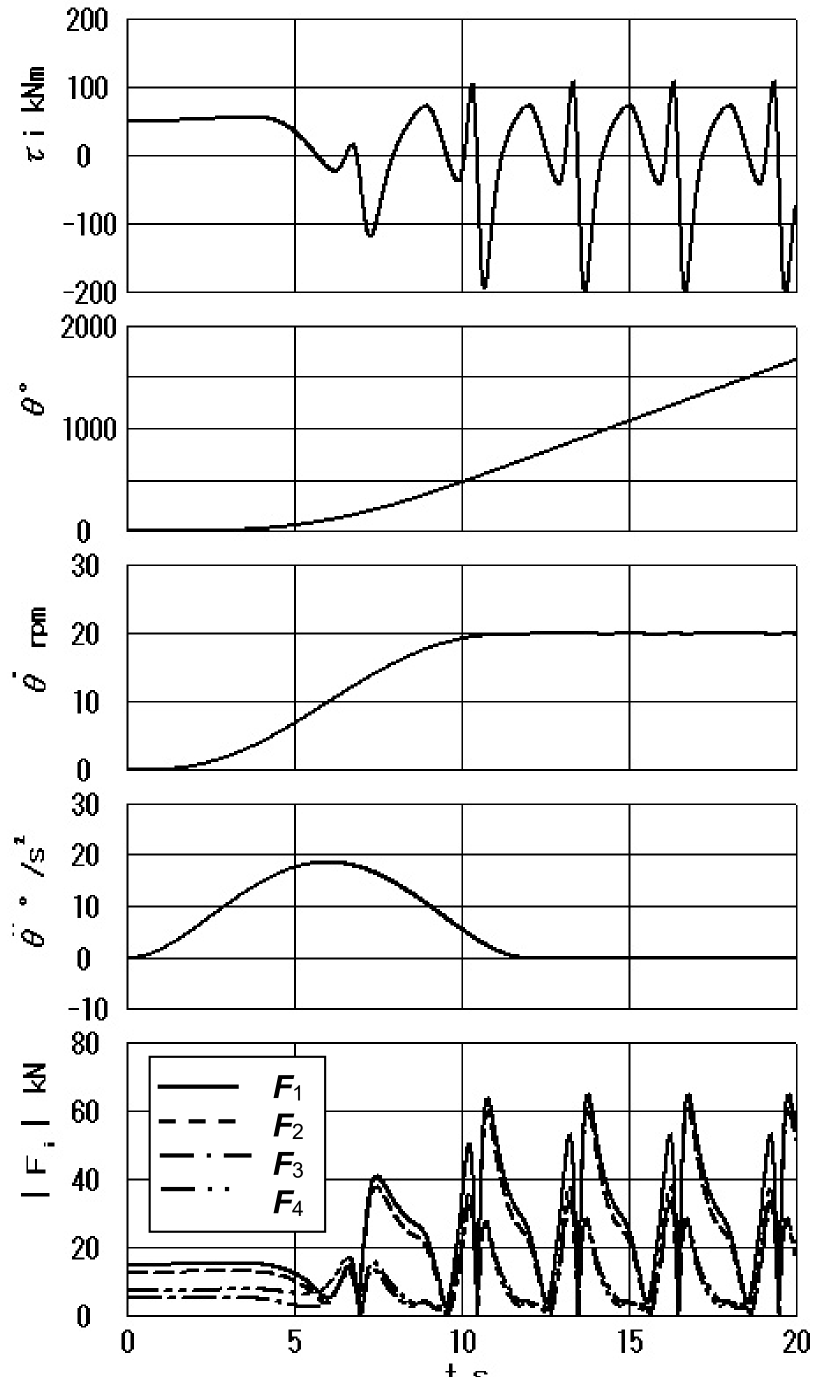
The obtained equations of motion

Kinemactical dimensions

L_2 m	4.0
L_3 m	8.0
L_4 m	8.0
(X_1, Y_1) m	(10.0,0.0)
(X_4, Y_4) m	(0.0,0.0)
(ξ, η) m	(5.0,5.0)

Mass and moment of inertia of each link

m_1 kg	247.0	
m_2 kg	1235.0	
m_3 kg	494.0	
$(\xi_{G,1}, \eta_{G,1})$ mm	(2.0,0.0)	J_1, J_2
$(\xi_{G,2}, \eta_{G,2})$ mm	(4.0,3.0)	J_3, J_2
$(\xi_{G,3}, \eta_{G,3})$ mm	(4.0,0.0)	J_4, J_3
I_1 kgm^2	329.0	
I_2 kgm^2	7905.0	
I_3 kgm^2	2635.0	
$(F_{E,X}, F_{E,Y})$ N	(100.0,0.0)	
τ_E Nm	-5.0	



Results of simulation

Driving torque is given by carrying out the inverse dynamics calculation.

Angular velocity of crank was given as a cycloid function.

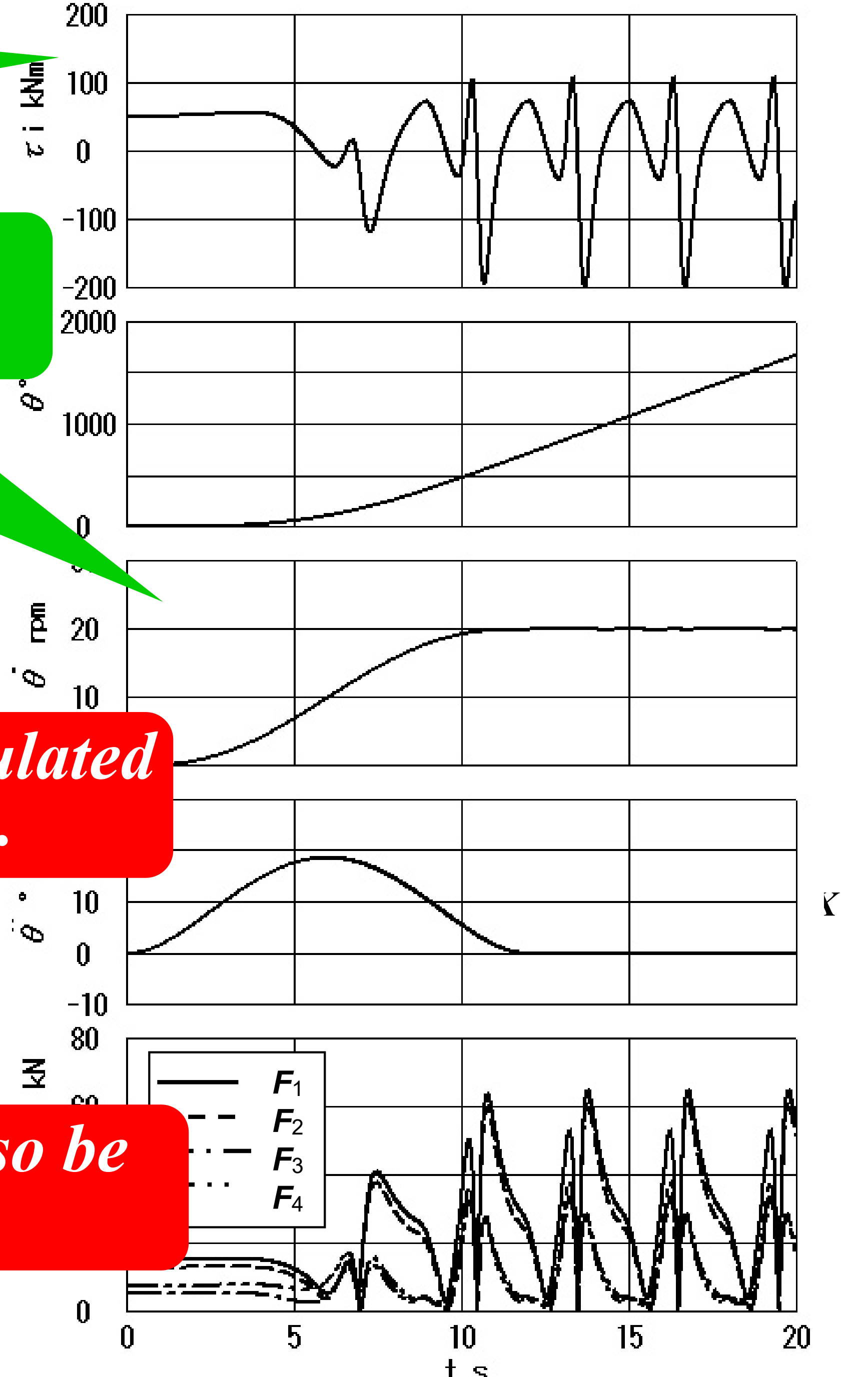
(X_4, Y_4) m	(0.0,0.0)
(ξ, η) m	(5.0,5.0)

Mass and moment of inertia of each link

m_1 kg	247.0	
m_2 kg	1235.0	
[illegible]	[illegible]	
$(\xi_{G,2}, \eta_{G,2})$ mm	(4.0,3.0)	J_3, J_2
$(\xi_{G,3}, \eta_{G,3})$ mm	(4.0,0.0)	J_4, J_3
I_1 kgm^2	329.0	
I_2 kgm^2	7905.0	
I_3 kgm^2	[illegible]	
$(F_{E,X}, F$[illegible]	[illegible]	
τ_E Nm	[illegible]	

