May 8, 2023

Advanced Mechanical Elements
(Lecture 4)

Motion control of redundant link mechanisms

- Optimum motion control to utilize redundancy -

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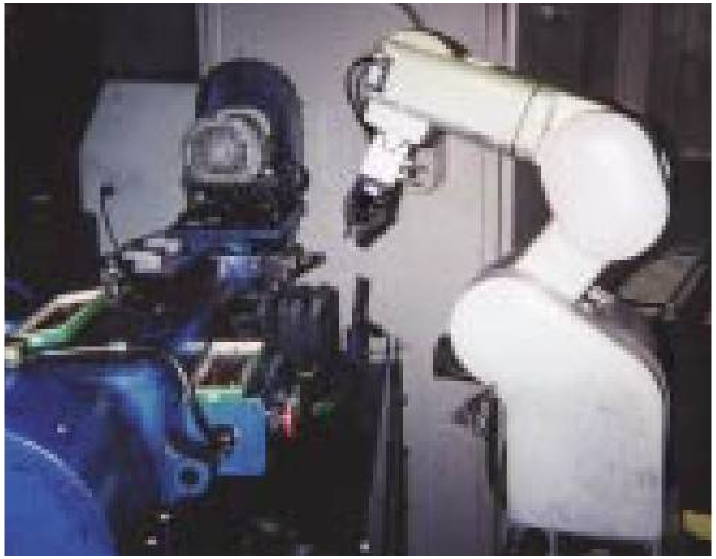
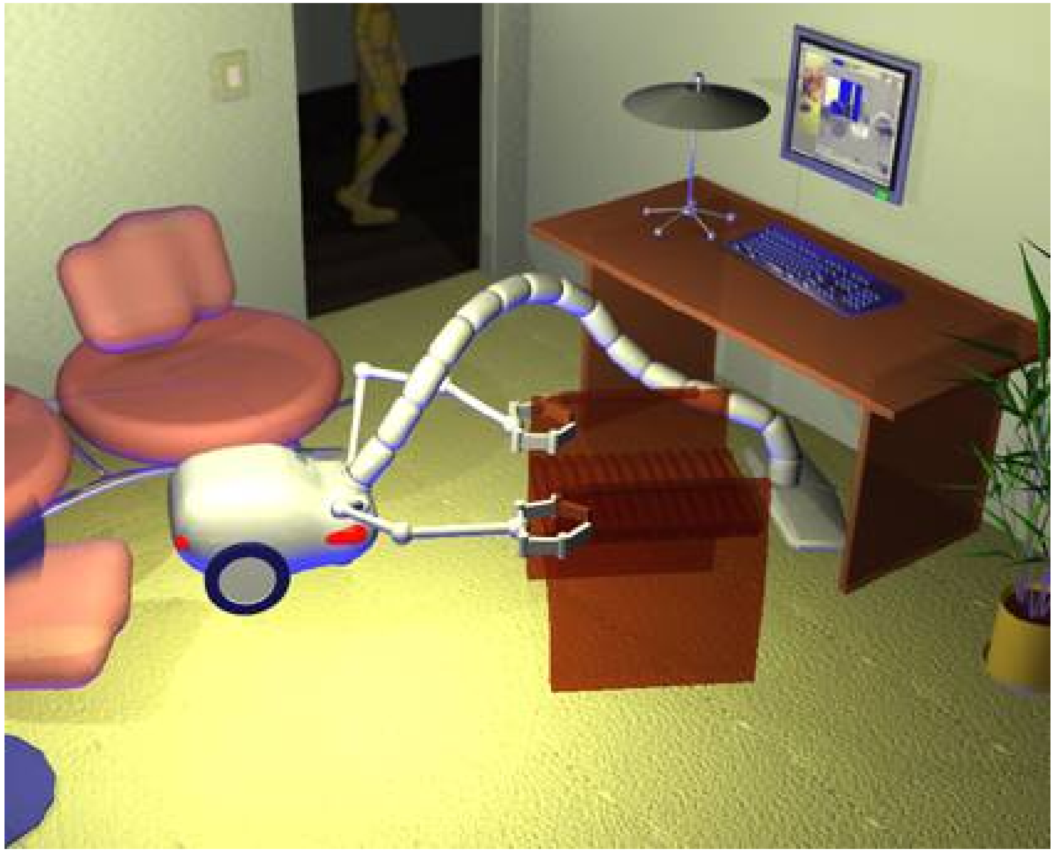
1. Advantages and issues to be solved for redundant robots



A robot in industry



A robot in human daily life

High performance robot which can generate intelligent and flexible motion should be required in the future.

"The robot may not be required in industries
but must be required *in human daily life*."

Remarkable advances in computer techniques (Hardware & Software)

- Microprocessor
- Neural computing

\+

Development of microactuators

- With microfabrication
- Functional materials

There will be a possibility to build a robot which has *hundreds of actuators* and can achieve intelligent motions with adequate flexibility.

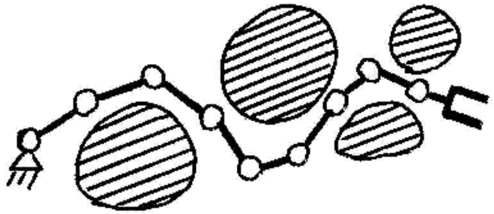
Objectives

Aiming to develop high performance robots which will work in human daily life, the methods to synthesize and control new robotic mechanisms with *hyper redundancy* (HR) should be established.

1.1 Merits of hyper redundant robots

(1) Flexibility

- ***Kinematical* flexibility**
  "To achieve complicated motion"
- ***Structural* flexibility**
  "Soft touch for a human"



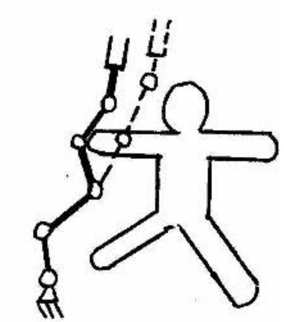
Flexible motion

(2) Cooperative motion

- **Small size**
- **Low power**
- **Simply controlled**

} **actuators**

can *cooperatively generate high power.*



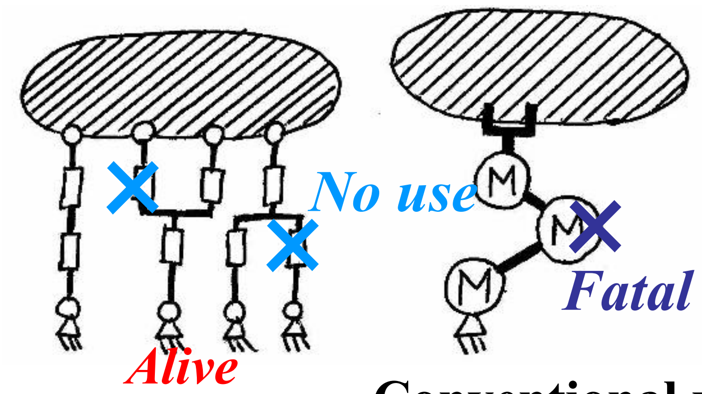
Flexible structure

(3) Reliability

"No problem if a few actuators will be broken down."

(4)Ambiguousness

"Accuracy won't be required."



HR robot **Conventional robot**

1.2 Issues to be solved

(1) How to develop practical and useful small/micro actuators

→・Development of new materials should be expected.

(2) How to synthesize HR mechanisms

→・Not only serial mechanism but also multi-loop mechanism should be systematically synthesized.

(3) How to manufacture HR mechanisms

→・Rigid links and joints won't be available.

・Flexible structural material and new manufacturing method should be expected.

(4) How to control so many Degree of Freedom

→・Optimum control of redundant DOF will be necessary

・Autonomous dispersion control should be adopted in stead of conventional integrated control.

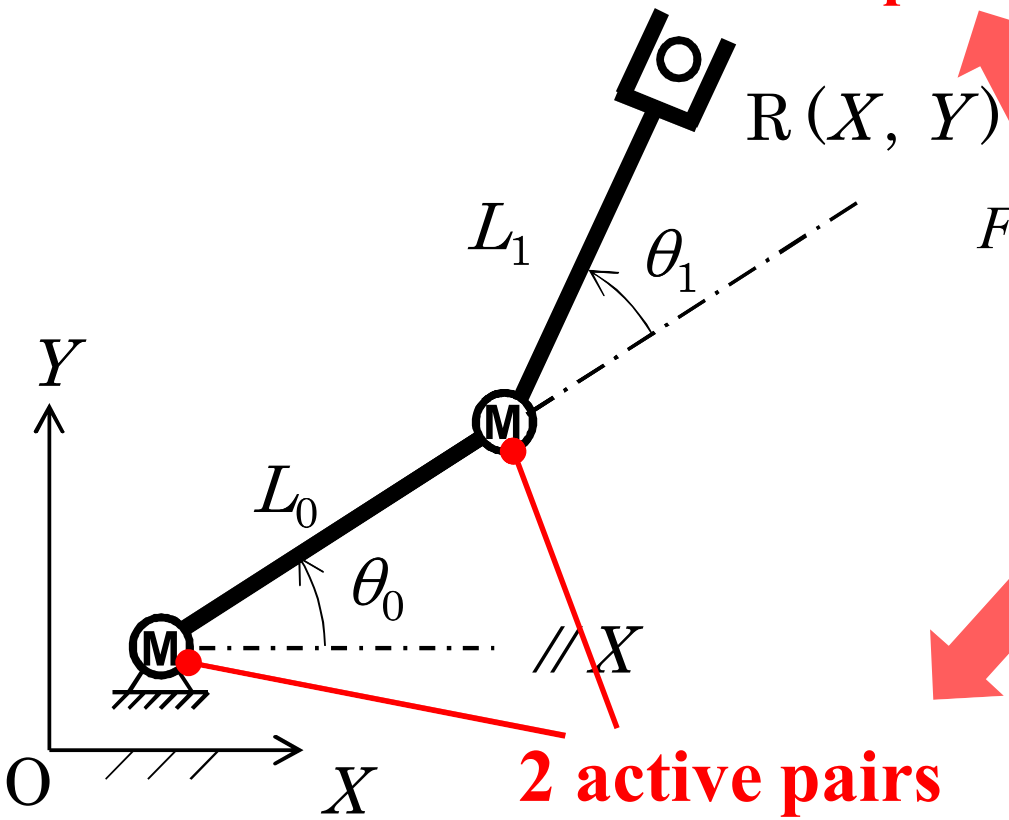
・Over-constraint and over actuator problem should be solved.

2. Forward/inverse kinematics analysis of redundant link mechanisms

- The simplest example : planar serial manipulator

Ex. Planar 2R manipulator (Non redundant)

2 outputs to be controled



2 active pairs (Actuators)

DOF of mechanism:

$$F = F_S(N-1) - \sum_{f=1}^{F_S-1}(F_S - f)J_f$$

$$= 3(3-1) - (3-1)\cdot 2$$

$= 2$ **DOF mechanism**

$F_S = 3$ **Planar motion**

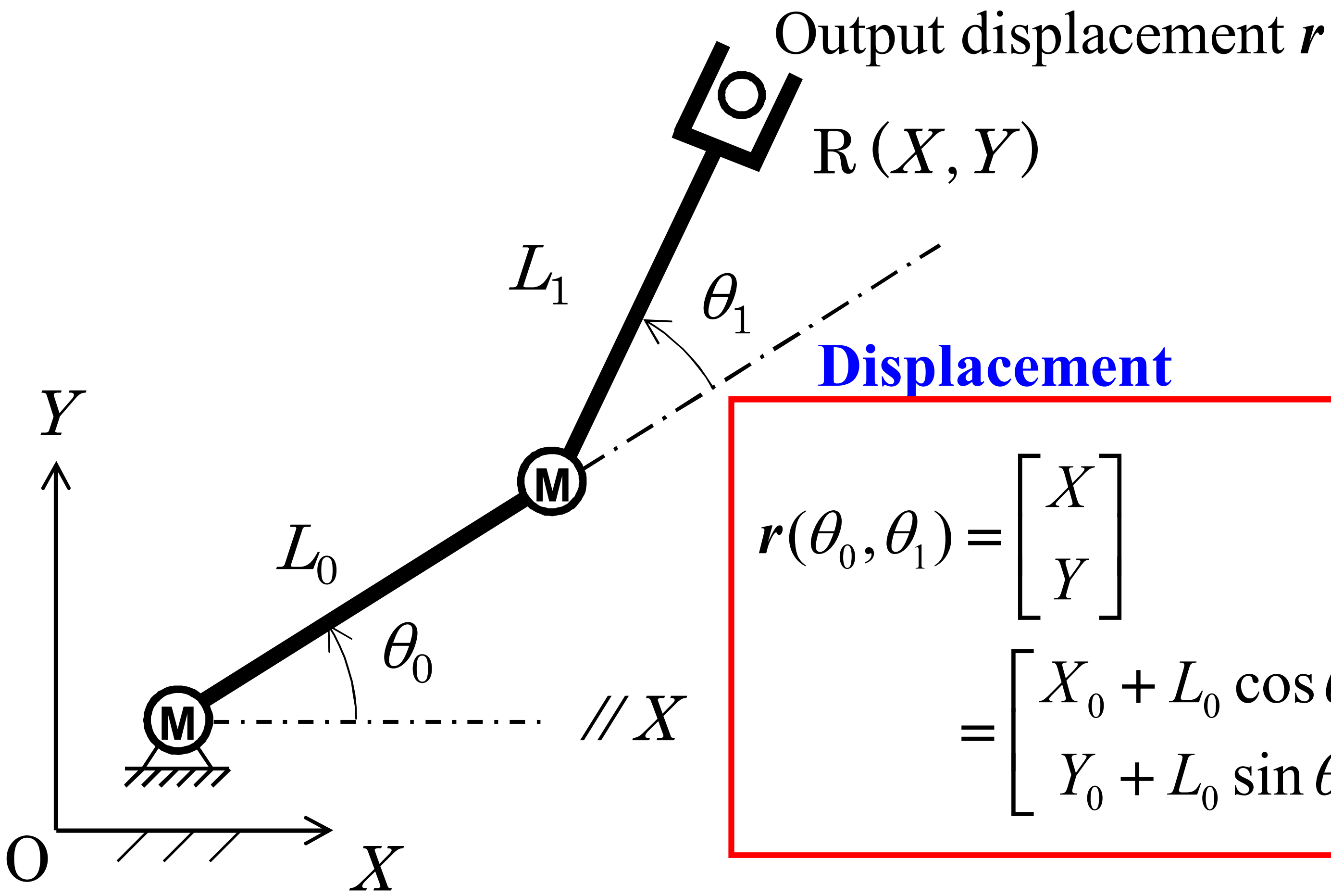
$N = 3$ **Including a frame**

$f = 1$ **Revolute pair (1DOF)**

$J_f = 2$ **2 Revolute pairs**

Ex. Planar 2R manipulator (Non-redundant)

Forward kinematics: ***…… Just review***



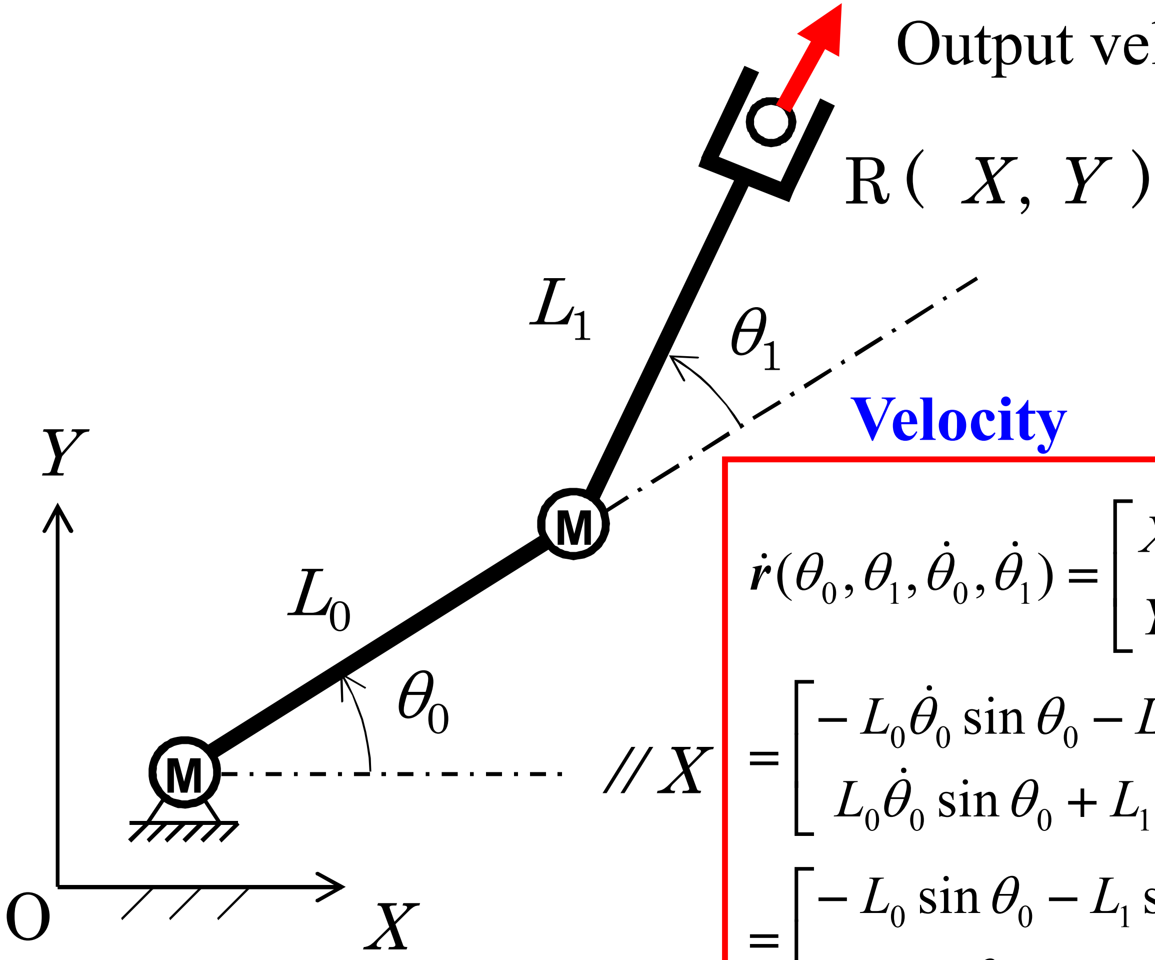
Displacement

$$\boldsymbol{r}(\theta_0, \theta_1) = \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$= \begin{bmatrix} X_0 + L_0 \cos\theta_0 + L_1 \cos(\theta_0 + \theta_1) \\ Y_0 + L_0 \sin\theta_0 + L_1 \sin(\theta_0 + \theta_1) \end{bmatrix}$$