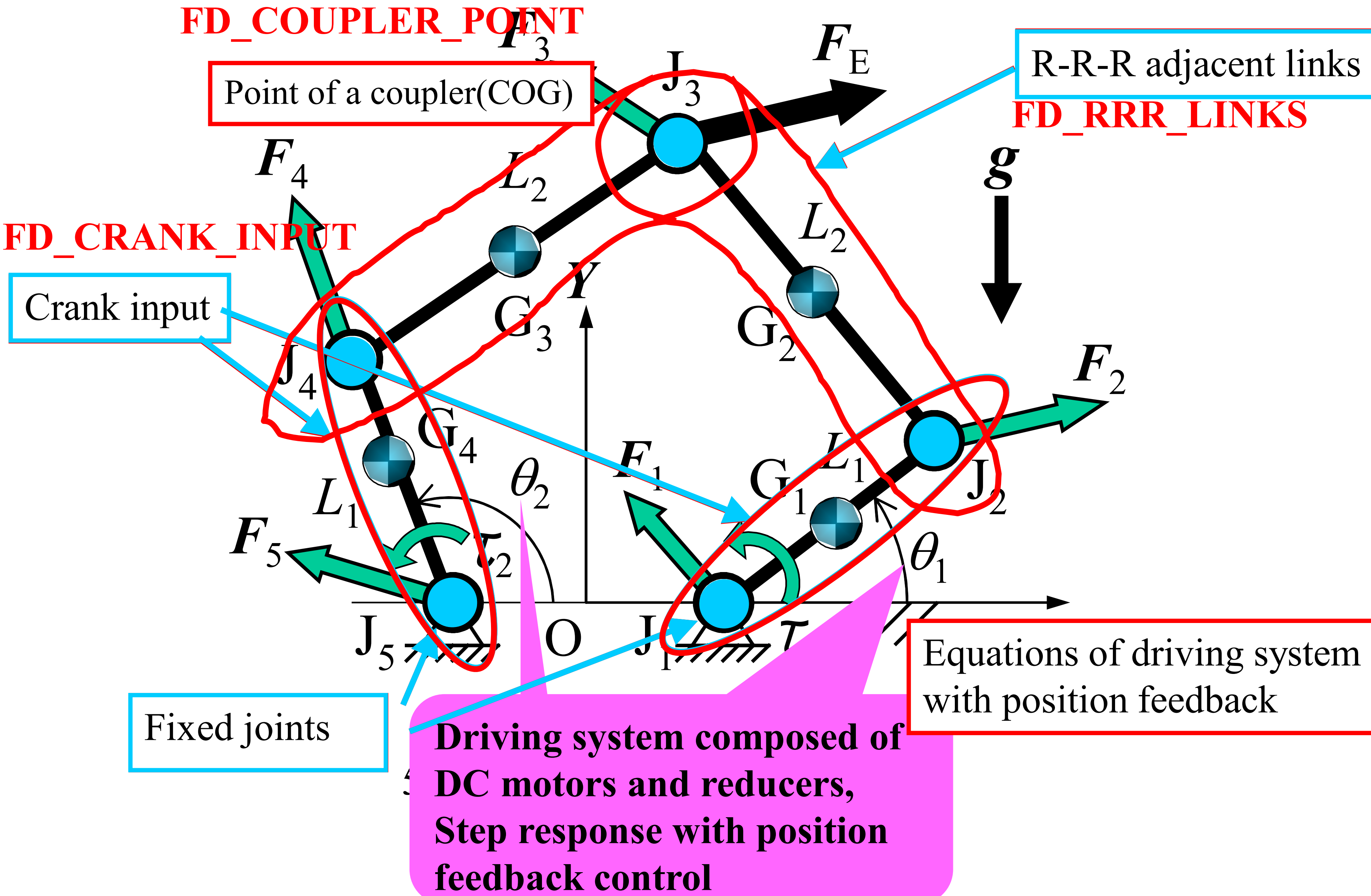
The starting process can be simulated with an adequate accuracy.

The joint forces can also be calculated.



Results of simulation

Example 2: Positioning process of 5-bar mechanism with 2DOF



Example 2: Positioning process of 5-bar mechanism with 2DOF

$$
\begin{bmatrix}
-m_1A_{G,1,1} & -m_1A_{G,1,2} & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-m_1C_{G,1,1} & -m_1C_{G,1,2} & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-I_1E_{G,1,1} & -I_1E_{G,1,2} & Y_1-Y_{G,1} & X_{G,1}-X_1 & Y_2-Y_{G,1} & X_{G,1}-X_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
-m_2A_{G,2,1} & -m_2A_{G,2,2} & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-m_2C_{G,2,1} & -m_2C_{G,2,2} & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
-I_2E_{G,2,1} & -I_2E_{G,2,2} & 0 & 0 & Y_{G,2}-X_2 & X_2-X_{G,2} & Y_3-Y_{G,2} & X_{G,2}-X_3 & 0 & 0 & 0 & 0 \\
-m_3A_{G,3,1} & -m_3A_{G,3,2} & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
-m_3C_{G,3,1} & -m_3C_{G,3,2} & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
-I_3E_{G,3,1} & -I_3E_{G,3,2} & 0 & 0 & 0 & 0 & Y_{G,3}-X_3 & X_3-X_{G,3} & Y_4-Y_{G,3} & X_{G,3}-X_4 & 0 & 0 \\
-m_4A_{G,4,1} & -m_4A_{G,4,2} & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
-m_4C_{G,4,1} & -m_4C_{G,4,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
-I_4E_{G,4,1} & -I_4E_{G,4,2} & 0 & 0 & 0 & 0 & 0 & 0 & Y_{G,4}-X_4 & X_4-X_{G,4} & Y_5-Y_{G,4} & X_{G,4}-X_5
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_1 \\ \ddot{\theta}_2 \\ F_{1,X} \\ F_{1,Y} \\ F_{2,X} \\ F_{2,Y} \\ F_{3,X} \\ F_{3,Y} \\ F_{4,X} \\ F_{4,Y} \\ F_{5,X} \\ F_{5,Y}
\end{bmatrix}
=
\begin{bmatrix}
m_1B_{G,1} \\
m_1(D_{G,1}+g) \\
I_1F_1-\tau_1 \\
m_2B_{G,2}-F_{E,X} \\
m_2(D_{G,2}+g)-F_{E,Y} \\
I_2F_2-(\boldsymbol{J}_3-\boldsymbol{G}_2)\times\boldsymbol{F}_E \\
m_3B_{G,3} \\
m_3(D_{G,3}+g) \\
I_3F_3 \\
m_4B_{G,4} \\
m_4(D_{G,4}+g) \\
I_4F_4-\tau_2
\end{bmatrix}
$$

The given driving torque
(Equations of motion of DC motors and feedback control system):

$$\tau_i = \frac{n\eta K_T[K_P(\theta_{d,i}-\theta_i)-K_E n\dot{\theta}_i]}{R_a}$$

The obtained equations of motion

Fixed joints

Driving system composed of DC motors and reducers, Step response with position feedback control

Kinematical dimensions

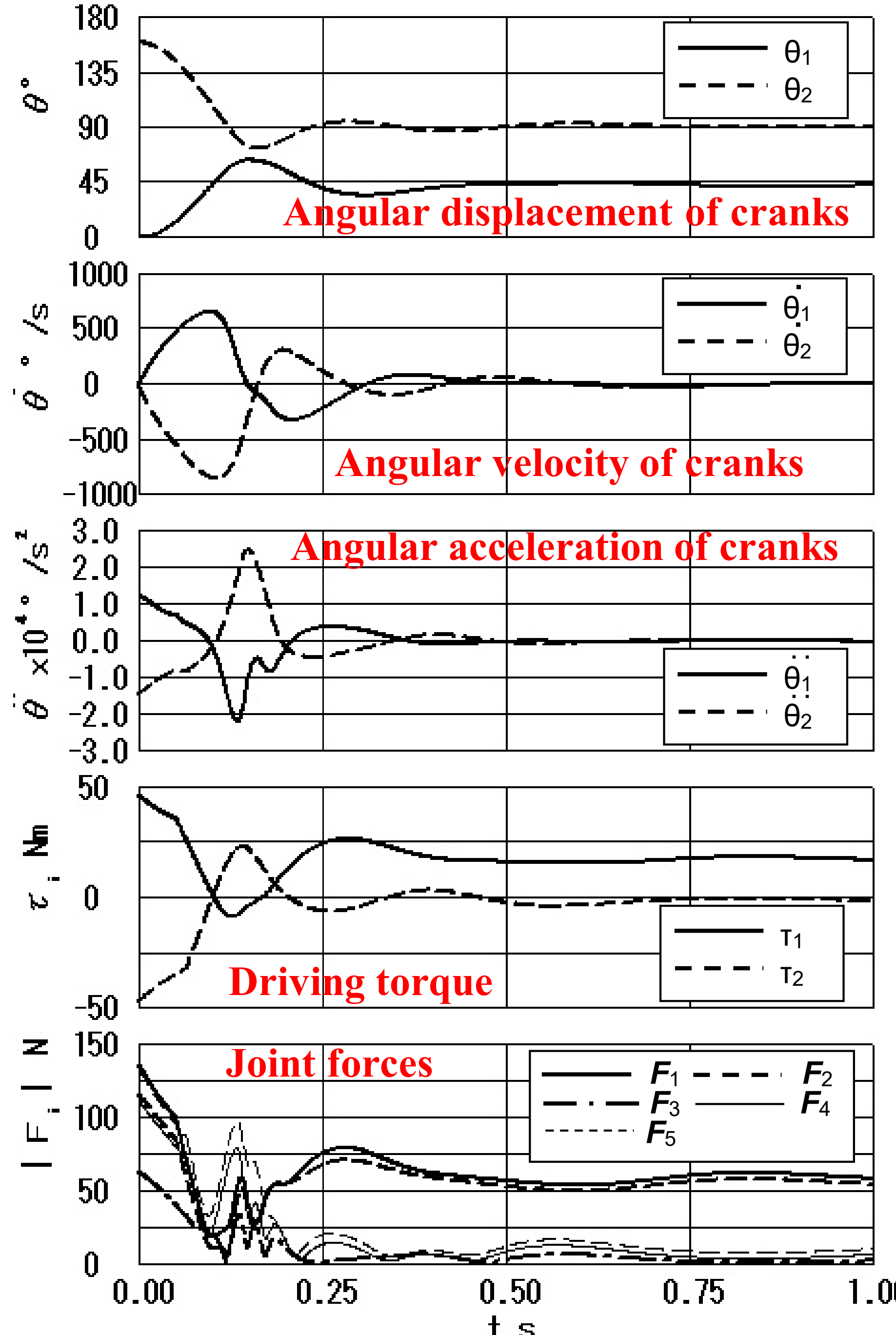
L_1 m	0.4
L_2 m	0.6
(X_1, Y_1) m	(0.05,0.0)
(X_5, Y_5) m	(-0.05,0.0)

Mass, COG and moment of inertia of each link

m_1 kg	0.2	$(\xi_{G,1}, \eta_{G,1})$ mm	(0.2,0.0)	J_1,J_2
m_2 kg	0.3	$(\xi_{G,2}, \eta_{G,2})$ mm	(0.3,0.0)	J_3,J_2
m_3 kg	0.3	$(\xi_{G,3}, \eta_{G,3})$ mm	(0.3,0.0)	J_4,J_3
m_4 kg	0.2	$(\xi_{G,4}, \eta_{G,4})$ mm	(0.2,0.0)	J_5,J_4
I_1 kgm²	5.33×10^{-3}	$(F_{E,X}, F_{E,Y})$ N	(0.0,-50.0)	
I_2 kgm²	18.0×10^{-3}			
I_3 kgm²	18.0×10^{-3}			
I_4 kgm²	5.33×10^{-3}			

Driving system

n	10.0
η	0.8
K_P	100.0
K_T Nm/A	0.22
K_E Vs/rad	0.22
R_a Ω	3.0



Results of simulation

Kinematical dimensions

L_1 m	0.4

Positioning process can be simulated with an adequate accuracy.

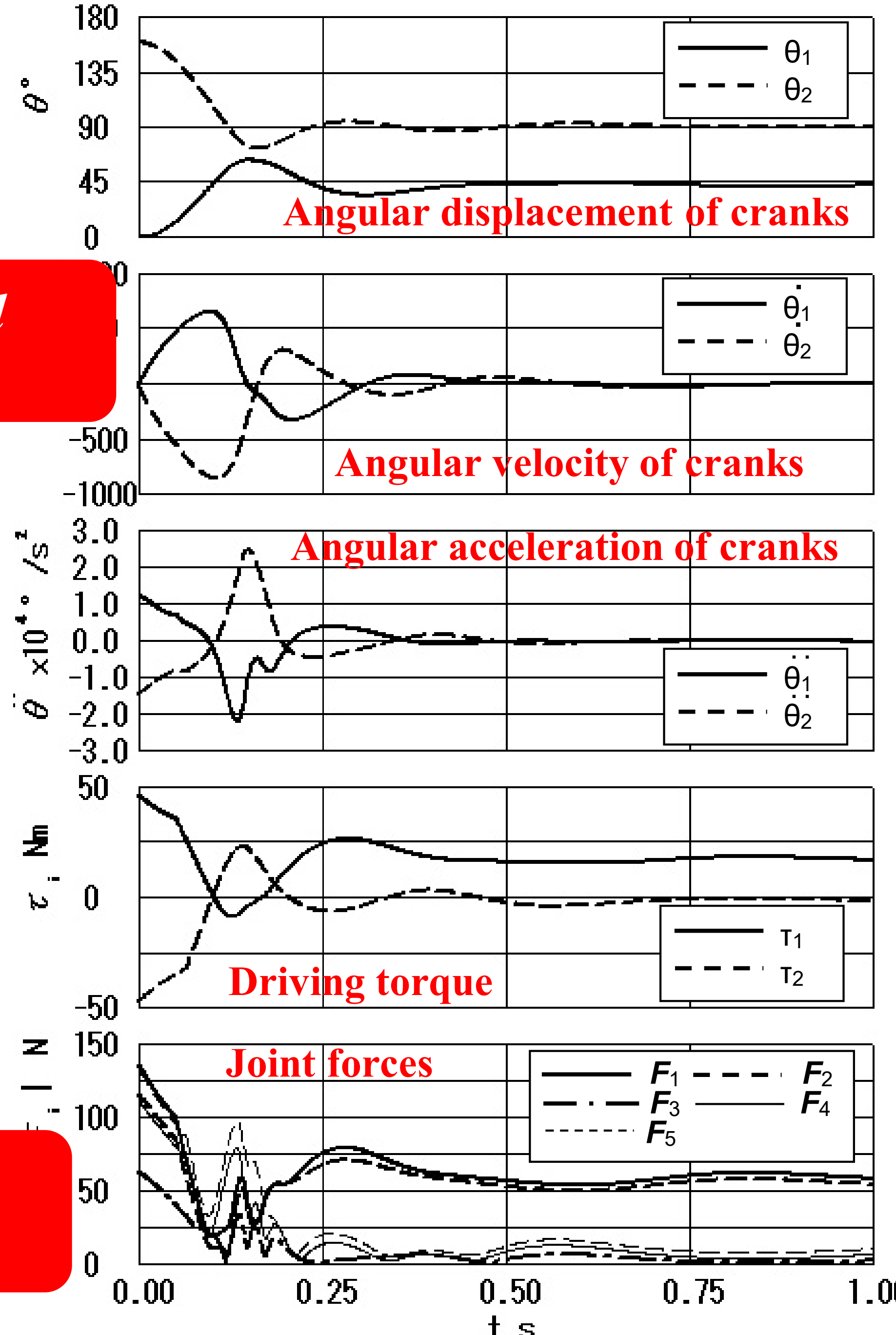
Mass, COG and moment of inertia of each link

m_1 kg	0.2	$(\xi_{G,1},\eta_{G,1})$ mm	(0.2,0.0)	J_1,J_2	
m_2 kg	0.3	$(\xi_{G,2},\eta_{G,2})$ mm	(0.3,0.0)	J_3,J_2	
m_3 kg	0.3	$(\xi_{G,3},\eta_{G,3})$ mm	(0.3,0.0)	J_4,J_3	
m_4 kg	0.2	$(\xi_{G,4},\eta_{G,4})$ mm	(0.2,0.0)	J_5,J_4	
I_1 kgm²	5.33×10^{-3}	$(F_{E,X},F_{E,Y})$ N	(0.0,-50.0)		
I_2 kgm²	18.0×10^{-3}				
I_3 kgm²	18.0×10^{-3}				
I_4 kgm²	5.33×10^{-3}				

Driving system

n	10.0
η	0.8
K_P	100.0

Joint forces can also be calculated.



Results of simulation

Equations to Derive Accelerations for Spatial Mechanisms

Crank input

Translational and angular accelerations of a revolute joint R and previous link are given as the linear combination of accelerations of other driving link inputs as

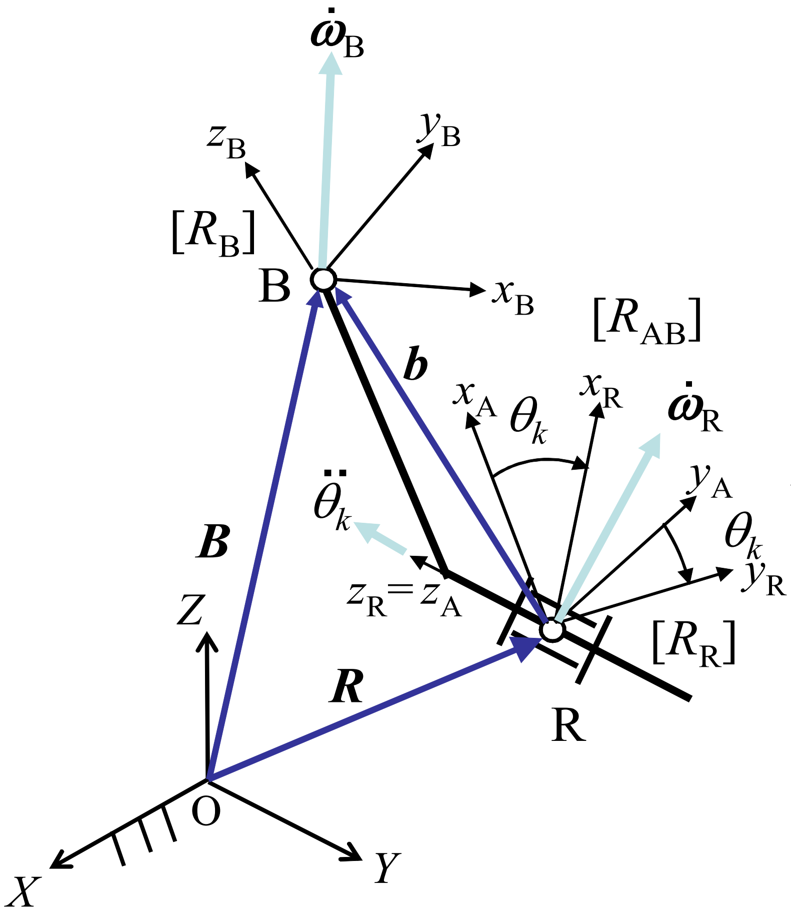
$$\ddot{\boldsymbol{R}} = [C_R]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_R$$

$$\dot{\boldsymbol{\omega}}_R = [C_{\omega R}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega R}\ , \ [\ddot{R}_R] = [C_{RR}]\ddot{\boldsymbol{\theta}} + [d_{RR}]$$