Ex. Planar 2R manipulator (Non-redundant)

Forward kinematics: …… *Just review*



Velocity

$$\dot{\boldsymbol{r}}(\theta_0, \theta_1, \dot{\theta}_0, \dot{\theta}_1) = \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix}$$

$$= \begin{bmatrix} -L_0\dot{\theta}_0 \sin\theta_0 - L_1(\dot{\theta}_0 + \dot{\theta}_1)\sin(\theta_0+\theta_1) \\ L_0\dot{\theta}_0 \sin\theta_0 + L_1(\dot{\theta}_0 + \dot{\theta}_1)\cos(\theta_0+\theta_1) \end{bmatrix}$$

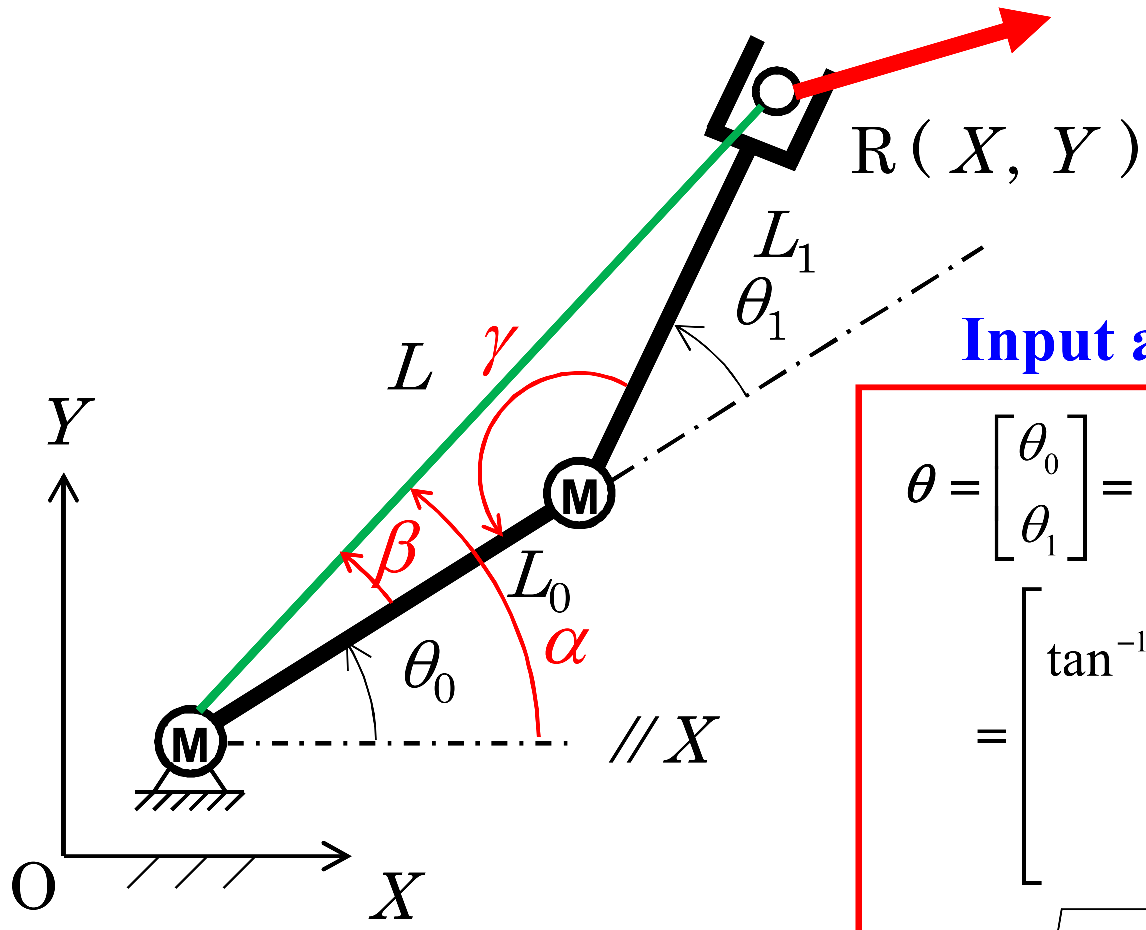
$$= \begin{bmatrix} -L_0 \sin\theta_0 - L_1 \sin(\theta_0+\theta_1) & -L_1 \sin(\theta_0+\theta_1) \\ L_0 \cos\theta_0 + L_1 \cos(\theta_0+\theta_1) & L_1 \cos(\theta_0+\theta_1) \end{bmatrix} \begin{bmatrix} \dot{\theta}_0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$= \boldsymbol{J}\dot{\boldsymbol{\theta}}$$

Jacobian matrix (2x2 Matrix)

Ex. Planar 2R manipulator (Non-redundant)

Inverse kinematics: …… *Just review*



Input angular displacement

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} \alpha - \beta \\ \pi - \gamma \end{bmatrix}$$

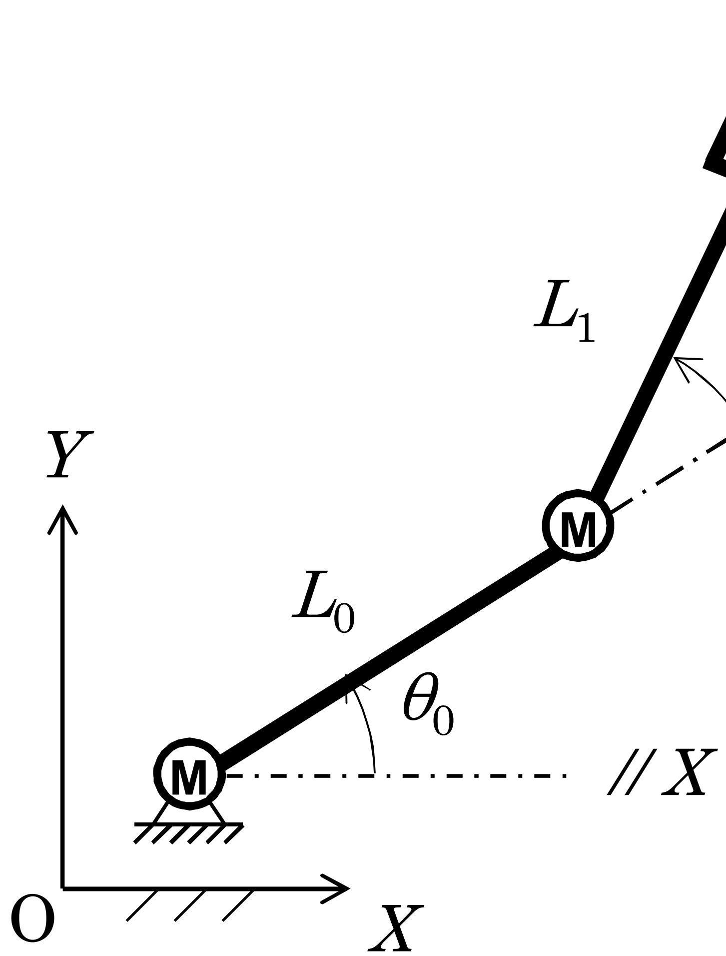
$$= \begin{bmatrix} \tan^{-1} \dfrac{Y - Y_0}{X - X_0} - \cos^{-1} \dfrac{L^2 + L_0^2 - L_1^2}{2 L L_0} \\ \pi - \cos^{-1} \dfrac{L_0^2 + L_1^2 - L^2}{2 L_0 L_1} \end{bmatrix}$$

$$L = \sqrt{(X - X_0)^2 + (Y - Y_0)^2}$$

Input joint angles become settled uniquely.

Ex. Planar 2R manipulator (Non-redundant)

Inverse kinematics: ***…… Just review***



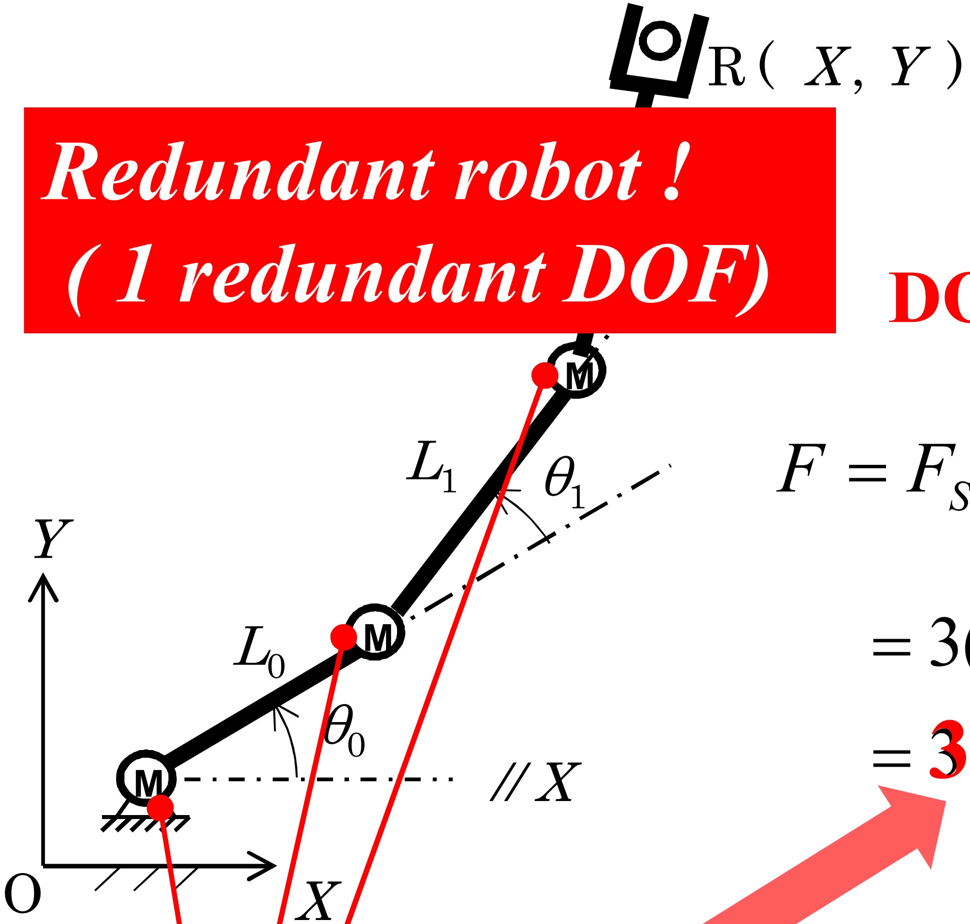
Input angular velocity

$$\dot{\boldsymbol{\theta}} = \boldsymbol{J}^{-1}\dot{\boldsymbol{r}}$$

Inverse of Jacobian matrix (2x2 Matrix)

Input joint angular velocities become settled uniquely.

Ex. Planar 3R manipulator (Redundant)



2 outputs to be controled

DOF of Mechanism:

$$F = F_S(N-1) - \sum_{f=1}^{F_S-1}(F_S - f)J_f$$

$$= 3(4-1) - (3-1)\cdot 3$$

$= 3$ **DOF mechanism**

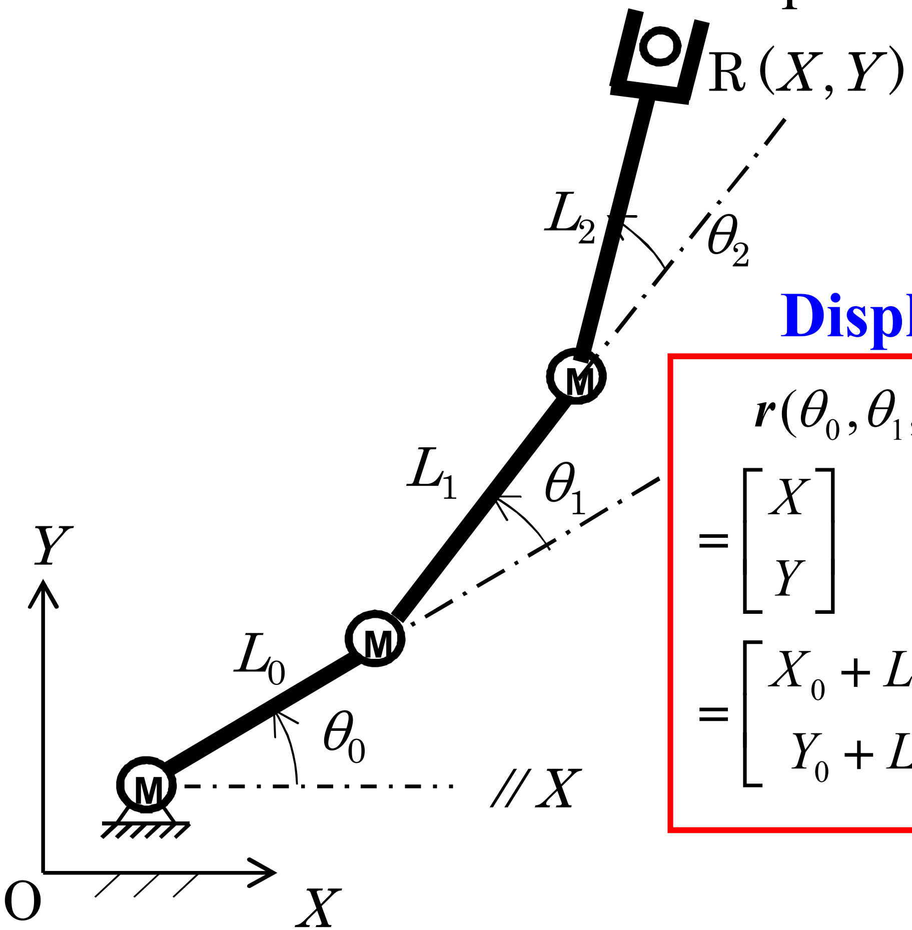
3 active pairs (Actuators)

$F_S = 3$	**Planar motion**
$N = 4$	**Including a frame**
$f = 1$	**Revolute pair (1DOF)**
$J_f = 3$	**3 Revolute pairs**

Ex. Planar 3R manipulator (Redundant)

Forward kinematics: …… *Just review*

Output displacement ***r***

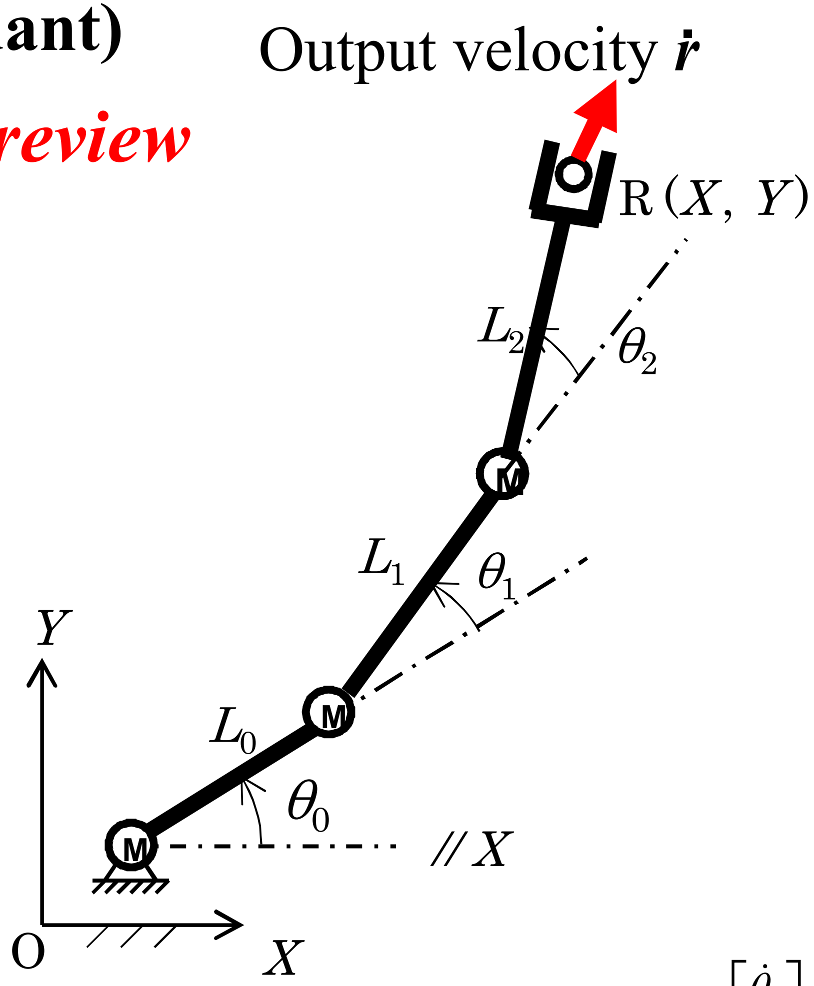


Displacement

$$\boldsymbol{r}(\theta_0, \theta_1, \theta_2) = \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} X_0 + L_0 \cos\theta_0 + L_1 \cos(\theta_0 + \theta_1) + L_2 \cos(\theta_0 + \theta_1 + \theta_2) \\ Y_0 + L_0 \sin\theta_0 + L_1 \sin(\theta_0 + \theta_1) + L_2 \sin(\theta_0 + \theta_1 + \theta_2) \end{bmatrix}$$

Ex. Planar 3R manipulator (Redundant)