A new crank input: θ_k

$$\ddot{\boldsymbol{B}} = [C_B]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_B$$

$$\dot{\boldsymbol{\omega}}_B = [C_{\omega B}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega B}, \quad [\ddot{R}_B] = [C_{RB}]\ddot{\boldsymbol{\theta}} + [d_{RB}]$$



A crank

where

$$[C_B] = ([C_{RR}][\theta] + [R_R][C_\theta])\boldsymbol{b} + [C_R],$$

$$\boldsymbol{d}_B = ([d_{RR}][\theta] + 2[\dot{R}_R][\dot{\theta}] + [R_R][d_\theta])\boldsymbol{b},$$

$$[C_{\omega B}] = [R_{AB}][C_{\omega R}], \quad \boldsymbol{d}_{\omega B} = [R_{AB}](\boldsymbol{d}_{\omega R} + [0 \quad 0 \quad 1]^T),$$

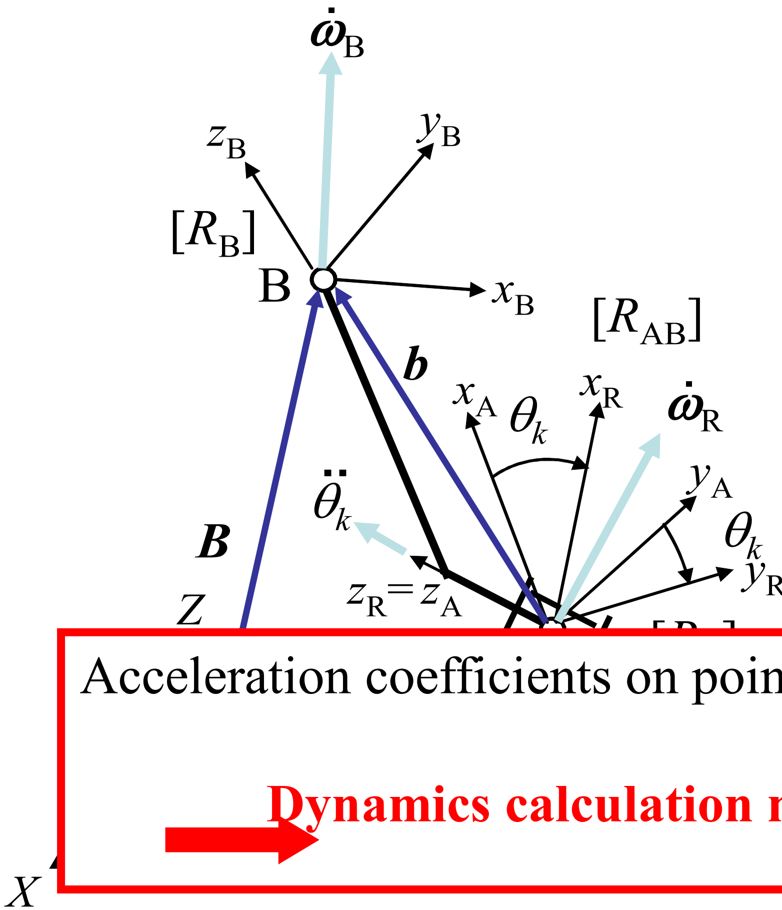
$$[C_{RB}] = ([C_{RR}][\theta] + [R_R][C_\theta])[R_{AB}],$$

$$[d_{RB}] = [d_{RR}][\theta] + 2[\dot{R}_R][\dot{\theta}] + [R_R][d_\theta],$$

$$[C_\theta] = \begin{bmatrix} -\sin\theta_k & -\cos\theta_k & 0 \\ \cos\theta_k & -\sin\theta_k & 0 \\ 0 & 0 & 0 \end{bmatrix}, [d_\theta] = \begin{bmatrix} -\dot{\theta}_k{}^2\cos\theta_k & \dot{\theta}_k{}^2\sin\theta & 0 \\ -\dot{\theta}_k{}^2\sin\theta & -\ddot{\theta}_k{}^2\cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Equations to Derive Accelerations for Spatial Mechanisms

Crank input



A crank

Translational and angular accelerations of a revolute joint R and previous link are given as the linear combination of accelerations of other driving link inputs as

$$\ddot{\boldsymbol{R}} = [C_R]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_R$$

$$\dot{\boldsymbol{\omega}}_R = [C_{\omega R}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega R}, \quad [\ddot{R}_R] = [C_{RR}]\ddot{\boldsymbol{\theta}} + [d_{RR}]$$

A new crank input: θ_k

$$\ddot{\boldsymbol{B}} = [C_B]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_B$$

$$\dot{\boldsymbol{\omega}}_B = [C_{\omega B}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega B}, \quad [\ddot{R}_B] = [C_{RB}]\ddot{\boldsymbol{\theta}} + [d_{RB}]$$

where

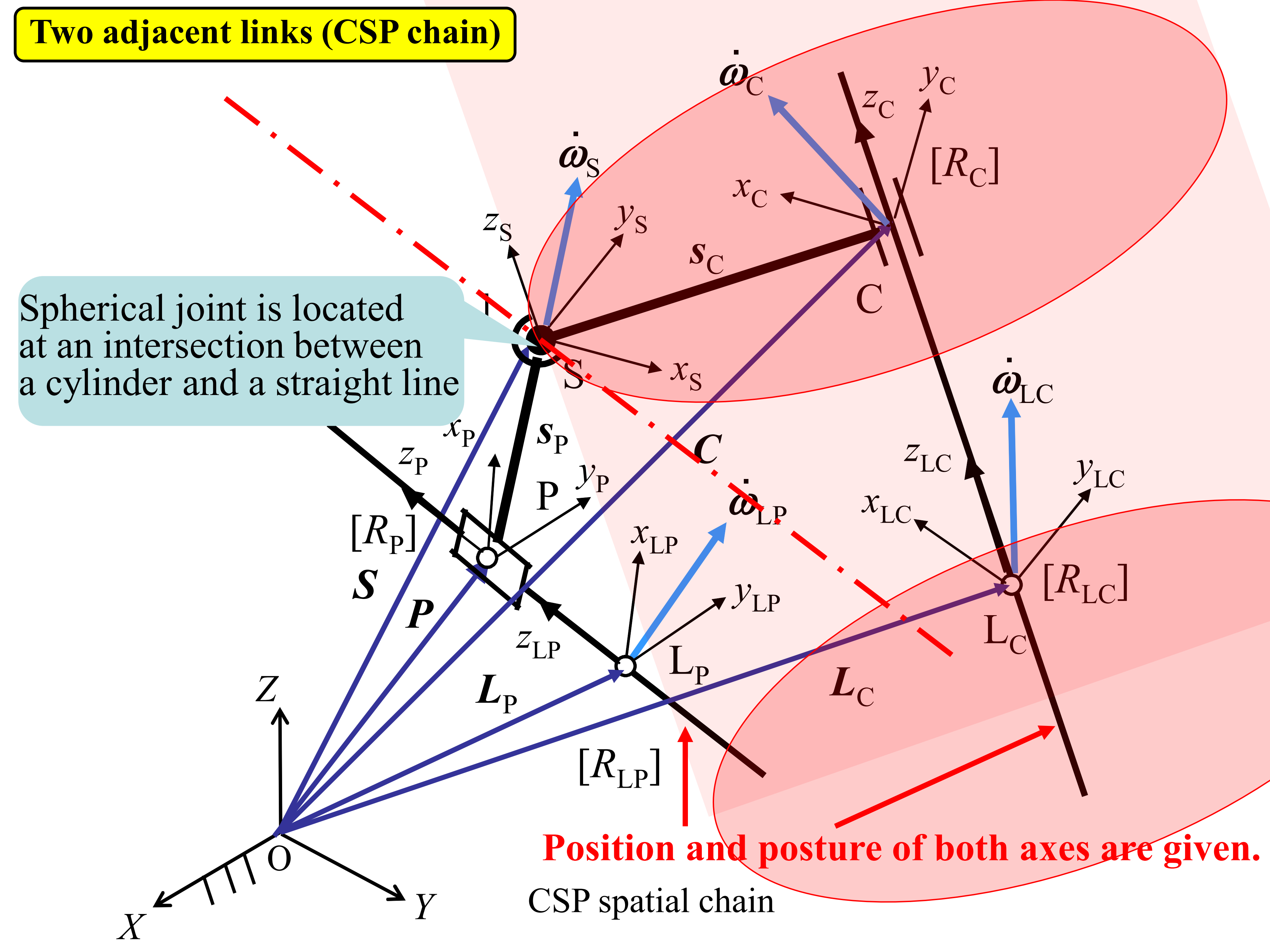
$$[C_B] = ([C_{RR}][\theta] + [R_R][C_\theta])\boldsymbol{b} + [C_R],$$

$$\boldsymbol{d}_B = ([d_{RR}][\theta] + 2[\dot{R}_{R}][\dot{\theta}] + [R_R][d_\theta])\boldsymbol{b} \ldots$$

Acceleration coefficients on point B can be recursively obtained.

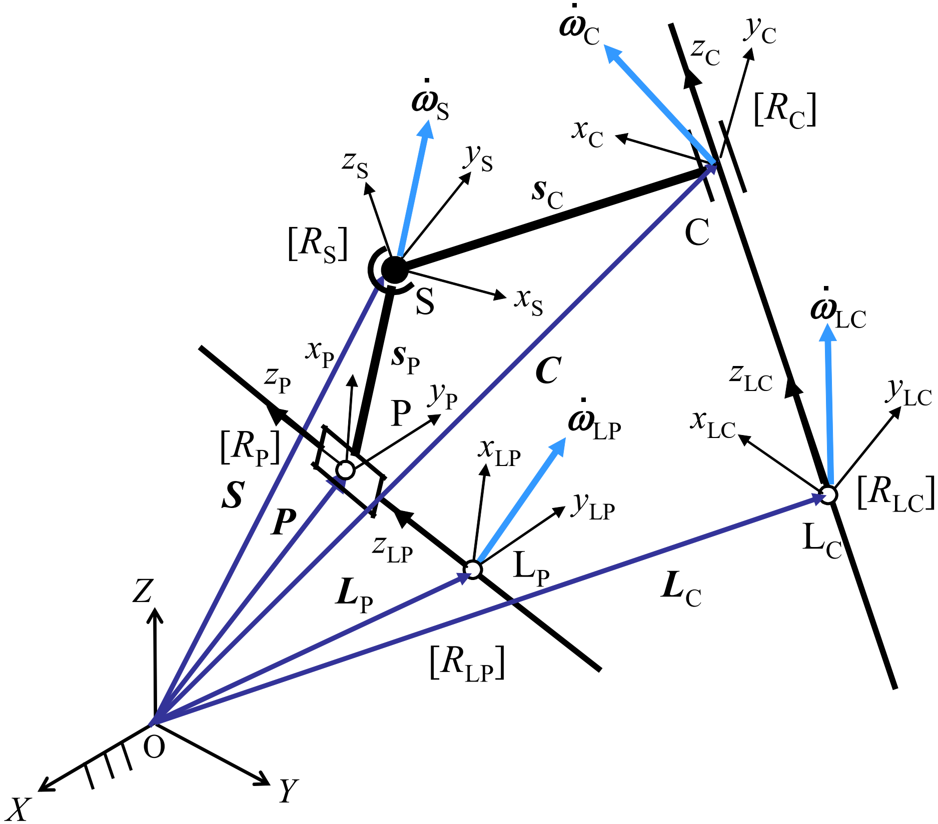
Dynamics calculation module: FD_S_CRANK_INPUT

$$[C_\theta] = \begin{bmatrix} -\sin\theta_k & -\cos\theta_k & 0 \\ \cos\theta_k & -\sin\theta_k & 0 \\ 0 & 0 & 0 \end{bmatrix}, [d_\theta] = \begin{bmatrix} -\theta_k \cos\theta_k & \theta_k \sin\theta & 0 \\ -\dot{\theta}_k^2 \sin\theta & -\ddot{\theta}_k^2 \cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



Two adjacent links (CSP chain)
Spherical joint is located at an intersection between a cylinder and a straight line
Position and posture of both axes are given.
CSP spatial chain
O
X
Y
Z
S
P
C
L_P
L_C
$\dot{\omega}_S$
$\dot{\omega}_C$
$\dot{\omega}_{LC}$
$\dot{\omega}_{LP}$
$[R_C]$
$[R_P]$
$[R_{LC}]$
$[R_{LP}]$
s_C
s_P

Translational and angular accelerations of axes of cylindrical and prismatic joints are given as the linear combination of accelerations of driving link inputs as



CSP two adjacent links

$$\ddot{\boldsymbol{L}}_C = [C_{LC}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{LC}, \quad \dot{\boldsymbol{\omega}}_{LC} = [C_{\omega LC}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega LC},$$

$$[\ddot{R}_{RLC}] = [C_{RLC}]\ddot{\boldsymbol{\theta}} + [d_{RLC}]$$

$$\ddot{\boldsymbol{L}}_P = [C_{LP}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{LP}, \quad \dot{\boldsymbol{\omega}}_{LP} = [C_{\omega LP}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega LP},$$

$$[\ddot{R}_{RLP}] = [C_{RLP}]\ddot{\boldsymbol{\theta}} + [d_{RLP}]$$

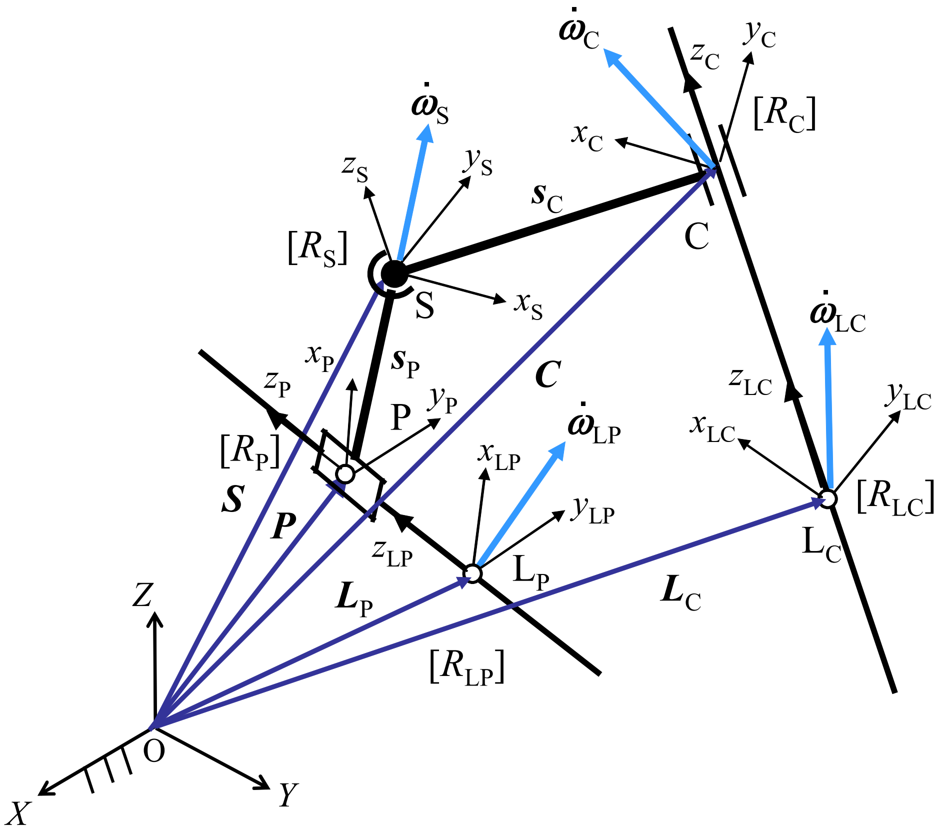
$$\ddot{\boldsymbol{C}} = [C_C]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_C,$$

$$\dot{\boldsymbol{\omega}}_C = [C_{\omega C}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega C}, \quad [\ddot{R}_C] = [C_{RC}]\ddot{\boldsymbol{\theta}} + [d_{RC}]$$

$$\ddot{\boldsymbol{S}} = [C_S]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_S,$$

$$\dot{\boldsymbol{\omega}}_S = [C_{\omega S}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega S}, \quad [\ddot{R}_S] = [C_{RS}]\ddot{\boldsymbol{\theta}} + [d_{RS}]$$

$$\ddot{\boldsymbol{P}} = [C_P]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_P$$



Translational and angular accelerations of axes of cylindrical and prismatic joints are given as the linear combination of accelerations of driving link inputs as

$$\ddot{\boldsymbol{L}}_C = [C_{LC}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{LC}, \quad \dot{\boldsymbol{\omega}}_{LC} = [C_{\omega LC}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega LC},$$

$$[\ddot{R}_{RLC}] = [C_{RLC}]\ddot{\boldsymbol{\theta}} + [d_{RLC}]$$

$$\ddot{\boldsymbol{L}}_P = [C_{LP}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{LP}, \quad \dot{\boldsymbol{\omega}}_{LP} = [C_{\omega LP}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega LP},$$

$$[\ddot{R}_{RLP}] = [C_{RLP}]\ddot{\boldsymbol{\theta}} + [d_{RLP}]$$

⬇

$$\ddot{\boldsymbol{C}} = [C_C]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_C,$$

$$\dot{\boldsymbol{\omega}}_C = [C_{\omega C}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega C}, \quad [\ddot{R}_C] = [C_{RC}]\ddot{\boldsymbol{\theta}} + [d_{RC}]$$