Forward kinematics: ...... *Just review*



Velocity

$$\dot{\boldsymbol{r}}(\theta_0,\theta_1,\theta_2,\dot{\theta}_0,\dot{\theta}_1,\dot{\theta}_2)=\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}$$

$$=\begin{bmatrix}-L_0\dot{\theta}_0\sin\theta_0-L_1(\dot{\theta}_0+\dot{\theta}_1)\sin(\theta_0+\theta_1)-L_2(\dot{\theta}_0+\dot{\theta}_1+\dot{\theta}_2)\sin(\theta_0+\theta_1+\theta_2)\\ L_0\dot{\theta}_0\sin\theta_0+L_1(\dot{\theta}_0+\dot{\theta}_1)\cos(\theta_0+\theta_1)+L_2(\dot{\theta}_0+\dot{\theta}_1+\dot{\theta}_2)\cos(\theta_0+\theta_1+\theta_2)\end{bmatrix}$$

$$=\begin{bmatrix}-L_0\sin\theta_0-L_1\sin(\theta_0+\theta_1)-L_2\sin(\theta_0+\theta_1+\theta_2) & -L_1\sin(\theta_0+\theta_1)-L_2\sin(\theta_0+\theta_1+\theta_2) & -L_2\sin(\theta_0+\theta_1+\theta_2)\\ L_0\cos\theta_0+L_1\cos(\theta_0+\theta_1)+L_2\cos(\theta_0+\theta_1+\theta_2) & L_1\cos(\theta_0+\theta_1)+L_2\cos(\theta_0+\theta_1+\theta_2) & L_2\cos(\theta_0+\theta_1+\theta_2)\end{bmatrix}\begin{bmatrix}\dot{\theta}_0\\ \dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$

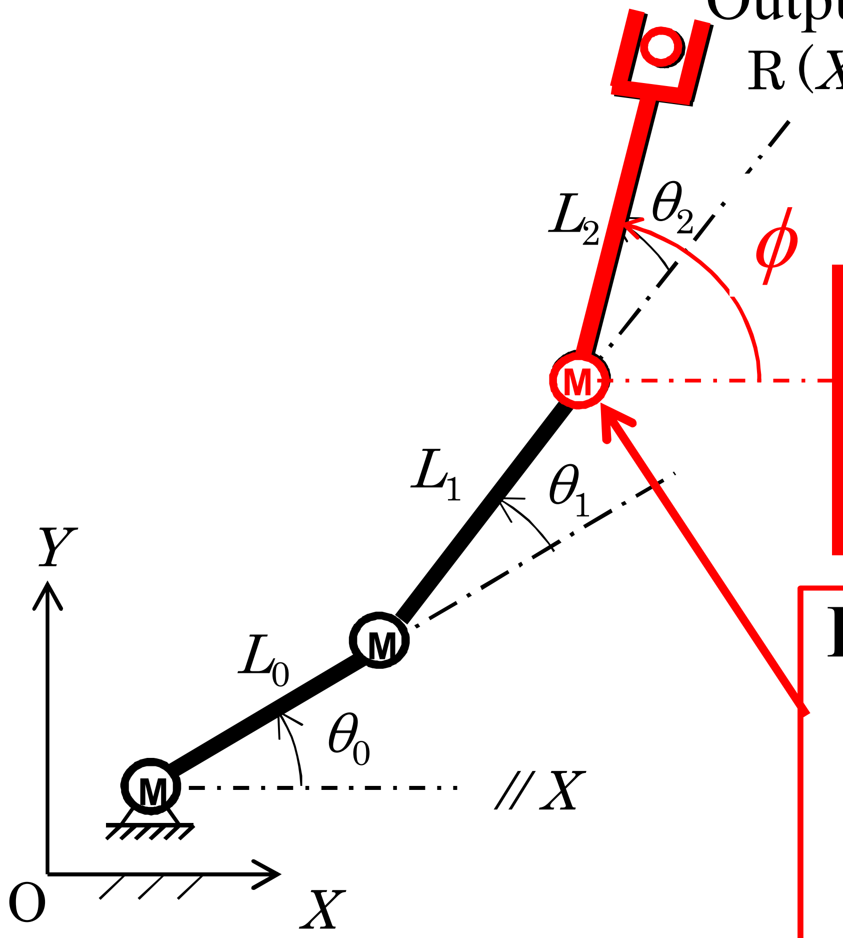
$$=\boldsymbol{J}\dot{\boldsymbol{\theta}}$$

Jacobian matrix

(2x3 Matrix)

Ex. Planar 3R manipulator (Redundant)

Inverse kinematics:



Input angular displacement

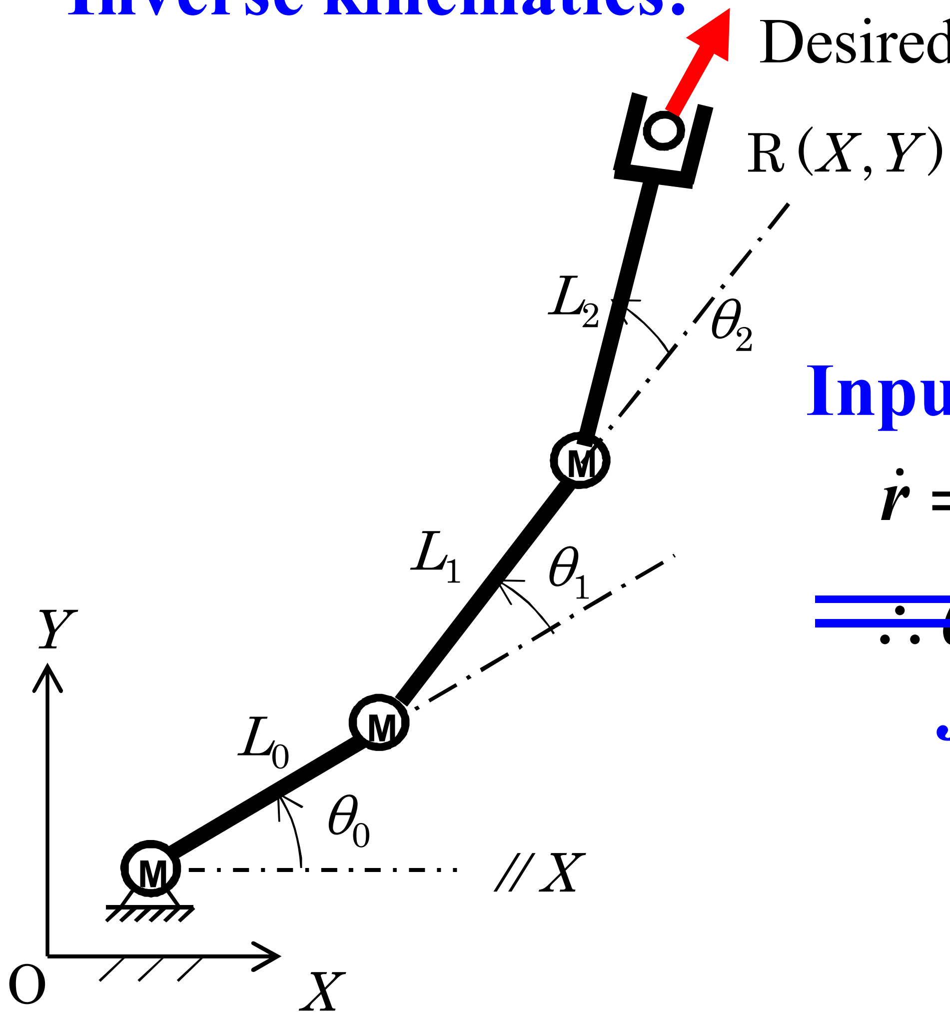
Since this robot has redundant 1 DOF, one joint input should be specified !

Posture angle of end-effector, ϕ, is assumed, the joint position J_2 is then calculated as

$$\boldsymbol{J}_2 = \begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X - L_2 \cos\phi \\ Y - L_2 \sin\phi \end{bmatrix}$$

Ex. Planar 3R manipulator (Redundant)

Inverse kinematics:



Input angular velocity

$$\dot{\boldsymbol{r}} = \boldsymbol{J}\dot{\boldsymbol{\theta}}$$

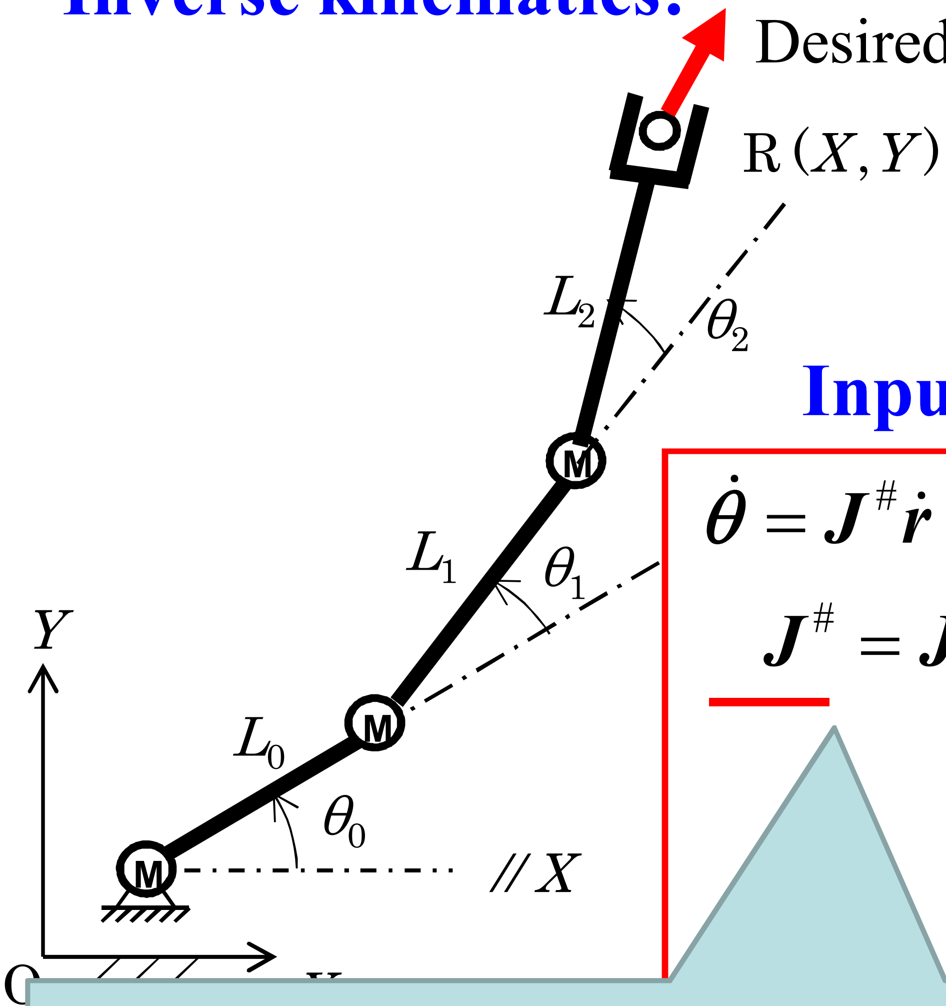
~~$\therefore \dot{\boldsymbol{\theta}} = \boldsymbol{J}^{-1}\dot{\boldsymbol{r}}$?~~

$\boldsymbol{J}^{-1}$ does not exist.

Ex. Planar 3R manipulator (Redundant)

Inverse kinematics:

Desired output velocity $\dot{\boldsymbol{r}}$



Input joint angular velocities are determined as a function of vector $\boldsymbol{a}$.

Input angular velocity

$$\dot{\boldsymbol{\theta}} = \boldsymbol{J}^{\#}\dot{\boldsymbol{r}} + (\boldsymbol{I} - \boldsymbol{J}^{\#}\boldsymbol{J})\boldsymbol{a}$$

$\boldsymbol{J}^{\#} = \boldsymbol{J}^{T}(\boldsymbol{J}\boldsymbol{J}^{T})^{-1}$: **Pseudo inverse of Jacobian matrix (3x2 Matrix)**

$\boldsymbol{a}$: **Arbitrary vector**

$$\boldsymbol{J}\dot{\boldsymbol{\theta}} = \boldsymbol{J}[\boldsymbol{J}^{\#}\dot{\boldsymbol{r}} + (\boldsymbol{I} - \boldsymbol{J}^{\#}\boldsymbol{J})\boldsymbol{a}]$$
$$= \boldsymbol{J}\{\boldsymbol{J}^{T}(\boldsymbol{J}\boldsymbol{J}^{T})^{-1}\dot{\boldsymbol{r}} + [\boldsymbol{I} - \boldsymbol{J}^{T}(\boldsymbol{J}\boldsymbol{J}^{T})^{-1}\boldsymbol{J}]\boldsymbol{a}\}$$
$$= \dot{\boldsymbol{r}}$$

3. How to determine redundant DOF

As seen in inverse kinematics of redundant serial manipulator, we can specify additional variable, namely, ϕ in displacement relationship or vector $\boldsymbol{a}$ in velocity relationship as redundant inputs.

⬇

In other words, we have to determine the redundant inputs based on some criteria.

⬇

We can determine the redundant inputs so as to realize more excellent control.

“The optimum motion control”

In order to utilize the redundant DOF, a certain objective function should be minimized or maximized by adjusting redundant inputs ϕ or a while the robot generate the desired output motion r or $\dot{r}$.

Various objective functions:

(1)Kinematical performances

- **Maximize speed**
- **Dexterous motion**

(2)Statics/dynamics performances

- **Maximize output force**
- **Minimize actuator torques**

4. Optimum motion control of serial manipulator based on dexterity

4.1 How to evaluate dexterity

Dexterity = Ability to achieve complicated motion

(1)Evaluation based on workspace

Limitation of end-effector motion

- **Workspace**
- **Accessible workspace**

  ○Easy to understand

  (How can end-effector reach desired position?)

  ▲Difficult to calculate

  ▲Strongly dependent on end-effector length

(2)Evaluation based on manipulability

Feasible velocity of end-effector

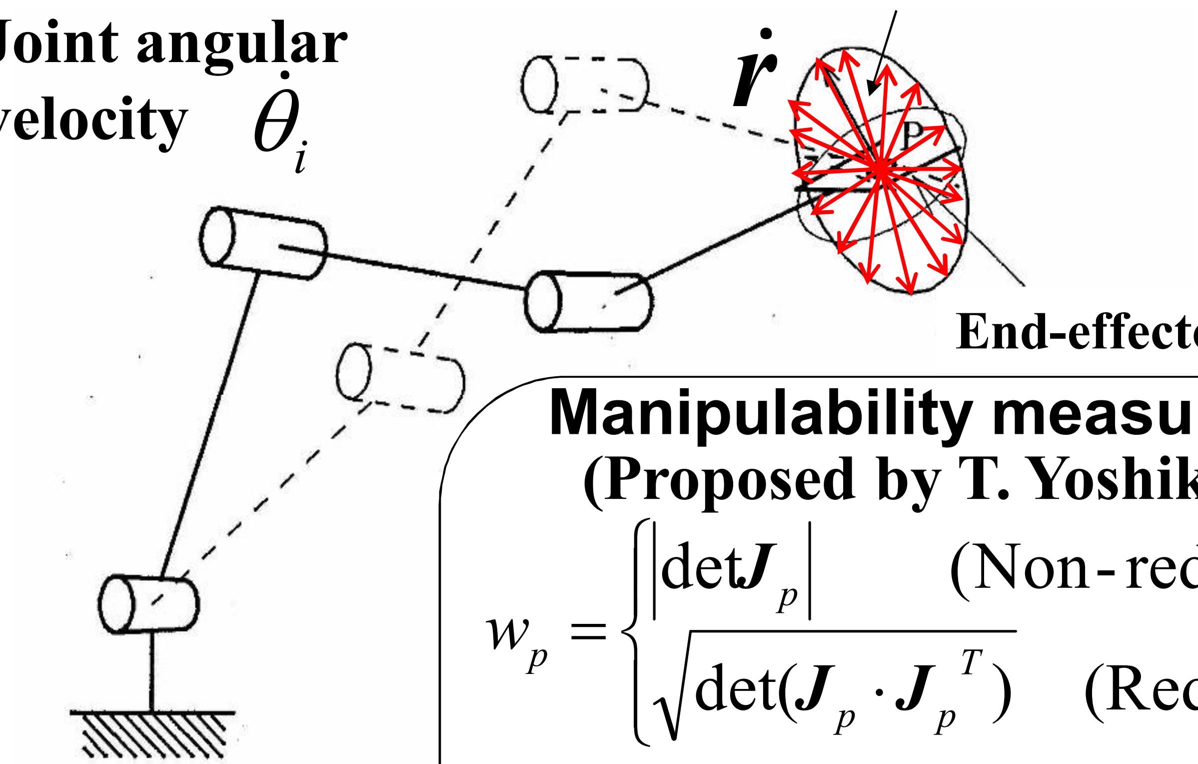
- **Manipulability measure**
- **Condition number of Jacobian matrix**

  ○Easy to calculate

  ▲Difficult to understand

4.2 Manipulability measure

Ellipsoid drawn by the output velocities
(Input joint velocity will be linearly converted with J_p)



Serial manipulator

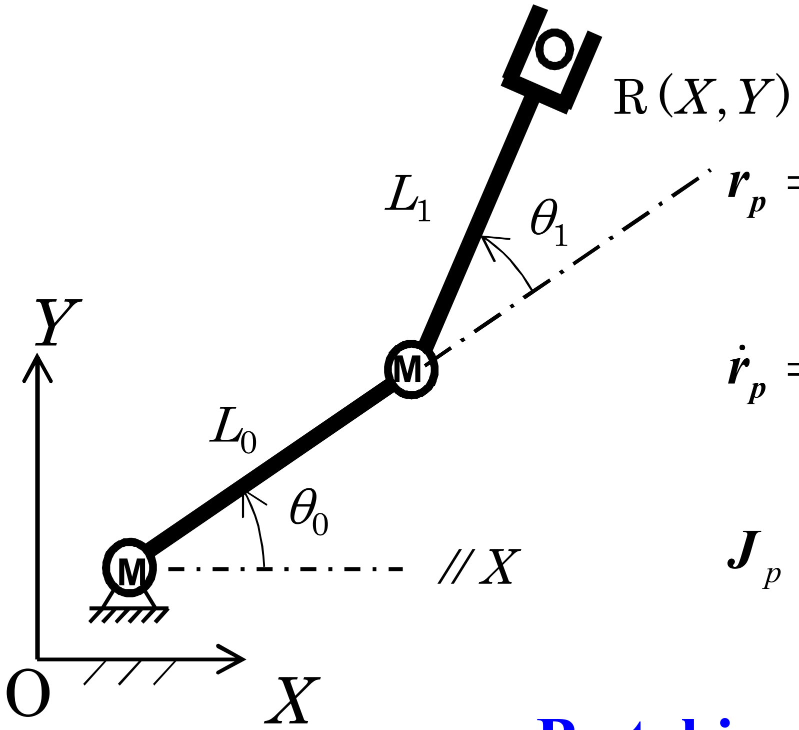
Manipulability measure :
(Proposed by T. Yoshikawa)

$$w_p = \begin{cases} \left|\det \boldsymbol{J}_p\right| & \text{(Non-redundant)} \\ \sqrt{\det(\boldsymbol{J}_p \cdot \boldsymbol{J}_p^{\,T})} & \text{(Redundant)} \end{cases}$$