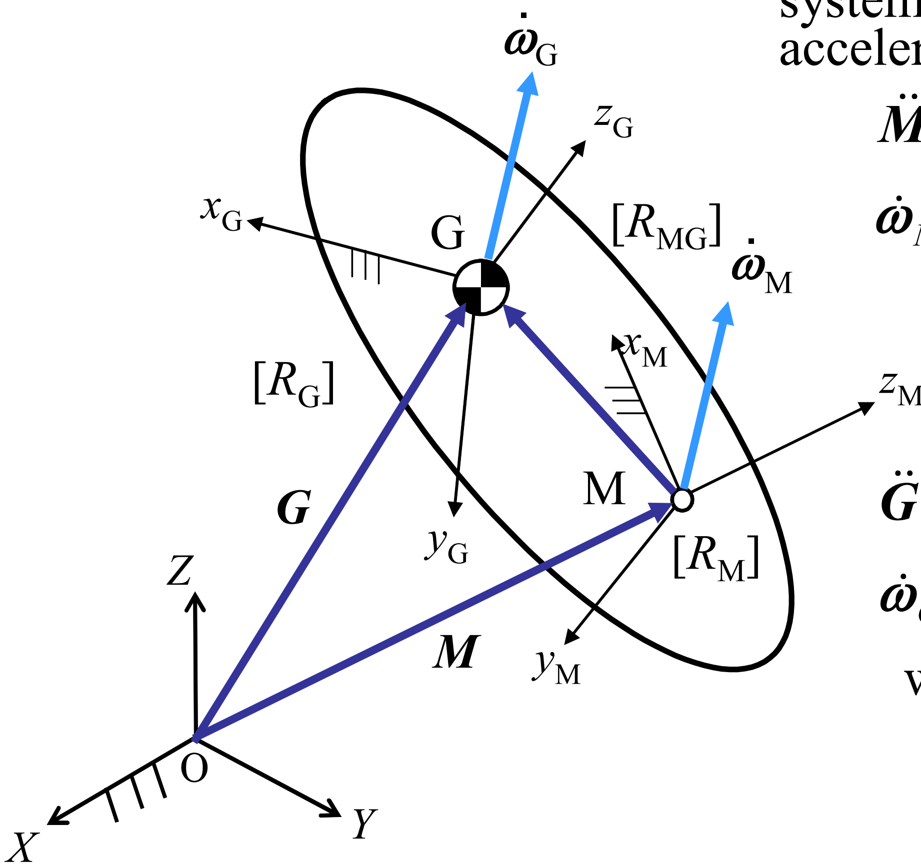
$$\ddot{\boldsymbol{S}} = [C_S]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_S,$$

$$\dot{\boldsymbol{\omega}}_S = [C_{\omega S}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega S}, \quad [\ddot{R}_S] = [C_{RS}]\ddot{\boldsymbol{\theta}} + [d_{RS}]$$

Acceleration coefficients on joints C, S, P can be recursively obtained.

➡ **Dynamics calculation module: FD_CSP_LINKS**

Motion of a coupler link



A coupler

Translational and angular accelerations of moving system are given as the linear combination of accelerations of other driving link inputs as

$$\ddot{\boldsymbol{M}} = [C_M]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_M,$$

$$\dot{\boldsymbol{\omega}}_M = [C_{\omega M}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega M}, \quad [\ddot{R}_M] = [C_{RM}]\ddot{\boldsymbol{\theta}} + [d_{RM}]$$

Coordinate transformation

$$\ddot{\boldsymbol{G}} = [C_G]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_G,$$

$$\dot{\boldsymbol{\omega}}_G = [C_{\omega G}]\ddot{\boldsymbol{\theta}} + \boldsymbol{d}_{\omega G}, \quad [\ddot{R}_G] = [C_{RG}]\ddot{\boldsymbol{\theta}} + [d_{RG}]$$

where

$$[C_G] = [C_M] + [C_{RM}]\boldsymbol{m}, \quad \boldsymbol{d}_G = \boldsymbol{d}_M + [d_{RM}]\boldsymbol{m},$$

$$[C_{\omega G}] = [R_{MG}]^T[C_{\omega M}], \quad \boldsymbol{d}_{\omega G} = [R_{MG}]^T\boldsymbol{d}_{\omega M},$$

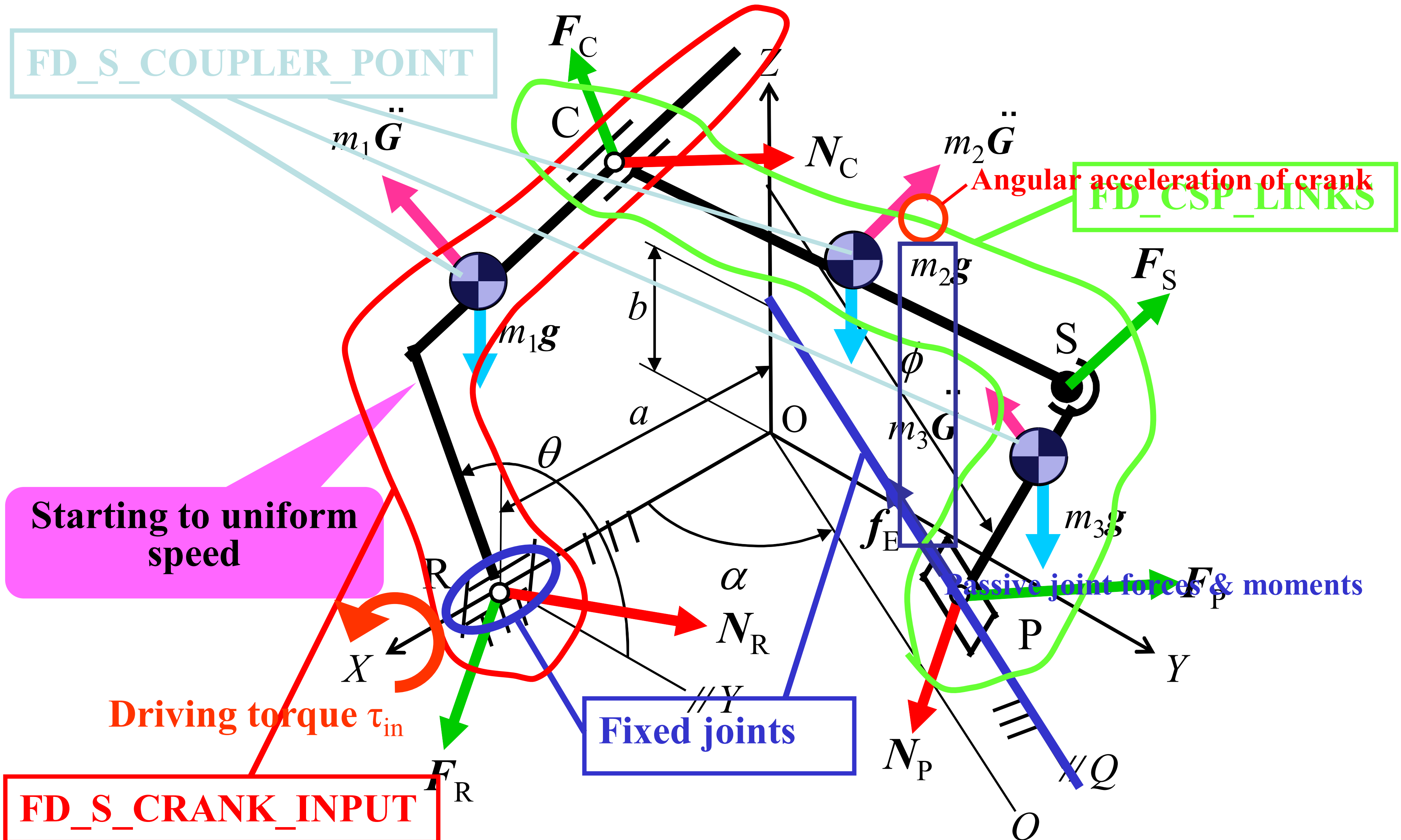
$$[C_{RG}] = [C_{RM}][R_{MG}], \quad [d_{RG}] = [d_{RM}][R_{MG}]$$

Acceleration coefficients on point G can be recursively obtained.

Dynamics calculation module: FD_S_COUPLER_POINT

Example of Forward Dynamics Analysis

Ex. Starting process of an RCSP spatial 4-bar mechanism



RCSP spatial 4-bar mechanism

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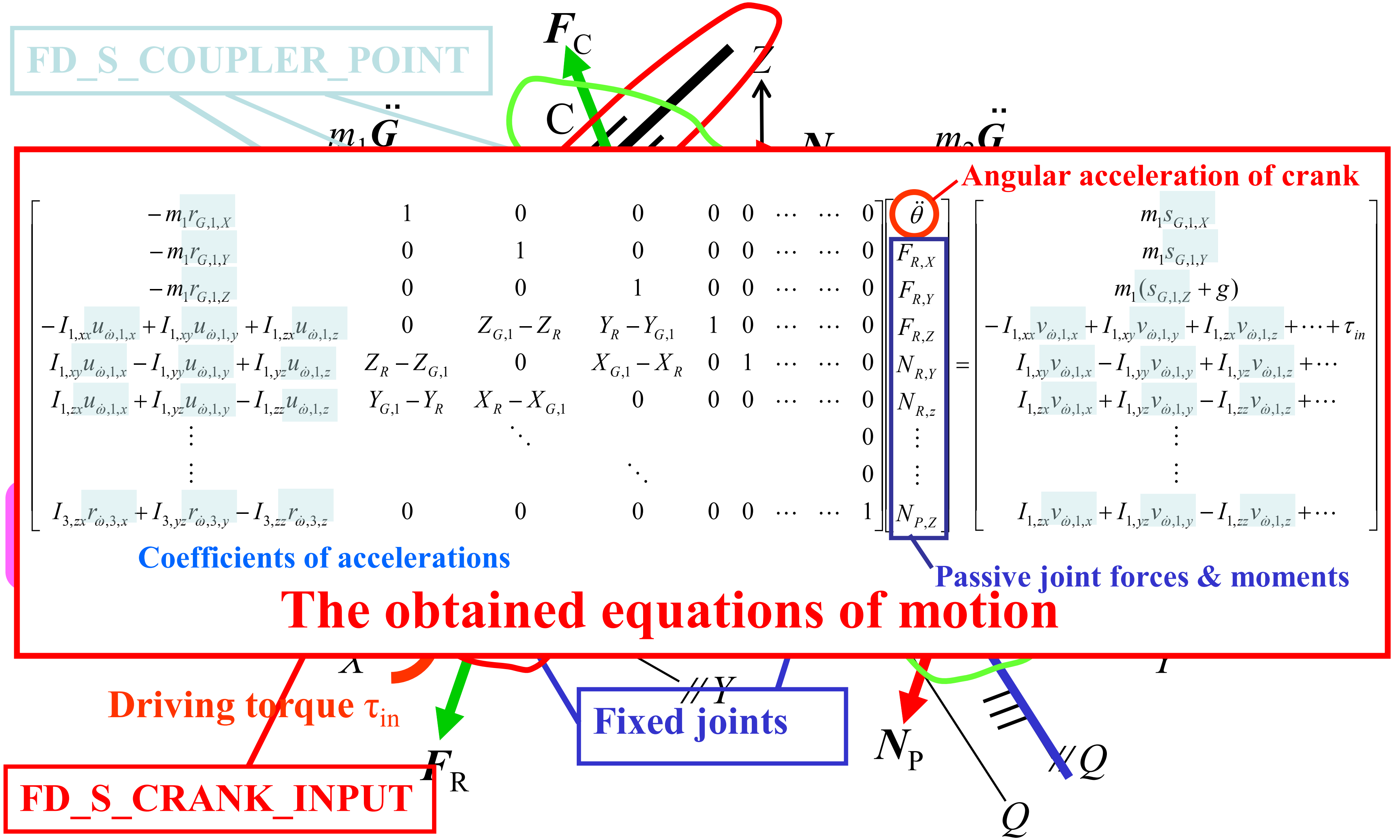