$\boldsymbol{J}_p$: Jacobian matrix $\quad (\dot{\boldsymbol{r}} = \boldsymbol{J}_p \dot{\boldsymbol{\theta}})$
taking account of posture

"Proportional to feasible velocities in all directions"

Ex. Planar 2R manipulator (Non-redundant)



$$\boldsymbol{r_p} = \begin{bmatrix} X \\ Y \\ \phi \end{bmatrix} = \begin{bmatrix} L_0 \cos\theta_0 + L_1 \cos(\theta_0 + \theta_1) \\ L_0 \sin\theta_0 + L_1 \sin(\theta_0 + \theta_1) \\ \theta_0 + \theta_1 \end{bmatrix}$$

$$\dot{\boldsymbol{r}}_{\boldsymbol{p}} = \boldsymbol{J}_p \dot{\boldsymbol{\theta}}$$

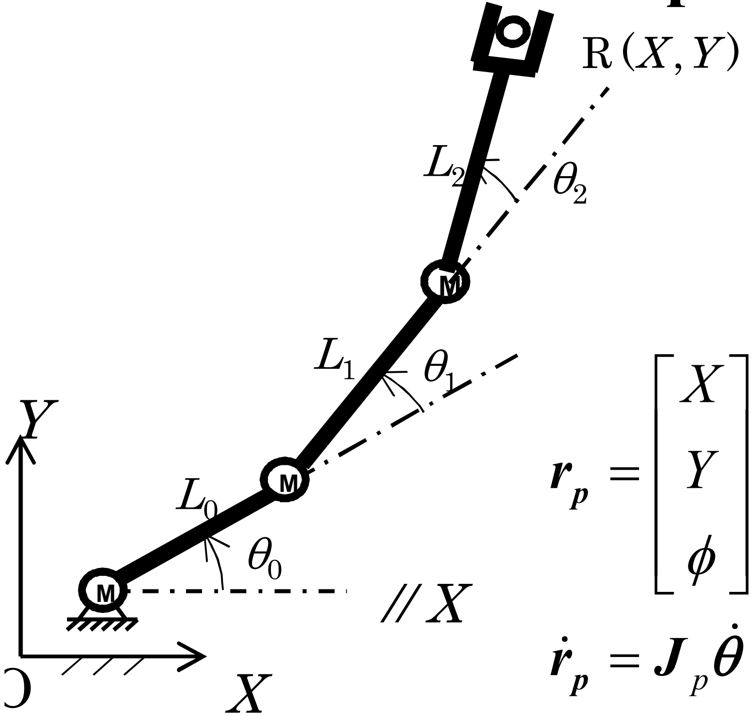
$$\boldsymbol{J}_p = \begin{bmatrix} -L_0 \sin\theta_0 - L_1 \sin(\theta_0 + \theta_1) & -L_1 \sin(\theta_0 + \theta_1) \\ L_0 \cos\theta_0 + L_1 \cos(\theta_0 + \theta_1) & L_1 \cos(\theta_0 + \theta_1) \\ 1 & 1 \end{bmatrix}$$

By taking account of end-effector's posture, this robot is assumed underactuator mechanism.

$$w_{\boldsymbol{p}} = 0$$

The 2R-serial manipulator has zero dexterity.

Ex. Planar 3R manipulator (redundant)



$$\boldsymbol{r}_p = \begin{bmatrix} X \\ Y \\ \phi \end{bmatrix} = \begin{bmatrix} L_0\cos\theta_0 + L_1\cos(\theta_0+\theta_1) + L_2\cos(\theta_0+\theta_1+\theta_2) \\ L_0\sin\theta_0 + L_1\sin(\theta_0+\theta_1) + L_2\sin(\theta_0+\theta_1+\theta_2) \\ \theta_0+\theta_1+\theta_2 \end{bmatrix}$$

$$\dot{\boldsymbol{r}}_p = \boldsymbol{J}_p \dot{\boldsymbol{\theta}}$$

$$\boldsymbol{J}_p = \begin{bmatrix} -L_0\sin\theta_0 - L_1\sin(\theta_0+\theta_1) - L_2\sin(\theta_0+\theta_1+\theta_2) & -L_1\sin(\theta_0+\theta_1) - L_2\sin(\theta_0+\theta_1+\theta_2) & -L_2\sin(\theta_0+\theta_1+\theta_2) \\ L_0\cos\theta_0 + L_1\cos(\theta_0+\theta_1) + L_2\cos(\theta_0+\theta_1+\theta_2) & L_1\cos(\theta_0+\theta_1) + L_2\cos(\theta_0+\theta_1+\theta_2) & L_2\cos(\theta_0+\theta_1+\theta_2) \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} w_p &= \left|\det \boldsymbol{J}_p\right| \\ &= L_0 L_1 \left|\cos\theta_0 \sin(\theta_0+\theta_1) - \sin\theta_0\cos(\theta_0+\theta_1)\right| \\ &= L_0 L_1 \left|\sin\theta_1\right| \end{aligned}$$

Therefore, the manipulator can take $\theta_1 = \pi/2$, w_p takes it maximum $w_{p,\max} = L_0 L_1$

4.3 Optimum configuration to maximize dexterity

For 3R-manipulator (1 redundant DOF), the configuration to reach desired output position can be calculated by giving ϕ.

↓

To search optimum ϕ to maximize w_p
(One-dimensional optimization)

As same procedure:
For 4R-manipulator (2 redundant DOF), the configuration to reach desired output position can be calculated by giving ϕ_1, ϕ_2, .

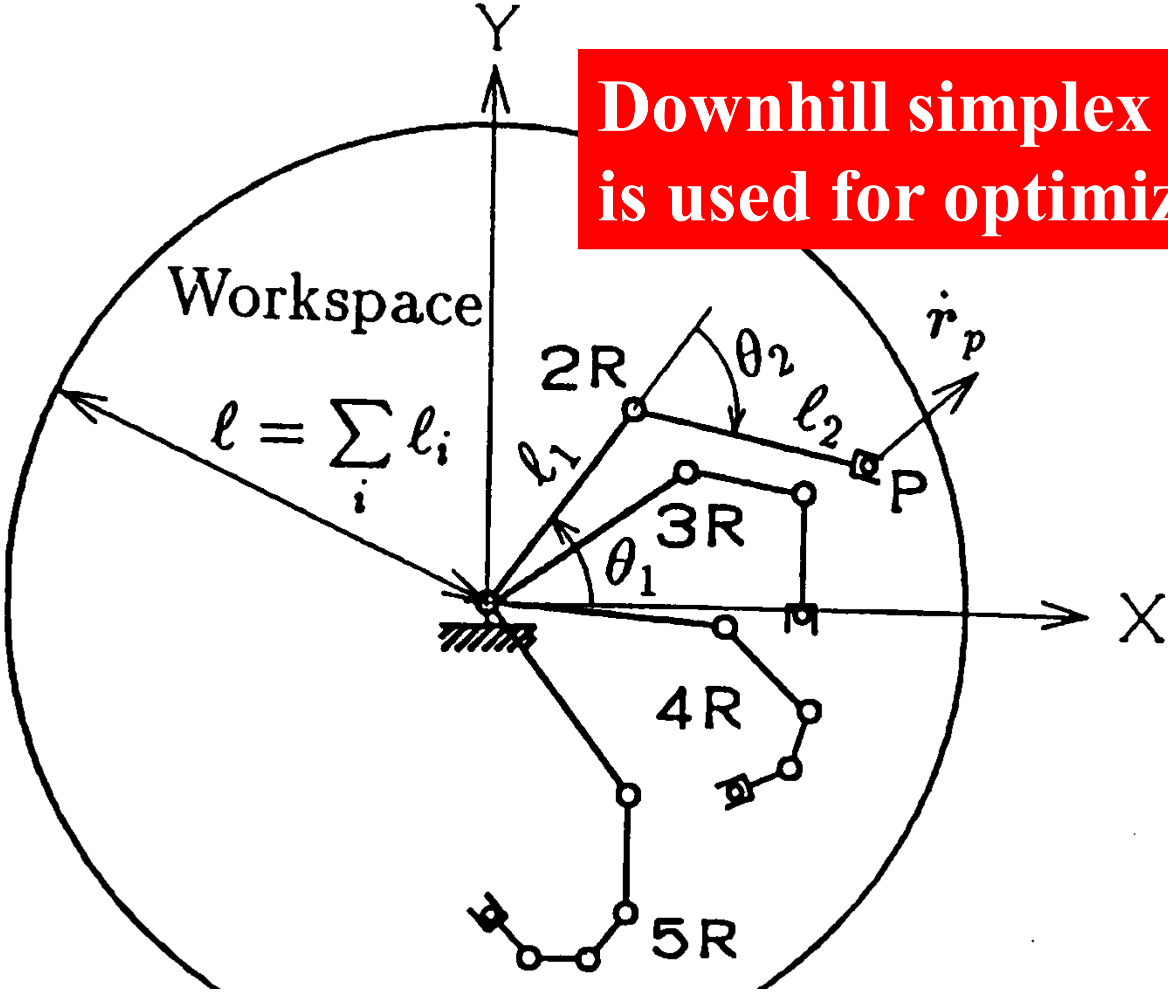
↓

To search optimum ϕ_1, ϕ_2, to maximize w_p
(Two-dimensional optimization)

⋮

For nR-manipulator

Downhill simplex method is used for optimization.



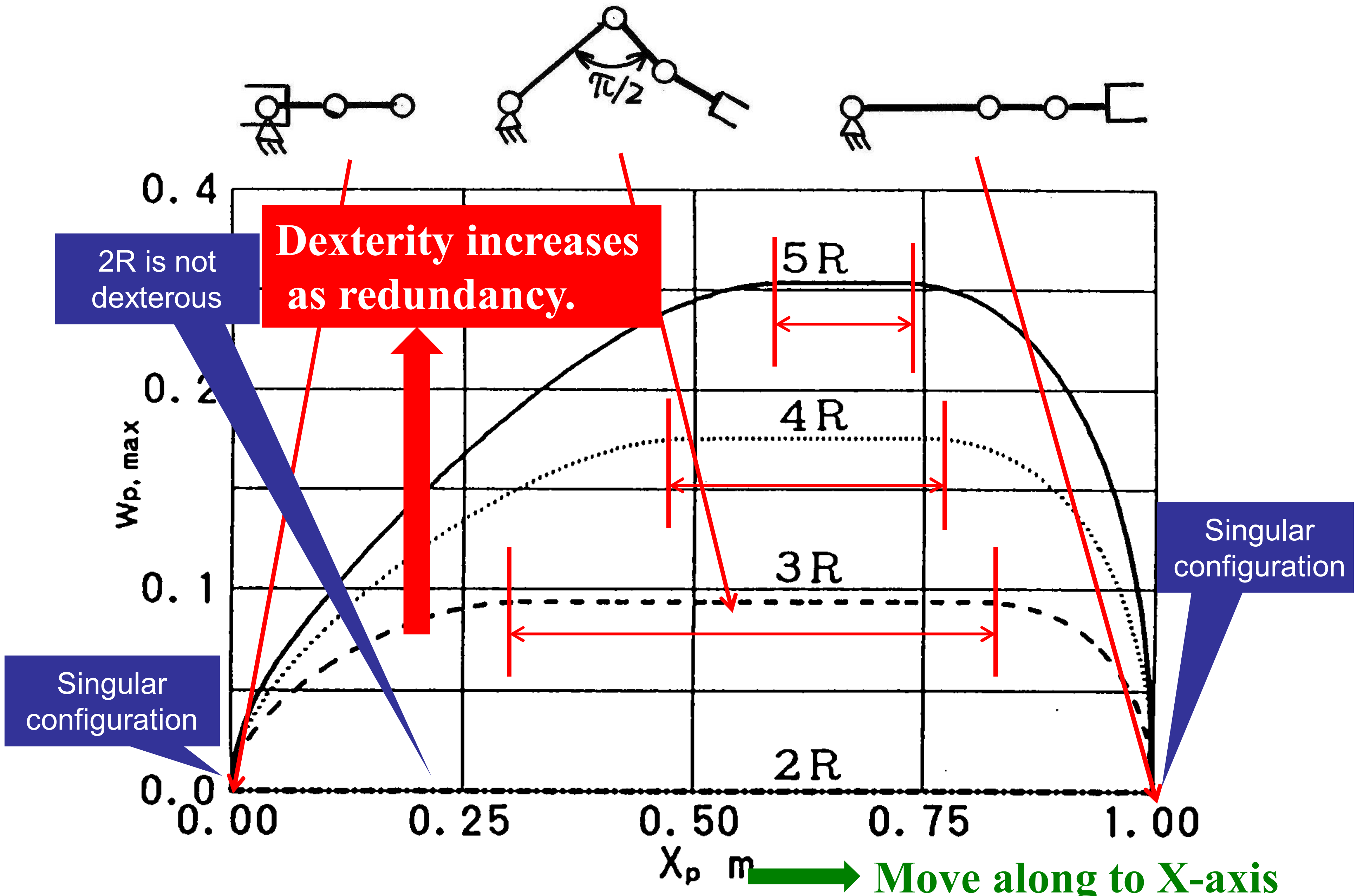
Tested manipulators with quite same workspace

$2R$: $\ell_1 : \ell_2 = 1 : 1$

$3R$: $\ell_1 : \ell_2 : \ell_3 = 1 : 0.5 : 0.5$

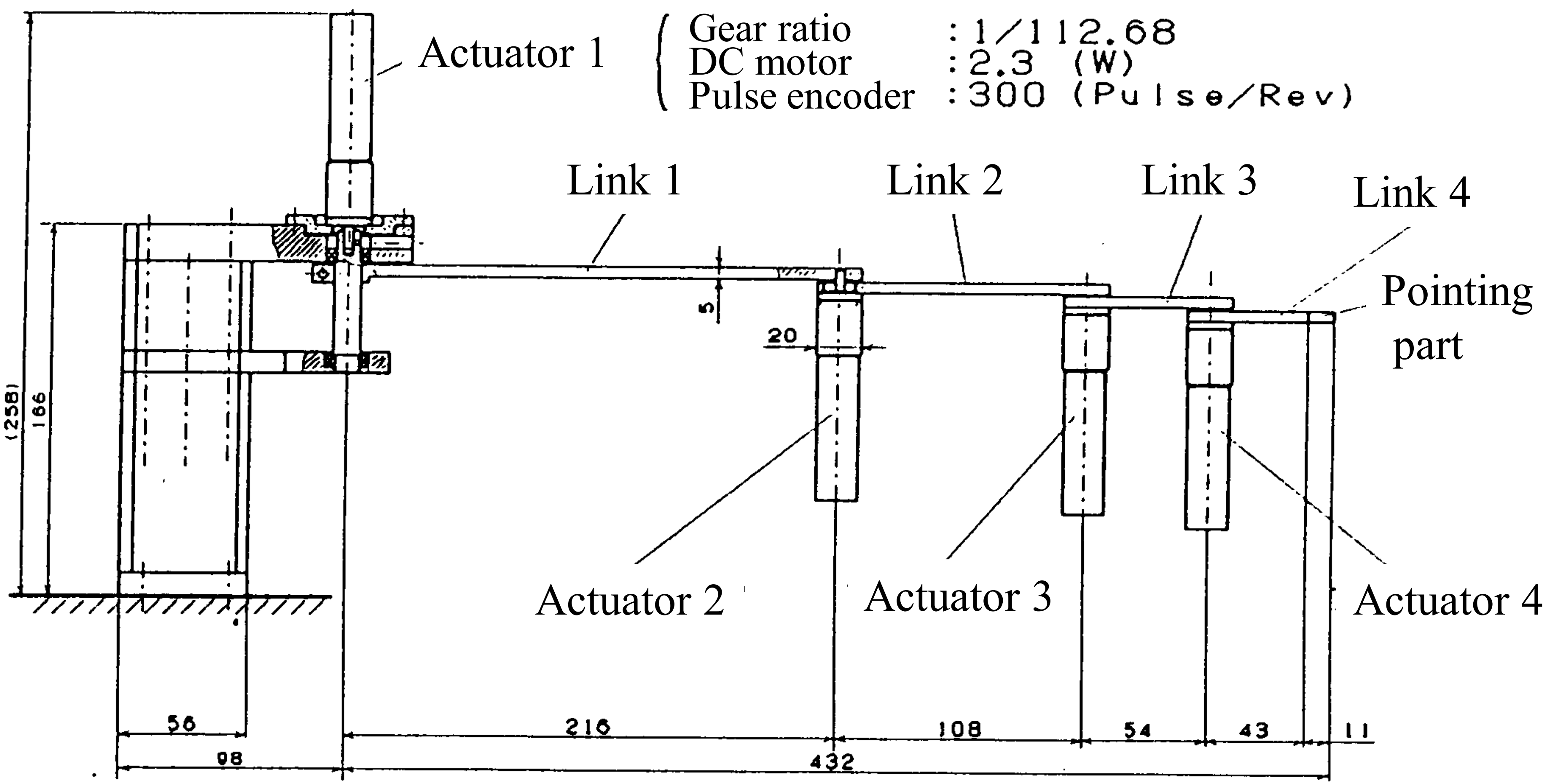
$4R$: $\ell_1 : \ell_2 : \ell_3 : \ell_4 = 1 : 0.5 : 0.25 : 0.25$

$5R$: $\ell_1 : \ell_2 : \ell_3 : \ell_4 : \ell_5 = 1 : 0.5 : 0.25 : 0.125 : 0.125$

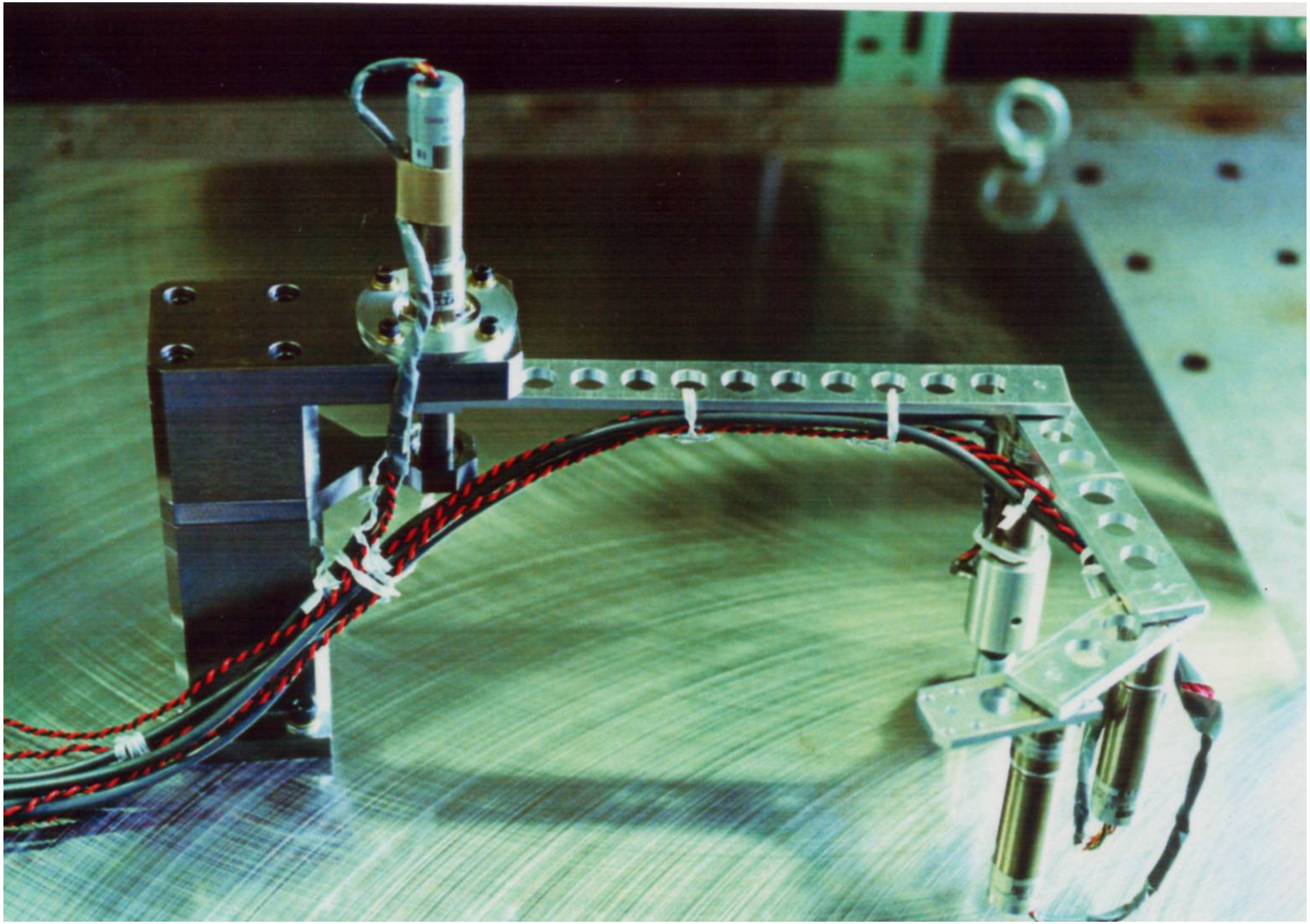


The maximum manipulability measure of serial planar manipulators

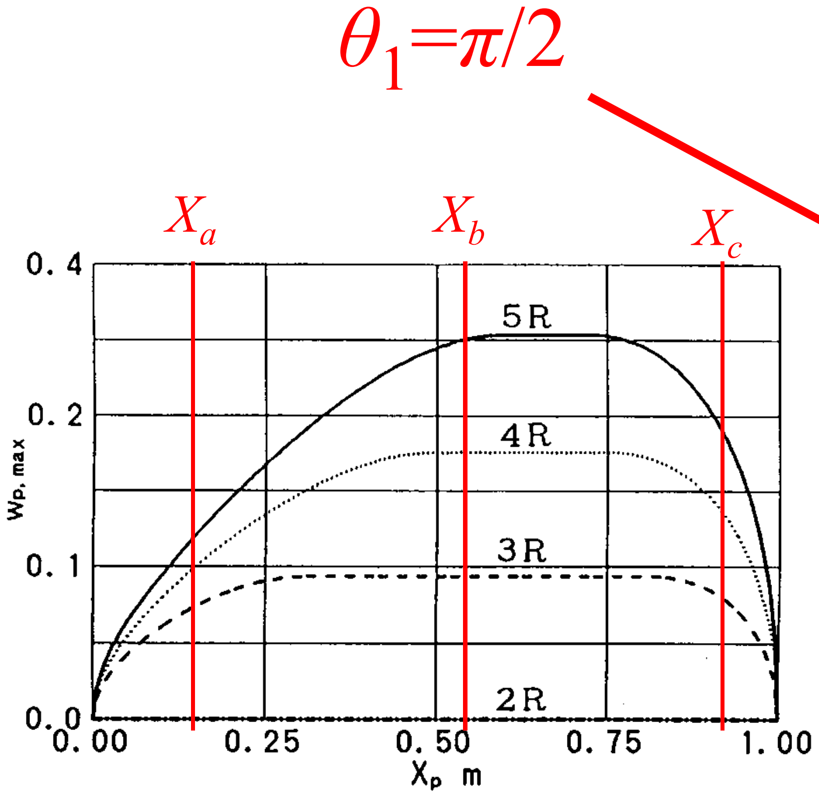
4.4 Motion control experiments

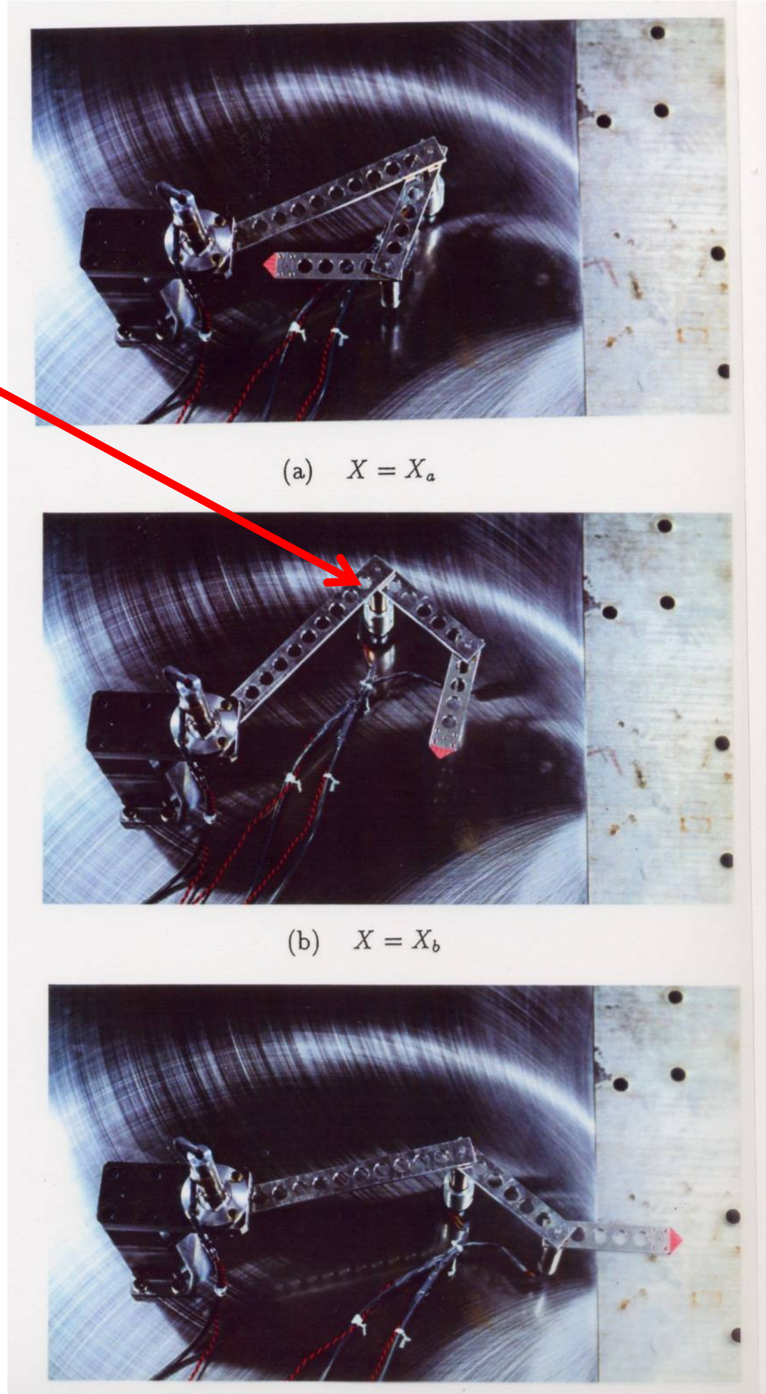


Experimental prototype (4R-manipulator)



Experimental prototype (4R-manipulator)



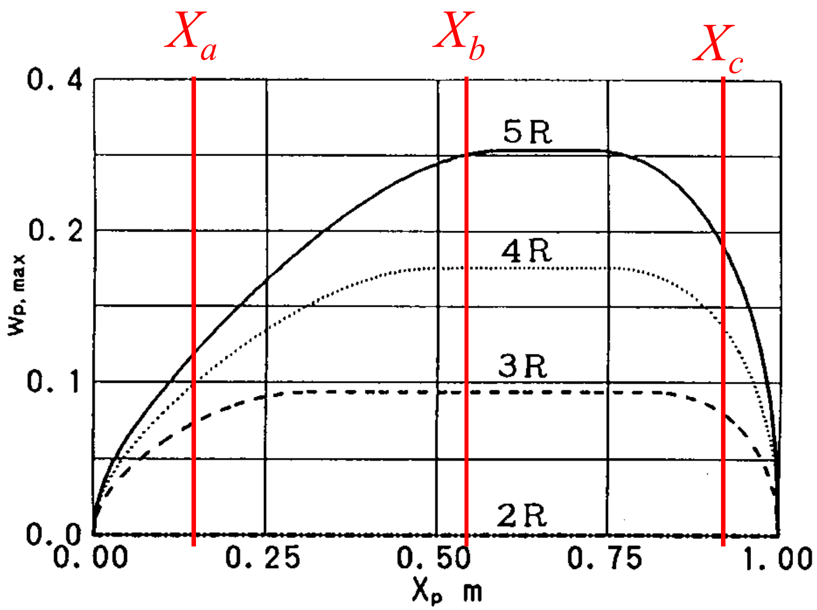


(a) $X = X_a$

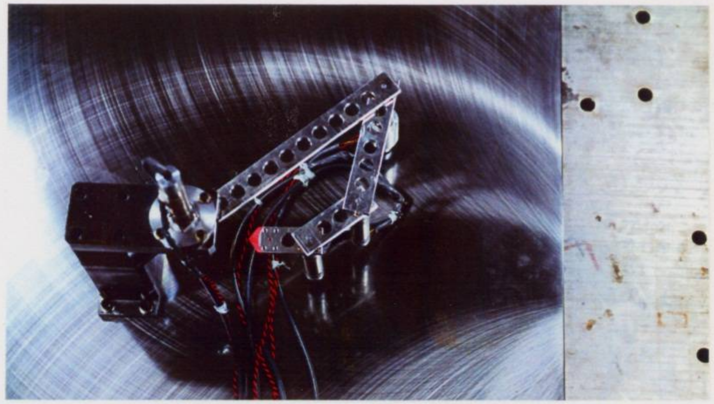
(b) $X = X_b$

(c) $X = X_c$

Dexterous configuration of 3R-manipulator



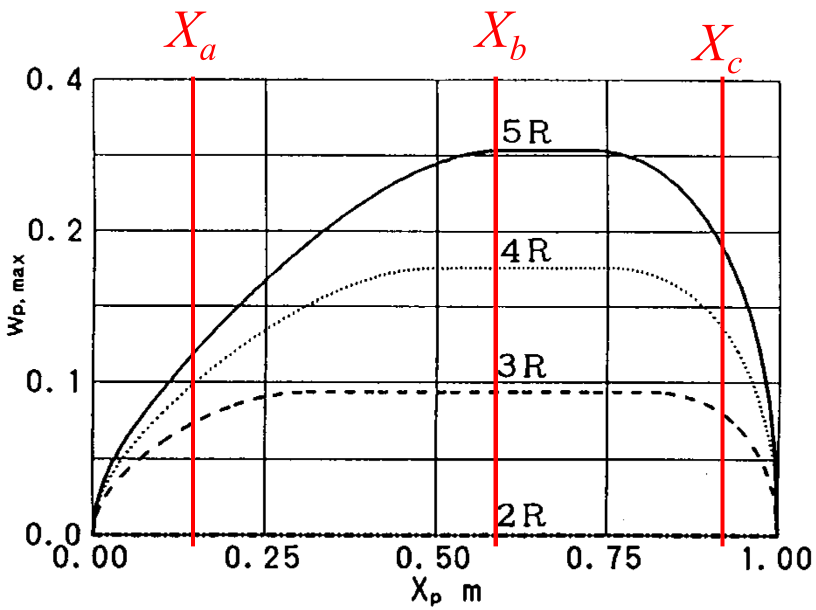
Dexterous configuration of 4R-manipulator



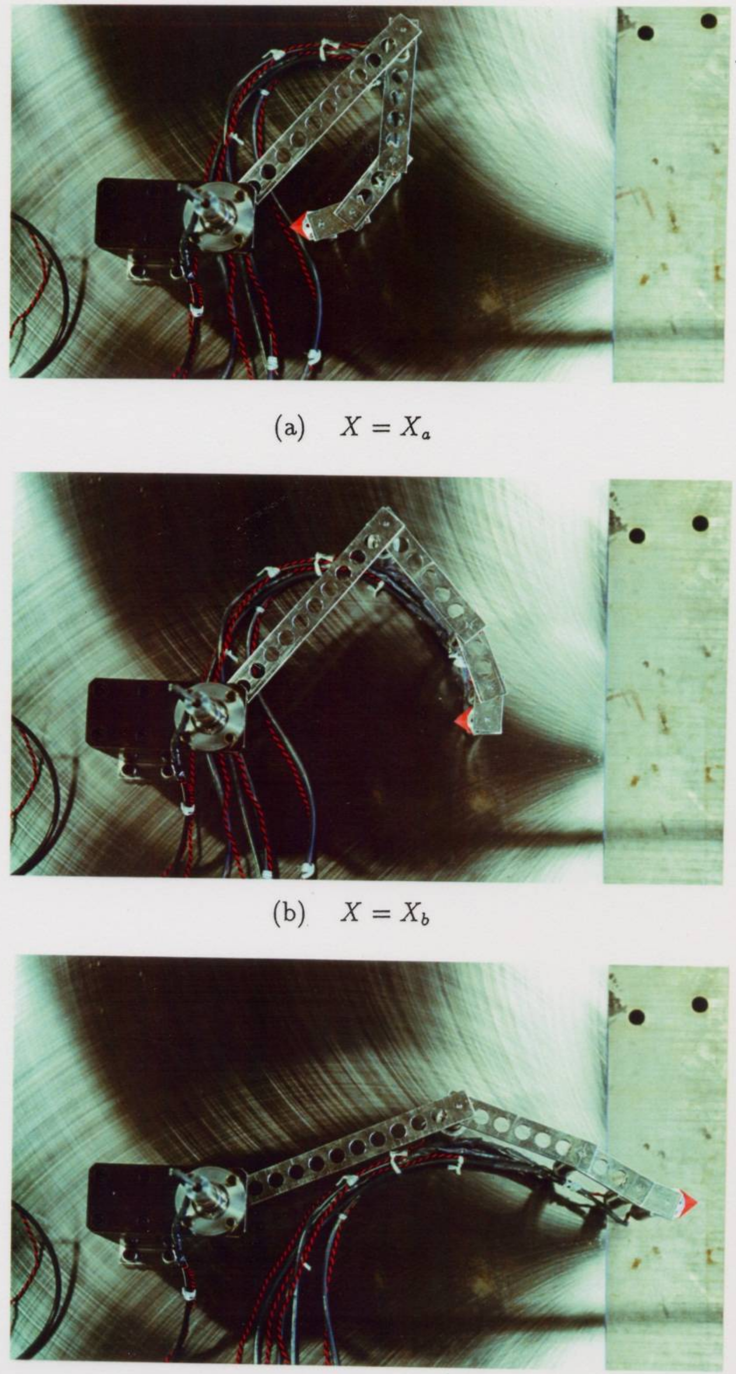
(a) $X = X_a$

(b) $X = X_b$

(c) $X = X_c$



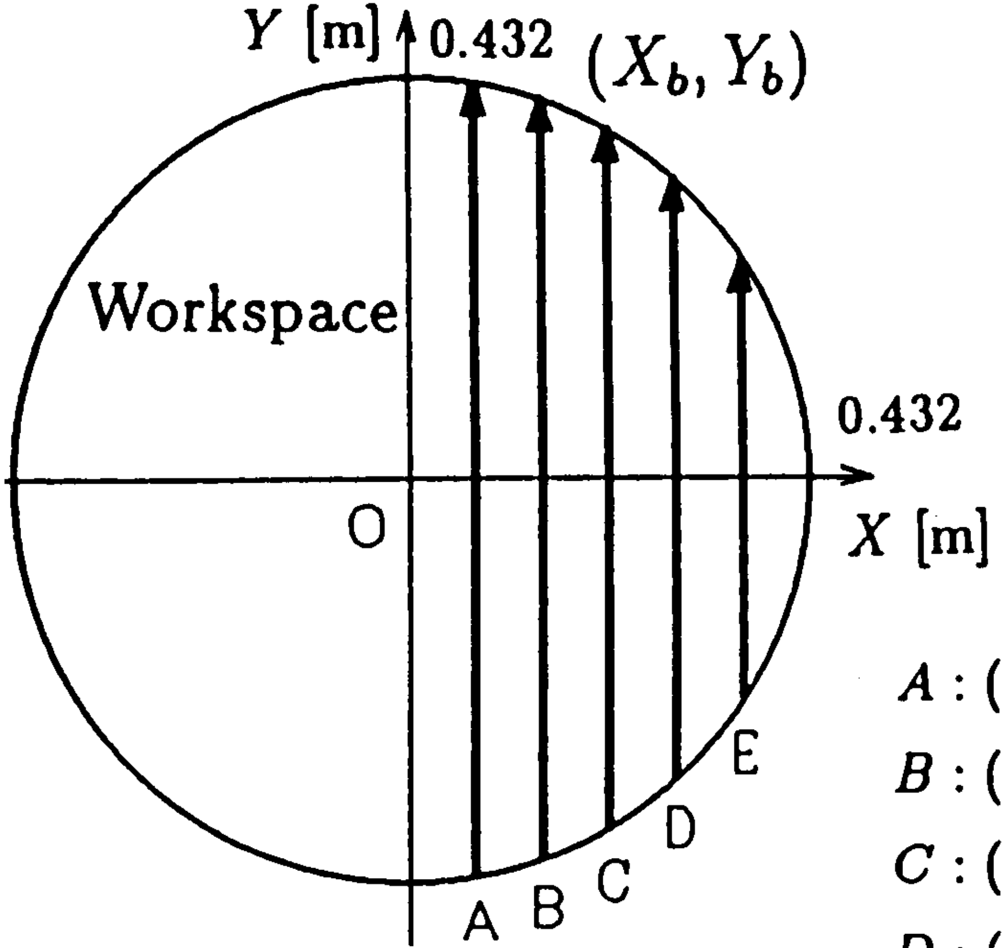
Dexterous configuration of 5R-manipulator