Angular acceleration of crank

$$
\begin{bmatrix}
-m_1 r_{G,1,X} & 1 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 \\
-m_1 r_{G,1,Y} & 0 & 1 & 0 & 0 & 0 & \cdots & \cdots & 0 \\
-m_1 r_{G,1,Z} & 0 & 0 & 1 & 0 & 0 & \cdots & \cdots & 0 \\
-I_{1,xx}u_{\dot{\omega},1,x}+I_{1,xy}u_{\dot{\omega},1,y}+I_{1,zx}u_{\dot{\omega},1,z} & 0 & Z_{G,1}-Z_R & Y_R-Y_{G,1} & 1 & 0 & \cdots & \cdots & 0 \\
I_{1,xy}u_{\dot{\omega},1,x}-I_{1,yy}u_{\dot{\omega},1,y}+I_{1,yz}u_{\dot{\omega},1,z} & Z_R-Z_{G,1} & 0 & X_{G,1}-X_R & 0 & 1 & \cdots & \cdots & 0 \\
I_{1,zx}u_{\dot{\omega},1,x}+I_{1,yz}u_{\dot{\omega},1,y}-I_{1,zz}u_{\dot{\omega},1,z} & Y_{G,1}-Y_R & X_R-X_{G,1} & 0 & 0 & 0 & \cdots & \cdots & 0 \\
\vdots & & \ddots & & & & & & 0 \\
\vdots & & & \ddots & & & & & 0 \\
I_{3,zx}r_{\dot{\omega},3,x}+I_{3,yz}r_{\dot{\omega},3,y}-I_{3,zz}r_{\dot{\omega},3,z} & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta} \\ F_{R,X} \\ F_{R,Y} \\ F_{R,Z} \\ N_{R,Y} \\ N_{R,z} \\ \vdots \\ \vdots \\ N_{P,Z}
\end{bmatrix}
=
\begin{bmatrix}
m_1 s_{G,1,X} \\
m_1 s_{G,1,Y} \\
m_1 (s_{G,1,Z}+g) \\
-I_{1,xx}v_{\dot{\omega},1,x}+I_{1,xy}v_{\dot{\omega},1,y}+I_{1,zx}v_{\dot{\omega},1,z}+\cdots+\tau_{in} \\
I_{1,xy}v_{\dot{\omega},1,x}-I_{1,yy}v_{\dot{\omega},1,y}+I_{1,yz}v_{\dot{\omega},1,z}+\cdots \\
I_{1,zx}v_{\dot{\omega},1,x}+I_{1,yz}v_{\dot{\omega},1,y}-I_{1,zz}v_{\dot{\omega},1,z}+\cdots \\
\vdots \\
\vdots \\
I_{1,zx}v_{\dot{\omega},1,x}+I_{1,yz}v_{\dot{\omega},1,y}-I_{1,zz}v_{\dot{\omega},1,z}+\cdots
\end{bmatrix}
$$

Coefficients of accelerations

Passive joint forces & moments

The obtained equations of motion

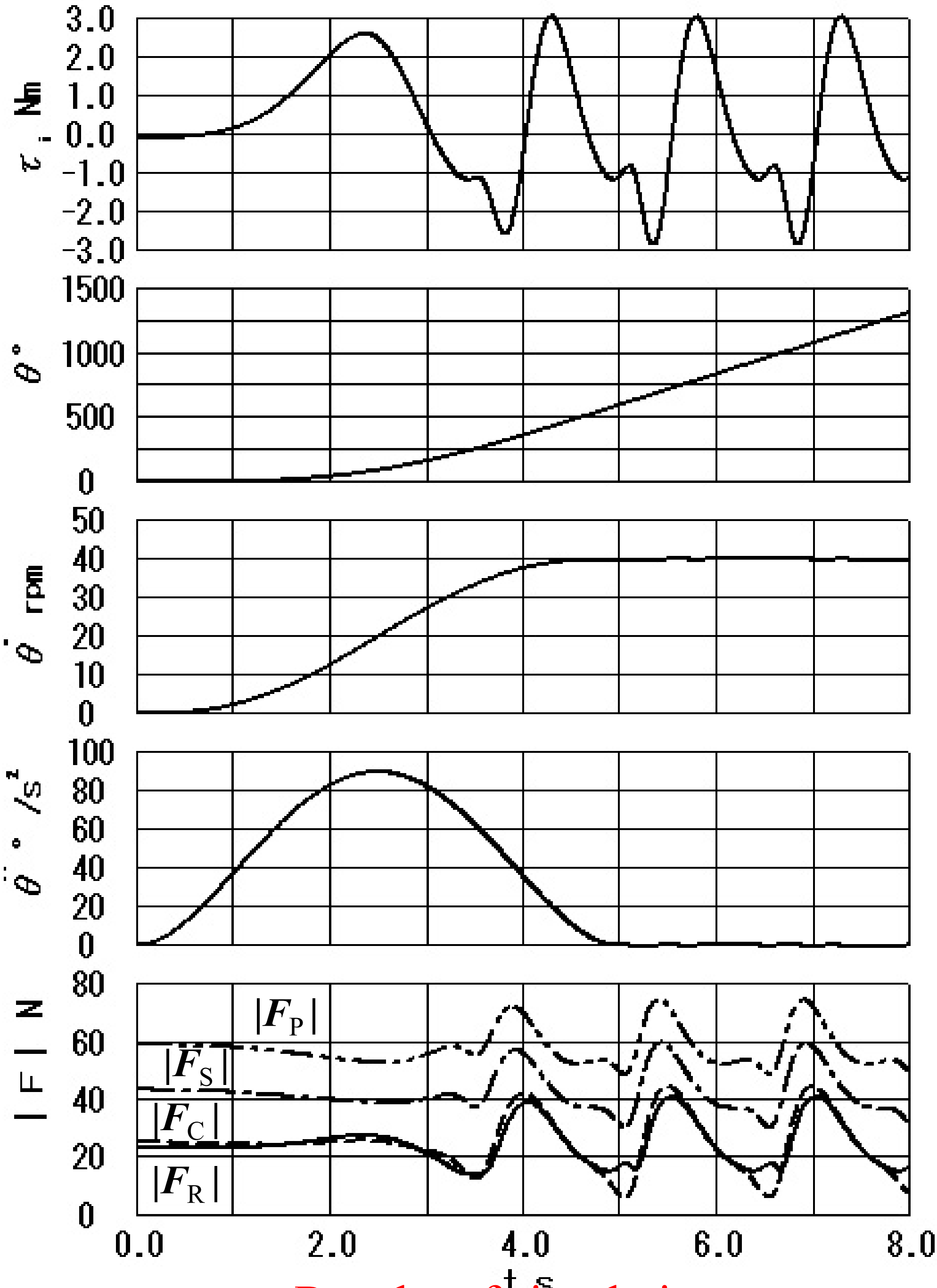
RCSP spatial 4-bar mechanism

Table 1 Mechanical dimensions of an R-C-S-P spatial 4-bar link mechanism

a [m]	0.10	$\boldsymbol{R}$ [m]	$(0.01 \quad 0 \quad 0)^T$
b [m]	0.11	$\boldsymbol{L}_P$ [m]	$(-0.2 \quad 0.3464 \quad 0.11)^T$
$\alpha°$	120	$\boldsymbol{l}_C$ [m]	$(0.1 \quad 0.0 \quad 0.0)^T$
		$\boldsymbol{s}_C$ [m]	$(0.6 \quad 0.0 \quad 0.0)^T$
		$\boldsymbol{s}_C$ [m]	$(0.0 \quad 0.3 \quad 0.0)^T$

Table 2 Mass and inertia parameters of an R-C-S-P spatial 4-bar link mechanism

Link	Mass [kg]	C.O.G [m]	Coordinate system	Inertia tensor [kgm²]
crank	1.524	$\begin{bmatrix} 0.0646 \\ 0.0000 \\ -0.0820 \end{bmatrix}$	$[R_A]$	$\begin{bmatrix} 0.0293 & 0.0000 & -0.0047 \\ 0.0000 & 0.0319 & 0.0000 \\ -0.0047 & 0.0000 & 0.0029 \end{bmatrix}$
coupler	2.698	$\begin{bmatrix} 0.3467 \\ 0.0000 \\ 0.0000 \end{bmatrix}$	$[R_C]$	$\begin{bmatrix} 0.0008 & 0.0000 & 0.0000 \\ 0.0000 & 0.1496 & 0.0000 \\ 0.0000 & 0.0000 & 0.1492 \end{bmatrix}$
output	2.147	$\begin{bmatrix} 0.0000 \\ 0.1189 \\ 0.0000 \end{bmatrix}$	$[R_P]$	$\begin{bmatrix} 0.0340 & 0.0000 & 0.0000 \\ 0.0000 & 0.0015 & 0.0000 \\ 0.0000 & 0.0000 & 0.0334 \end{bmatrix}$



Results of simulation

Driving torque is given by carrying out the inverse dynamics calculation.