(a) $X = X_a$

(b) $X = X_b$

(c) $X = X_c$



$A : (0.072, -0.425) \rightarrow (0.072, 0.425)$

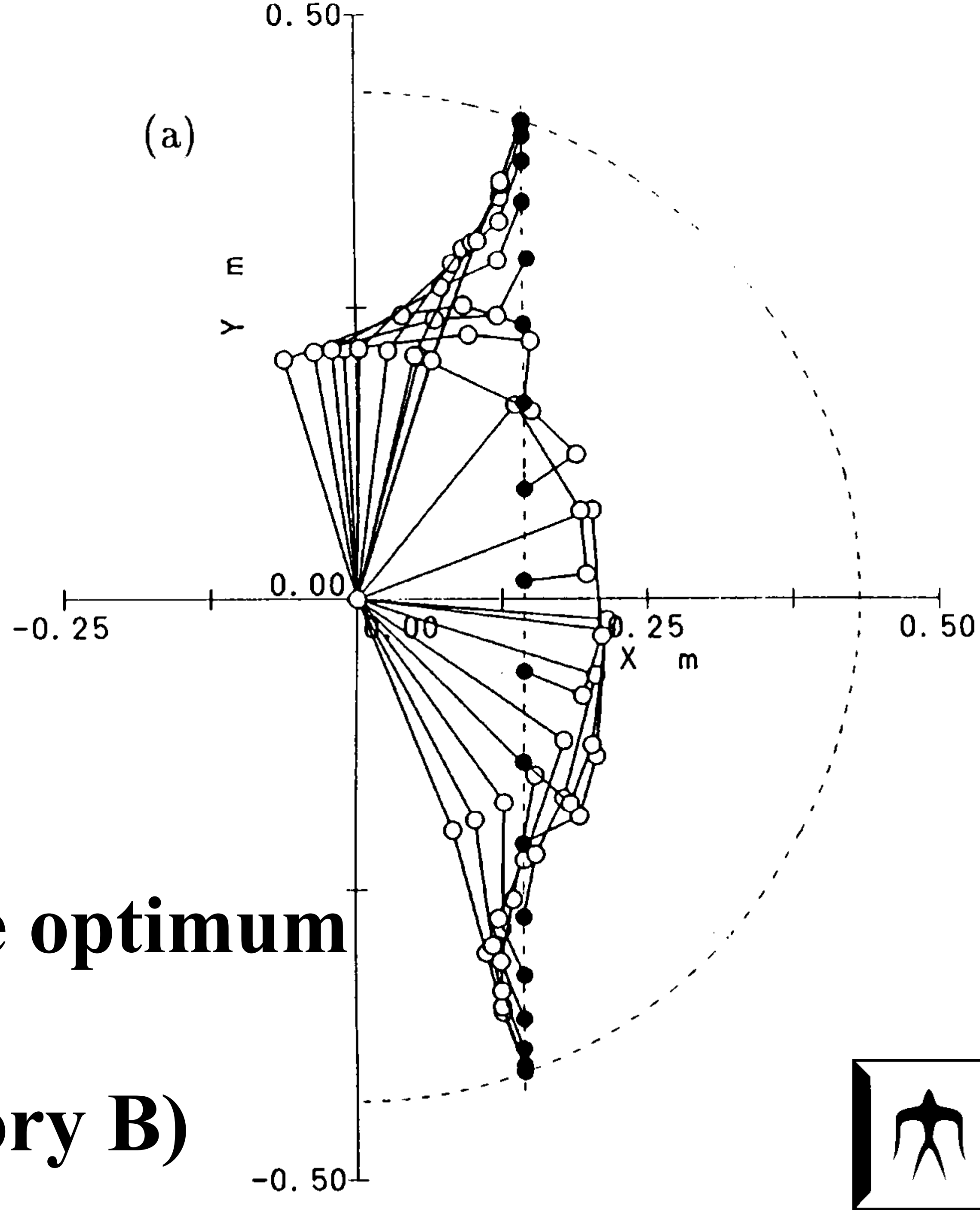
$B : (0.144, -0.407) \rightarrow (0.144, 0.407)$

$C : (0.216, -0.374) \rightarrow (0.216, 0.374)$

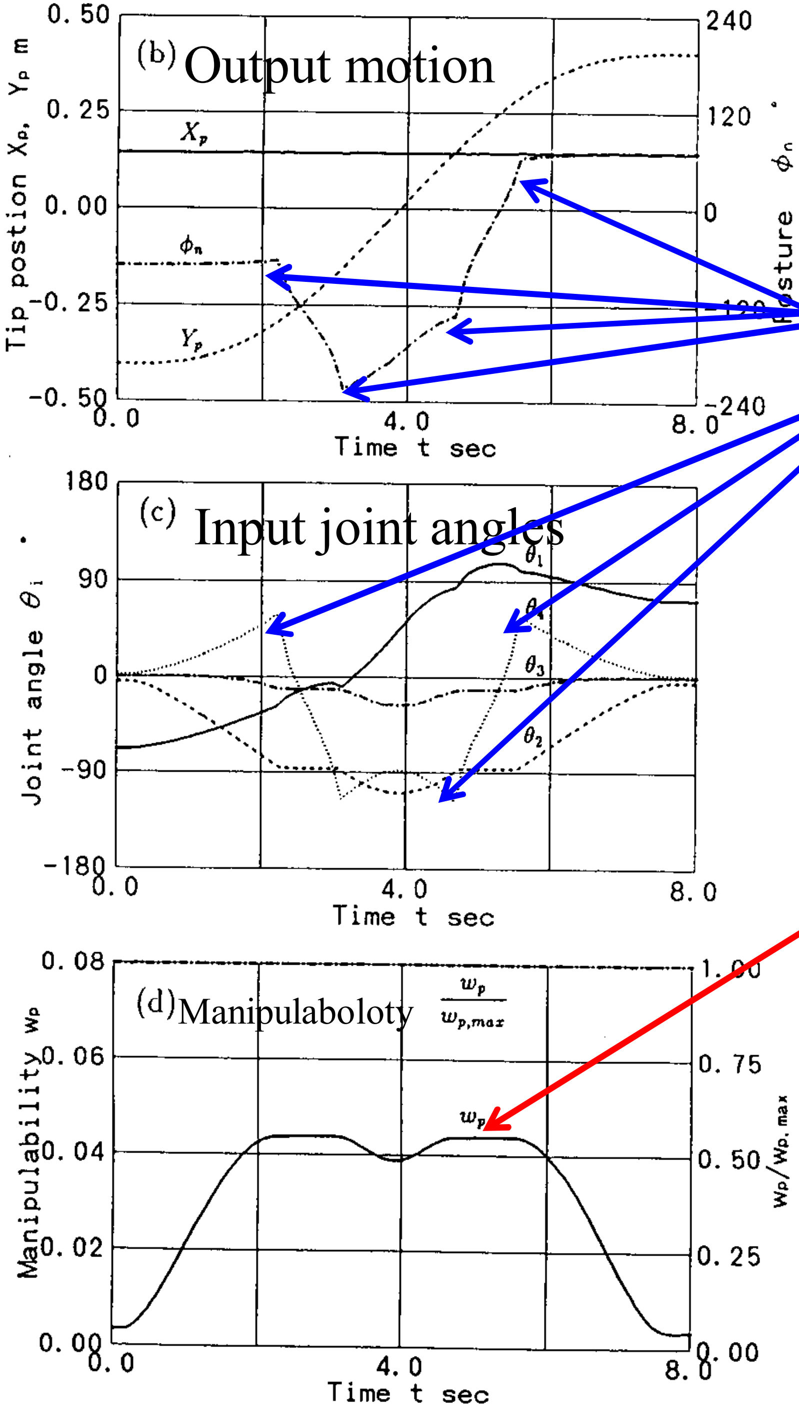
$D : (0.288, -0.321) \rightarrow (0.288, 0.321)$

$E : (0.360, -0.238) \rightarrow (0.360, 0.238)$

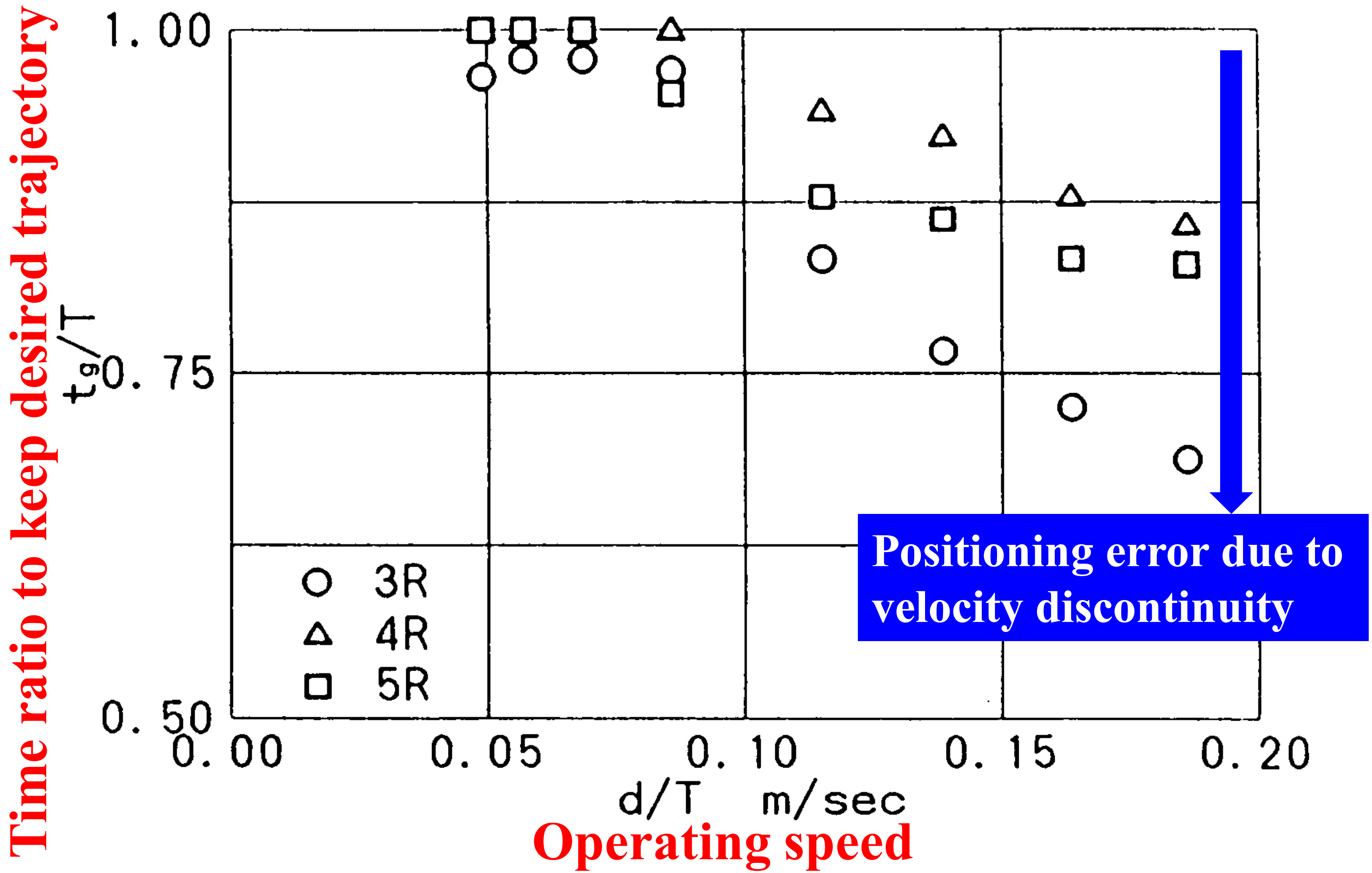
Tested trajectories



Result of the optimum CP control (4R, trajectory B)



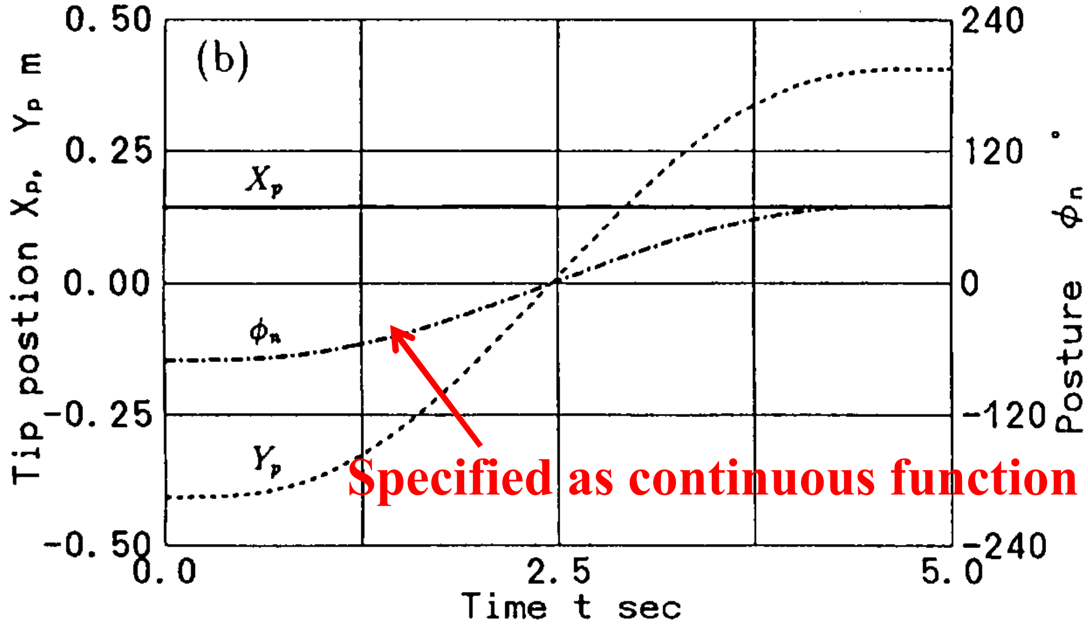
Result of the optimum CP control (4R, trajectory B)

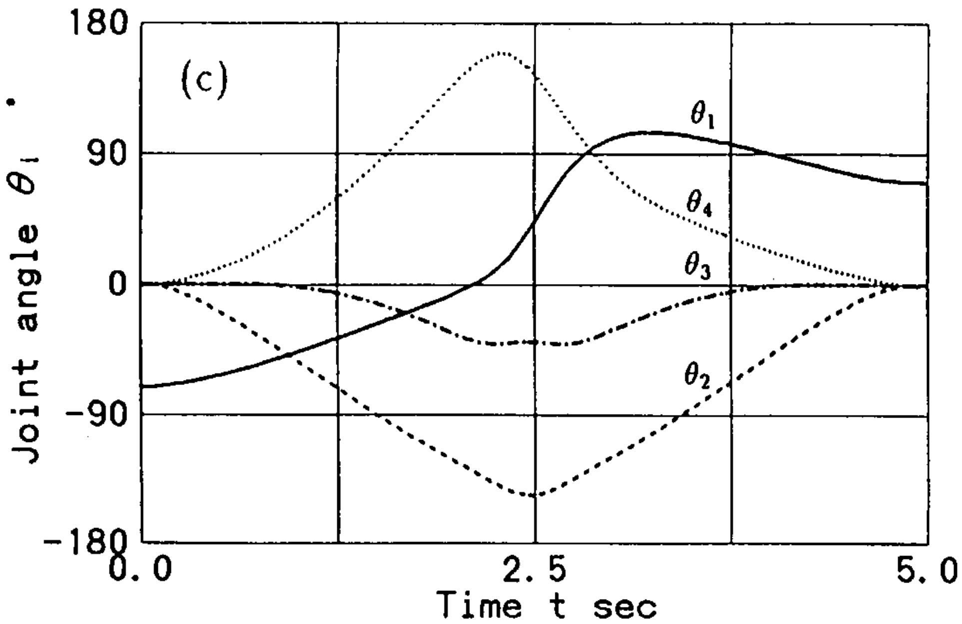


Performance on response (Various manipulators)

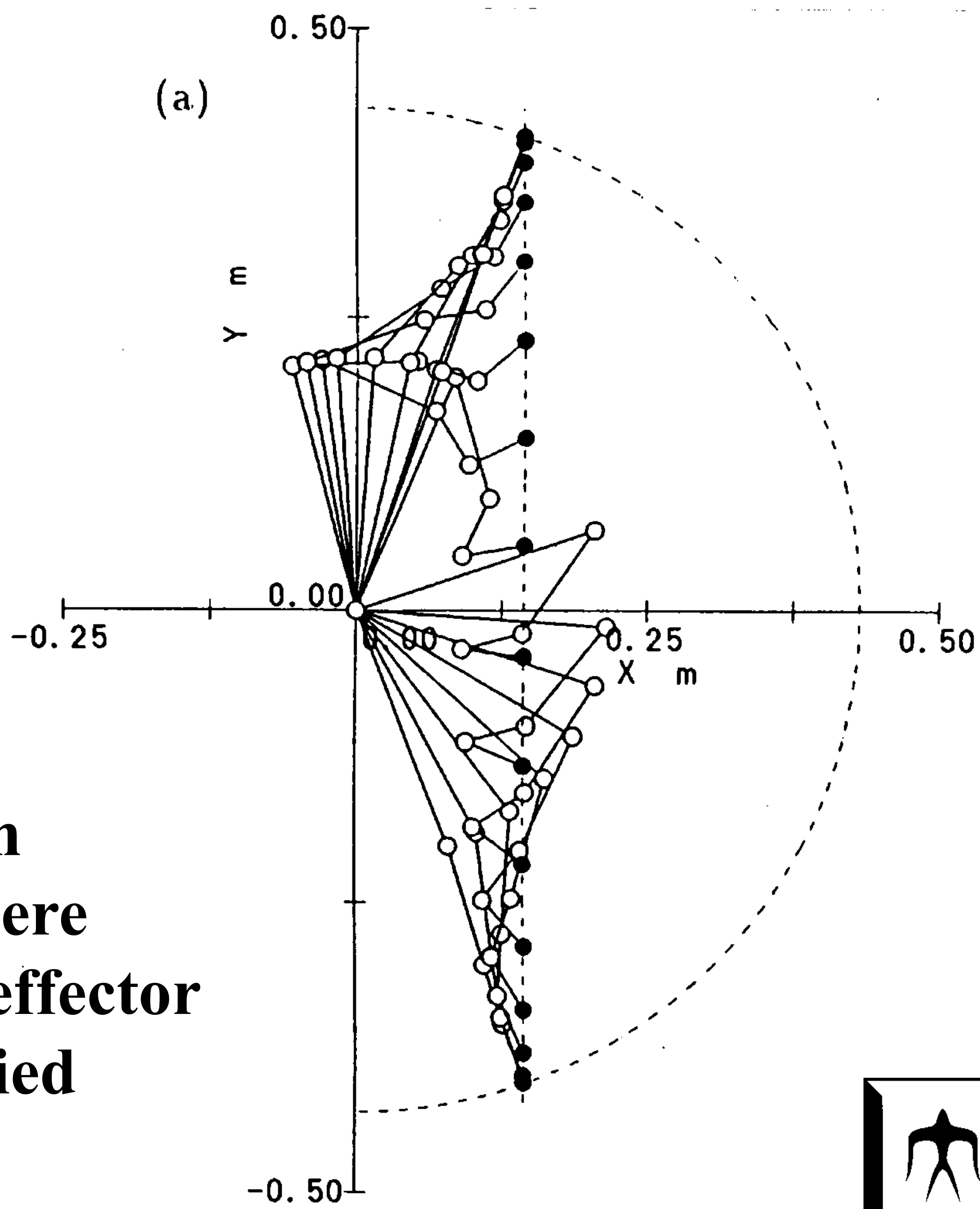
4.5 Improved optimum motion control

Results of optimum CP control in case where posture angle of end-effector is continuously specified

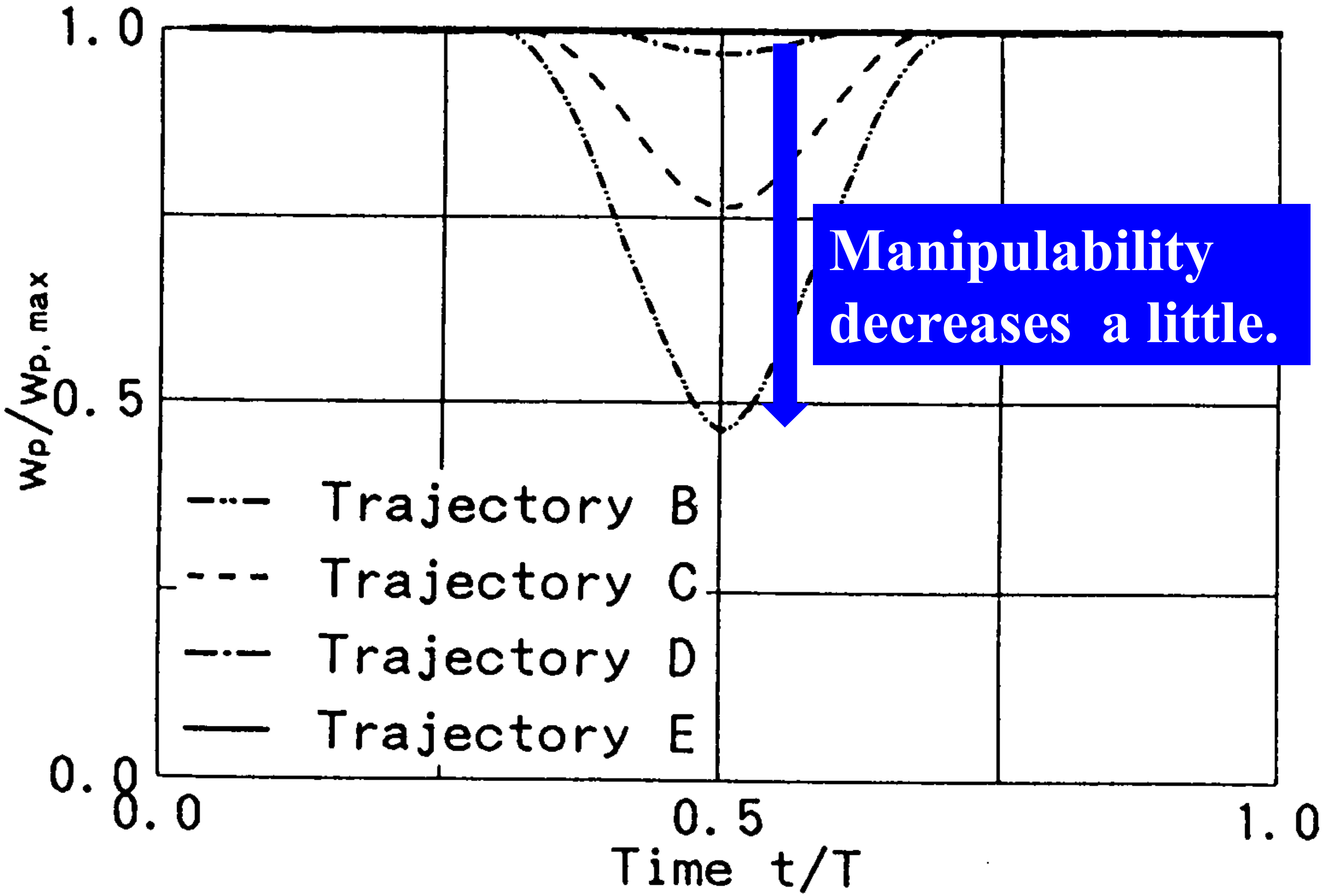




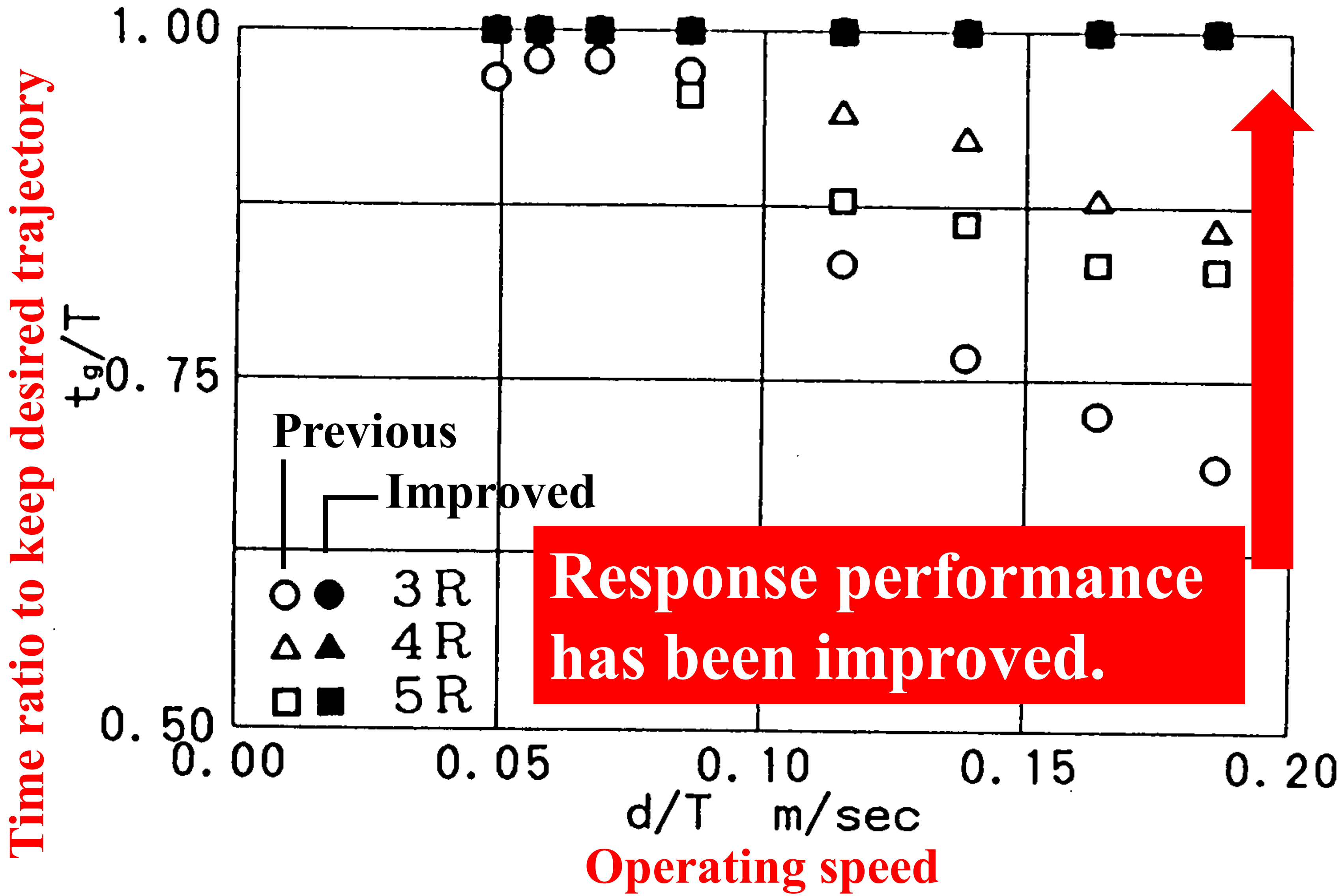
Input joint angles become smooth.



Result of the optimum CP control in case where posture angle of end-effector is continuously specified (4R, trajectory B)



Manipulability measure in the optimum CP control in case where posture angle of end-effector is continuously specified (4R-manipulator)

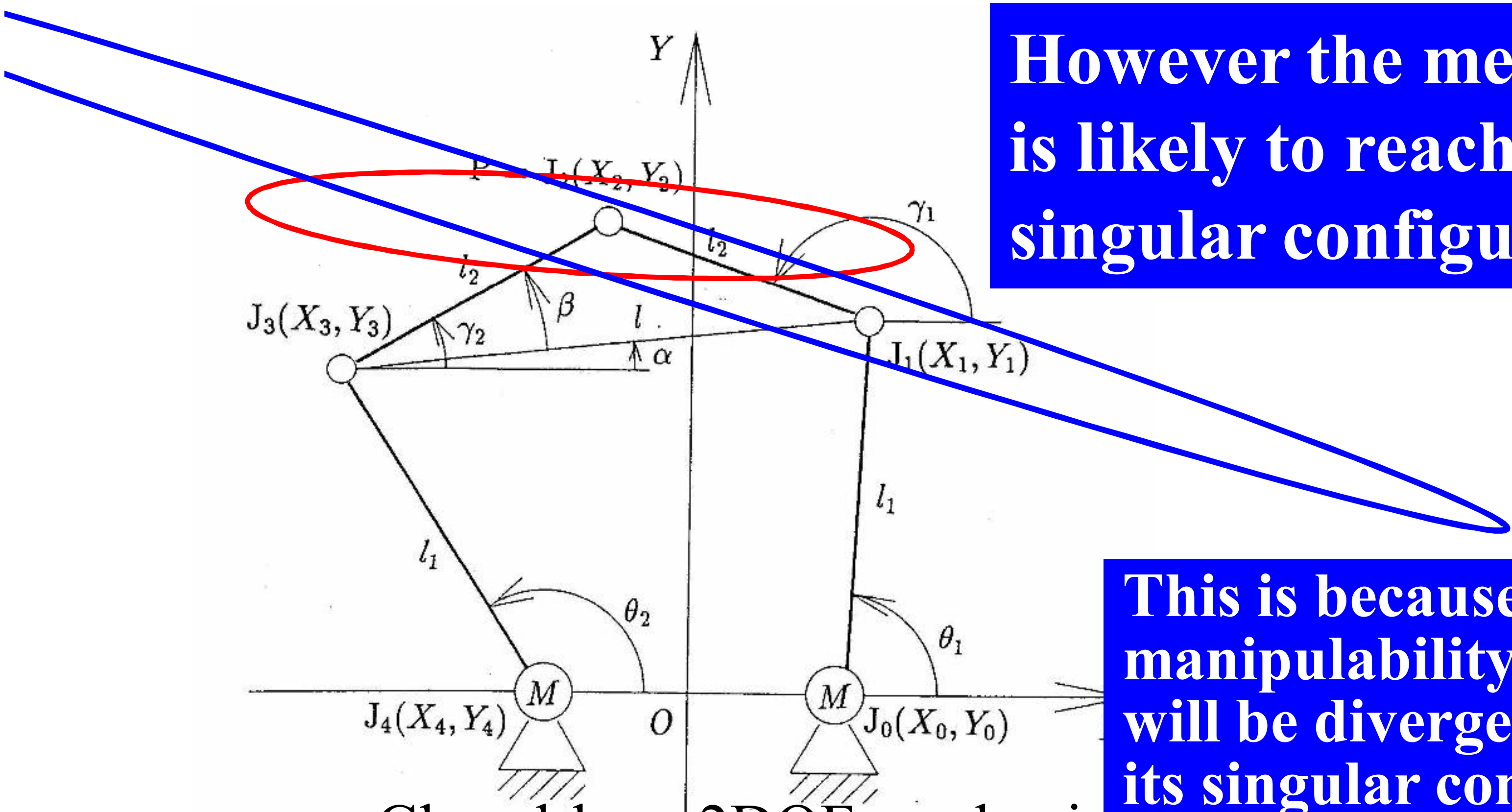


Performance on response in the optimum CP control in case where posture angle of end-effector is continuously specified (various manipulator)

5. Optimum motion control of closed-loop mechanisms based on dexterity

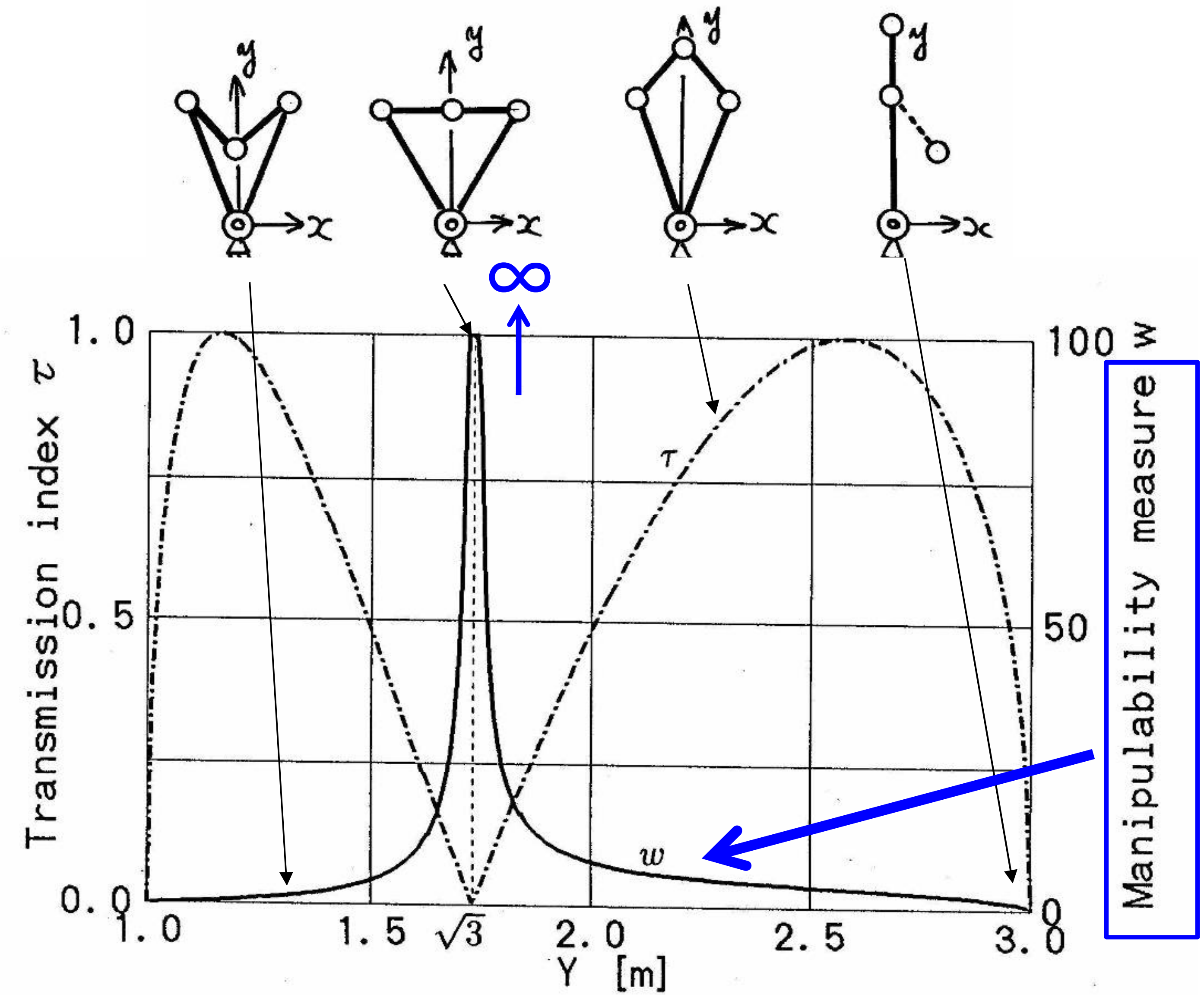
The optimum motion control with dexterity measure as an objective function.