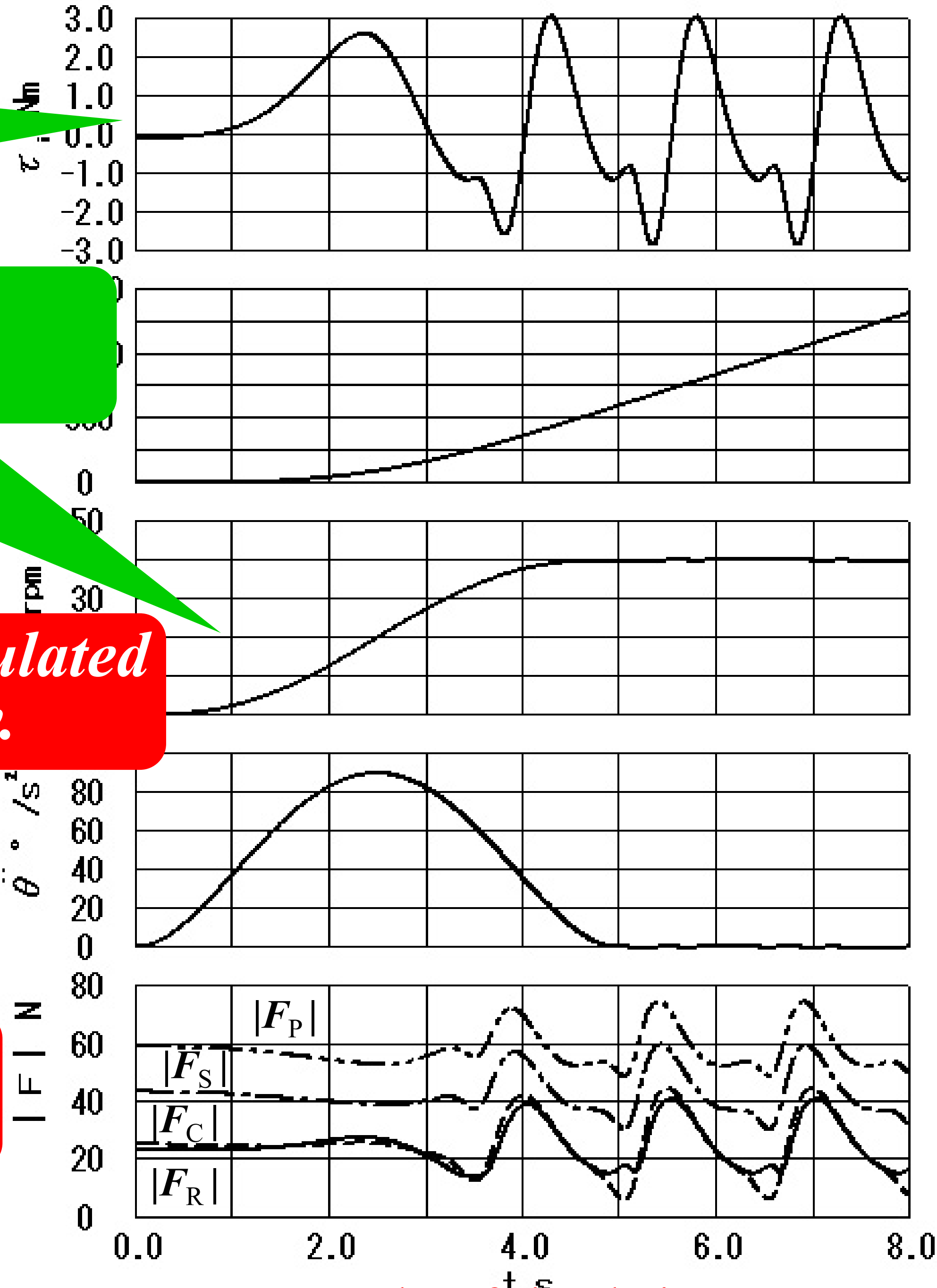
Angular velocity of crank was given as a cycloid function.

α°	120	$\boldsymbol{l}_C$ [m]	(0.1 0.0 [illegible]
		$\boldsymbol{s}_C$ [m]	$(0.6\quad 0.0\quad 0.0)^T$
		$\boldsymbol{s}_C$ [m]	$(0.0\quad 0.3\quad 0.0)^T$

The starting process can be simulated with an adequate accuracy.

crank	1.524	$\begin{bmatrix} 0.0646 \\ 0.0000 \\ -0.0820 \end{bmatrix}$	$[R_A]$	$\begin{bmatrix} 0.0293 & 0.0000 & -0.0047 \\ 0.0000 & 0.0319 & 0.0000 \\ -0.0047 & 0.0000 & 0.0029 \end{bmatrix}$
coupler	2.698	$\begin{bmatrix} 0.3467 \\ 0.0000 \\ \text{[illegible]} \end{bmatrix}$	$[R_C]$	$\begin{bmatrix} 0.0008 & 0.0000 & 0.0000 \\ 0.0000 & 0.1496 & 0.0000 \\ \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \end{bmatrix}$
[illegible]	[illegible]	$\begin{bmatrix} \text{[illegible]} \\ \text{[illegible]} \\ 0.0000 \end{bmatrix}$	[illegible]	$\begin{bmatrix} \text{[illegible]} \\ 0.0000 & 0.0000 & 0.0334 \end{bmatrix}$

The joint forces can also be calculated.



Results of simulation

Forward dynamics analysis can approximately be achieved with the improved systematic kinematics analysis method!

4. Concluding remarks

Dynamics analyses of planar/spatial link mechanisms can be achieved.

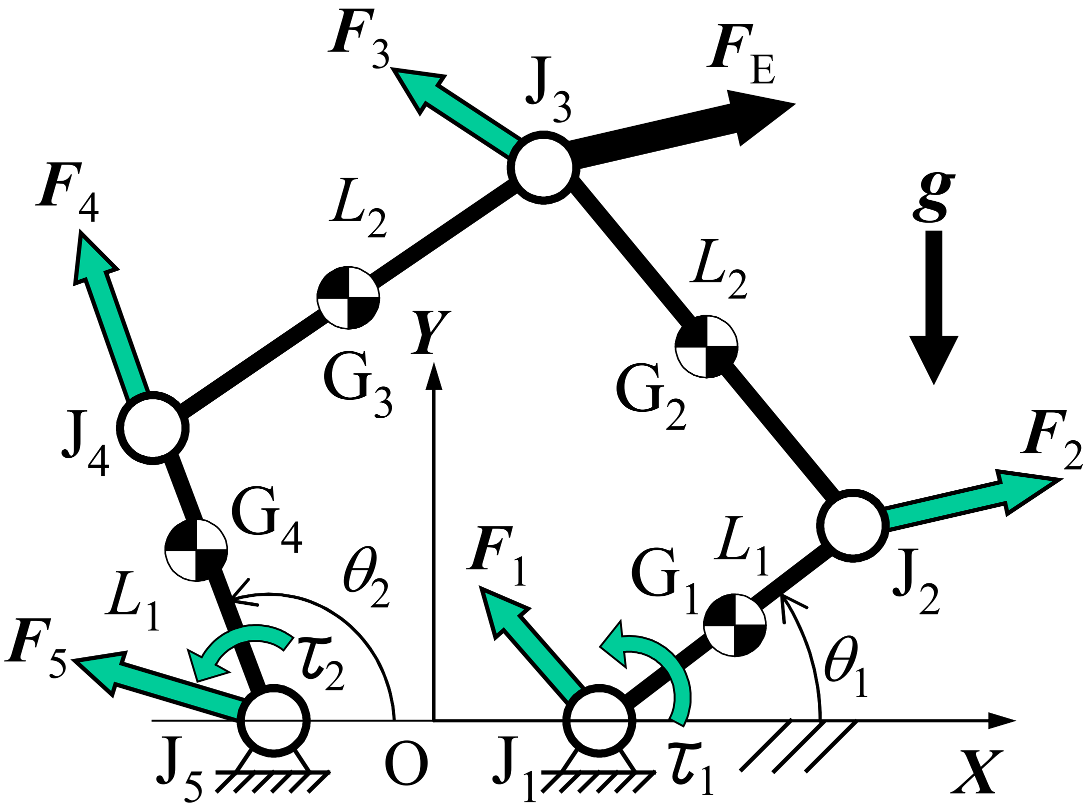
(1)The systematic kinematics analysis method is very useful to calculate translational and angular acceleration of moving link.

(2)Inverse dynamics analysis of planar/spatial link mechanism can easily be achieved only by solving a system of linear equations.

(3)Forward dynamics analysis can approximately be achieved with the improved systematic kinematics analysis method.

Homework 3

Derive system of linear equations for inverse dynamics of a planar parallel mechanism with 2 DOF.



The result will be summarized in A4 size PDF with less than 5 pages and sent to Prof. Iwatsuki via T2SCHOLA by May 8, 2023.