However the mechanism is likely to reach its singular configuration!



Closed-loop 2DOF mechanism

This is because the manipulability measure will be diverged at its singular configuration.



Manipulability measure and Transmission index

Manipulability measure becomes infinite at kinematically singular configuration.

Transmission index is taken into account.

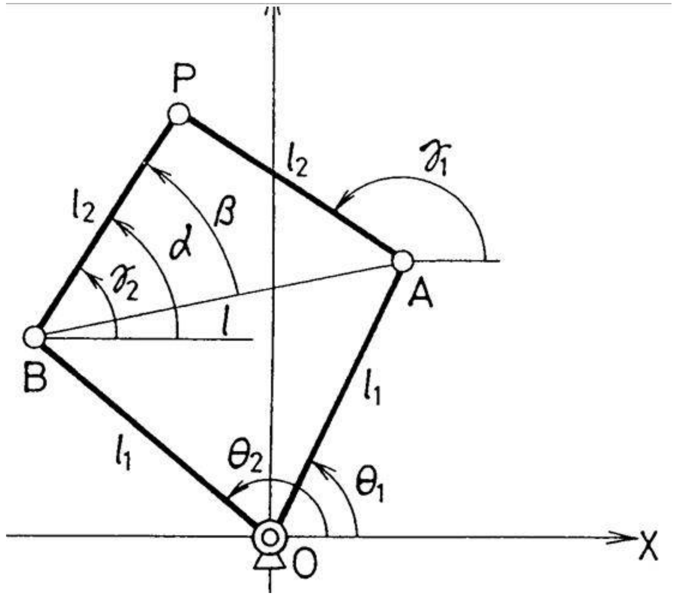
Transmission index:

"Traditional criterion to evaluate transmissibility of closed-loop mechanisms"

How to calculate the transmission index:

(1)Derive closed-loop equations with active and passive joint variables

$$\left.\begin{aligned} l_1\cos\theta_1 + l_2\cos\gamma_1 &= l_1\cos\theta_2 + l_2\cos\gamma_2 \\ l_1\sin\theta_1 + l_2\sin\gamma_1 &= l_1\sin\theta_2 + l_2\sin\gamma_2 \end{aligned}\right\}$$



Ex. Planar closed-loop mechanism with 2 DOF

(2)Consider output error due to parameter error

$$(l_1+\Delta l_1)\cos(\theta_1+\Delta\theta_1)+(l_2+\Delta l_2)\cos(\gamma_1+\Delta\gamma_1)$$
$$=(l_1+\Delta l_1)\cos(\theta_2+\Delta\theta_2)+(l_2+\Delta l_2)\cos(\gamma_2+\Delta\gamma_2)$$

$$(l_1+\Delta l_1)\sin(\theta_1+\Delta\theta_1)+(l_2+\Delta l_2)\sin(\gamma_1+\Delta\gamma_1)$$
$$=(l_1+\Delta l_1)\sin(\theta_2+\Delta\theta_2)+(l_2+\Delta l_2)\sin(\gamma_2+\Delta\gamma_2)$$

(3)Develop the equations

$$(l_1+\Delta l_1)(\cos\theta_1\cos\Delta\theta_1-\sin\theta_1\sin\Delta\theta_1)+(l_2+\Delta l_2)(\cos\gamma_1\cos\Delta\gamma_1-\sin\gamma_1\sin\Delta\gamma_1)$$
$$=(l_1+\Delta l_1)(\cos\theta_2\cos\Delta\theta_2-\sin\theta_2\sin\Delta\theta_2)+(l_2+\Delta l_2)(\cos\gamma_2\cos\Delta\gamma_2-\sin\gamma_2\sin\Delta\gamma_2)$$

$$(l_1+\Delta l_1)(\sin\theta_1\cos\Delta\theta_1+\cos\theta_1\sin\Delta\theta_1)+(l_2+\Delta l_2)(\sin\gamma_1\cos\Delta\gamma_1+\cos\gamma_1\sin\Delta\gamma_1)$$
$$=(l_1+\Delta l_1)(\sin\theta_2\cos\Delta\theta_2+\cos\theta_2\sin\Delta\theta_2)+(l_2+\Delta l_2)(\sin\gamma_2\cos\Delta\gamma_2+\cos\gamma_2\sin\Delta\gamma_2)$$

(4)Approximation

$$\cos\Delta\theta_1\cong 1,\ \sin\Delta\theta_1\cong\Delta\theta_1,\ \cos\Delta\theta_2\cong 1,\ \sin\Delta\theta_2\cong\Delta\theta_2,$$
$$\cos\Delta\gamma_1\cong 1,\ \sin\Delta\gamma_1\cong\Delta\gamma_1,\ \cos\Delta\gamma_2\cong 1,\ \sin\Delta\gamma_2\cong\Delta\gamma_2$$

$$(l_1+\Delta l_1)(\cos\theta_1-\Delta\theta_1\sin\theta_1)+(l_2+\Delta l_2)(\cos\gamma_1-\Delta\gamma_1\sin\gamma_1)$$
$$=(l_1+\Delta l_1)(\cos\theta_2-\Delta\theta_2\sin\theta_2)+(l_2+\Delta l_2)(\cos\gamma_2-\Delta\gamma_2\sin\gamma_2)$$
$$(l_1+\Delta l_1)(\sin\theta_1+\Delta\theta_1\cos\theta_1)+(l_2+\Delta l_2)(\sin\gamma_1+\Delta\gamma_1\cos\gamma_1)$$
$$=(l_1+\Delta l_1)(\sin\theta_2+\Delta\theta_2\cos\theta_2)+(l_2+\Delta l_2)(\sin\gamma_2+\Delta\gamma_2\cos\gamma_2)$$

$$\Delta l_1 \Delta \theta_1 \cong 0, \quad \Delta l_1 \Delta \theta_2 \cong 0, \quad \Delta l_2 \Delta \gamma_1 \cong 0, \quad \Delta l_2 \Delta \theta_2 \cong 0$$

$$(l_1 + \Delta l_1)\cos\theta_1 - l_1 \Delta\theta_1 \sin\theta_1 + (l_2 + \Delta l_2)\cos\gamma_1 - l_2 \Delta\gamma_1 \sin\gamma_1$$
$$= (l_1 + \Delta l_1)\cos\theta_2 - l_1 \Delta\theta_2 \sin\theta_2 + (l_2 + \Delta l_2)\cos\gamma_2 - l_2 \Delta\gamma_2 \sin\gamma_2$$
$$(l_1 + \Delta l_1)\sin\theta_1 + l_1 \Delta\theta_1 \cos\theta_1 + (l_2 + \Delta l_2)\sin\gamma_1 + l_2 \Delta\gamma_1 \cos\gamma_1$$
$$= (l_1 + \Delta l_1)\sin\theta_2 + l_1 \Delta\theta_2 \cos\theta_2 + (l_2 + \Delta l_2)\sin\gamma_2 + l_2 \Delta\gamma_2 \cos\gamma_2$$

(5)Deform and summarize by focusing passive joint error

$$-l_2 \underline{\Delta\gamma_1} \sin\gamma_1 + l_2 \underline{\Delta\gamma_2} \sin\gamma_2$$
$$= -(l_1 + \Delta l_1)\cos\theta_1 + l_1 \Delta\theta_1 \sin\theta_1 - (l_2 + \Delta l_2)\cos\gamma_1$$
$$+ (l_1 + \Delta l_1)\cos\theta_2 - l_1 \Delta\theta_2 \sin\theta_2 + (l_2 + \Delta l_2)\cos\gamma_2$$

$$l_2 \underline{\Delta\gamma_1} \cos\gamma_1 - l_2 \underline{\Delta\gamma_2} \cos\gamma_2$$
$$= -(l_1 + \Delta l_1)\sin\theta_1 - l_1 \Delta\theta_1 \cos\theta_1 - (l_2 + \Delta l_2)\sin\gamma_1$$
$$+ (l_1 + \Delta l_1)\sin\theta_2 + l_1 \Delta\theta_2 \cos\theta_2 + (l_2 + \Delta l_2)\sin\gamma_2$$

$$\Delta l_1 \Delta\theta_1 \cong 0, \quad \Delta l_1 \Delta\theta_2 \cong 0, \quad \Delta l_2 \Delta\gamma_1 \cong 0, \quad \Delta l_2 \Delta\theta_2 \cong 0$$

$$(l_1+\Delta l_1)\cos\theta_1 - l_1\Delta\theta_1 \sin\theta_1 + (l_2+\Delta l_2)\cos\gamma_1 - l_2\Delta\gamma_1 \sin\gamma_1$$
$$= (l_1+\Delta l_1)\cos\theta_2 - l_1\Delta\theta_2 \sin\theta_2 + (l_2+\Delta l_2)\cos\gamma_2 - l_2\Delta\gamma_2 \sin\gamma_2$$
$$(l_1+\Delta l_1)\sin\theta_1 + l_1\Delta\theta_1 \cos\theta_1 + (l_2+\Delta l_2)\sin\gamma_1 + l_2\Delta\gamma_1 \cos\gamma_1$$
$$= (l_1+\Delta l_1)\sin\theta_2 + l_1\Delta\theta_2 \cos\theta_2 + (l_2+\Delta l_2)\sin\gamma_2 + l_2\Delta\gamma_2 \cos\gamma_2$$

(5)Deform and summarize by focusing passive joint error

$$-l_2\underline{\Delta\gamma_1}\sin\gamma_1 + l_2\underline{\Delta\gamma_2}\sin\gamma_2$$
$$= -(l_1+\Delta l_1)\cos\theta_1 + l_1\Delta\theta_1 \sin\theta_1 - (l_2+\Delta l_2)\cos\gamma_1$$
$$+ (l_1+\Delta l_1)\cos\theta_2 - l_1\Delta\theta_2 \sin\theta_2 + (l_2+\Delta l_2)\cos\gamma_2$$

$$l_2\underline{\Delta\gamma_1}\cos\gamma_1 - l_2\underline{\Delta\gamma_2}\cos\gamma_2$$
$$= -(l_1+\Delta l_1)\sin\theta_1 - l_1\Delta\theta_1 \cos\theta_1 - (l_2+\Delta l_2)\sin\gamma_1$$
$$+ (l_1+\Delta l_1)\sin\theta_2 + l_1\Delta\theta_2 \cos\theta_2 + (l_2+\Delta l_2)\sin\gamma_2$$

$$\begin{bmatrix} -l_2 \sin\gamma_1 & l_2 \sin\gamma_2 \\ l_2 \cos\gamma_1 & -l_2 \cos\gamma_2 \end{bmatrix} \begin{bmatrix} \Delta\gamma_1 \\ \Delta\gamma_2 \end{bmatrix}$$

$$= \begin{bmatrix} -(l_1+\Delta l_1)\cos\theta_1 + l_1\Delta\theta_1 \sin\theta_1 - (l_2+\Delta l_2)\cos\gamma_1 \\ \quad +(l_1+\Delta l_1)\cos\theta_2 - l_1\Delta\theta_2 \sin\theta_2 + (l_2+\Delta l_2)\cos\gamma_2 \\ \\ -(l_1+\Delta l_1)\sin\theta_1 - l_1\Delta\theta_1 \cos\theta_1 - (l_2+\Delta l_2)\sin\gamma_1 \\ \quad +(l_1+\Delta l_1)\sin\theta_2 + l_1\Delta\theta_2 \cos\theta_2 + (l_2+\Delta l_2)\sin\gamma_2 \end{bmatrix}$$

$$\Downarrow$$

$$\left[U(l_1,l_2,\gamma_1,\gamma_2)\right] \begin{bmatrix} \Delta\gamma_1 \\ \Delta\gamma_2 \end{bmatrix} = v(l_1,l_2,\Delta l_1,\Delta l_2,\theta_1,\theta_2,\Delta\theta_1,\Delta\theta_2,\gamma_1,\gamma_2)$$

"A system of linear equations to solve motion error due to mechanical parameter error"

$$\begin{bmatrix} \Delta\gamma_1 \\ \Delta\gamma_2 \end{bmatrix} = \left[U(l_1,l_2,\gamma_1,\gamma_2)\right]^{-1} \cdot v(l_1,l_2,\Delta l_1,\Delta l_2,\theta_1,\theta_2,\Delta\theta_1,\Delta\theta_2,\gamma_1,\gamma_2)$$

(6)Transmission index

When a mechanism takes its singular configuration, the motion cannot be calculated, in other words, output motion error will be diverged.

We can evaluate the singular configuration by using the determinant of the coefficient matirix of the system of linear equations to solve output motion error.

Definition of transmission index :

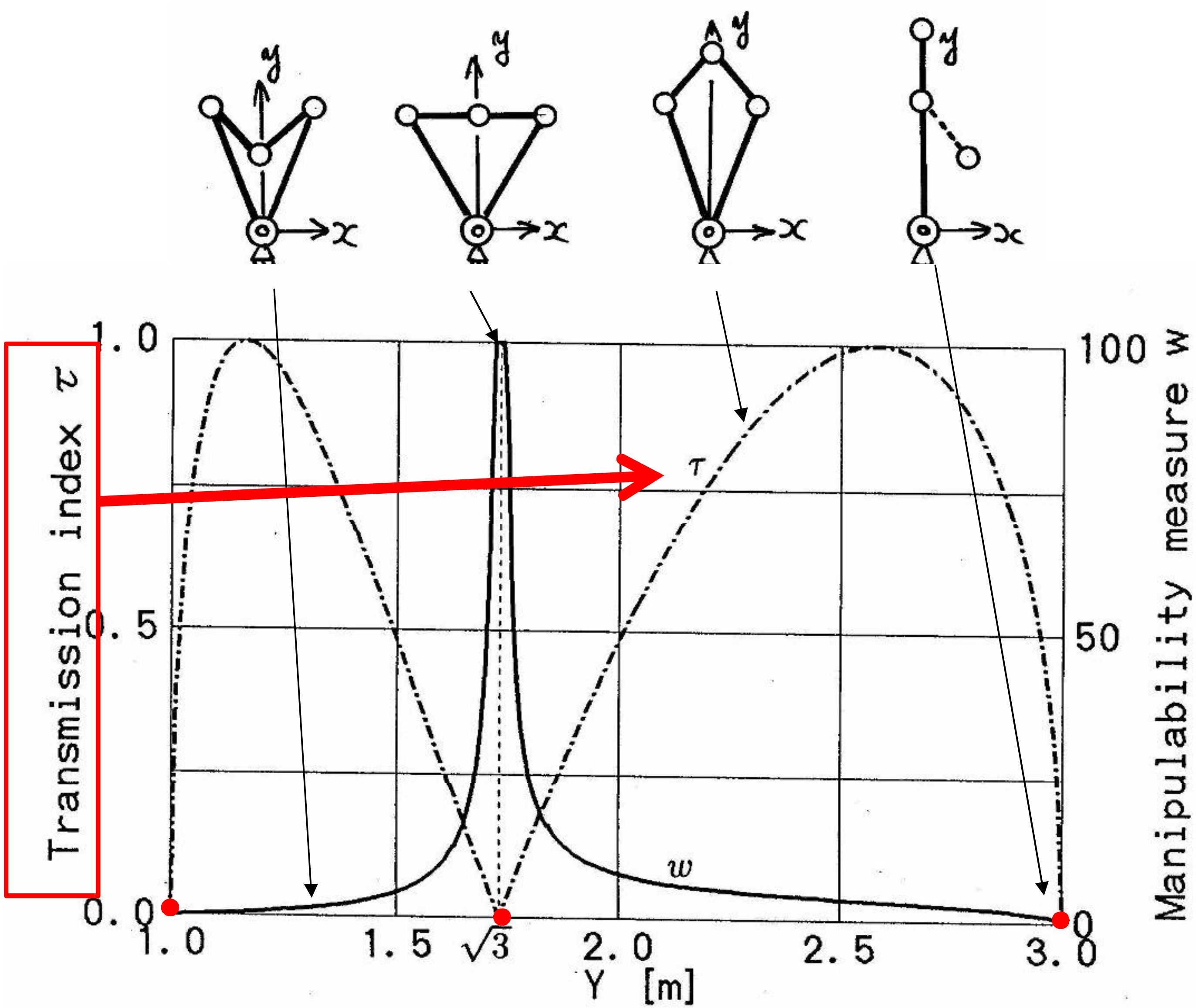
$$\tau = \frac{\left|\det([U])\right|}{\max\left[\left|\det([U])\right|\right]}$$

Normalized with its maximum value

For the example of planar closed-loop mechanism with 2DOF:

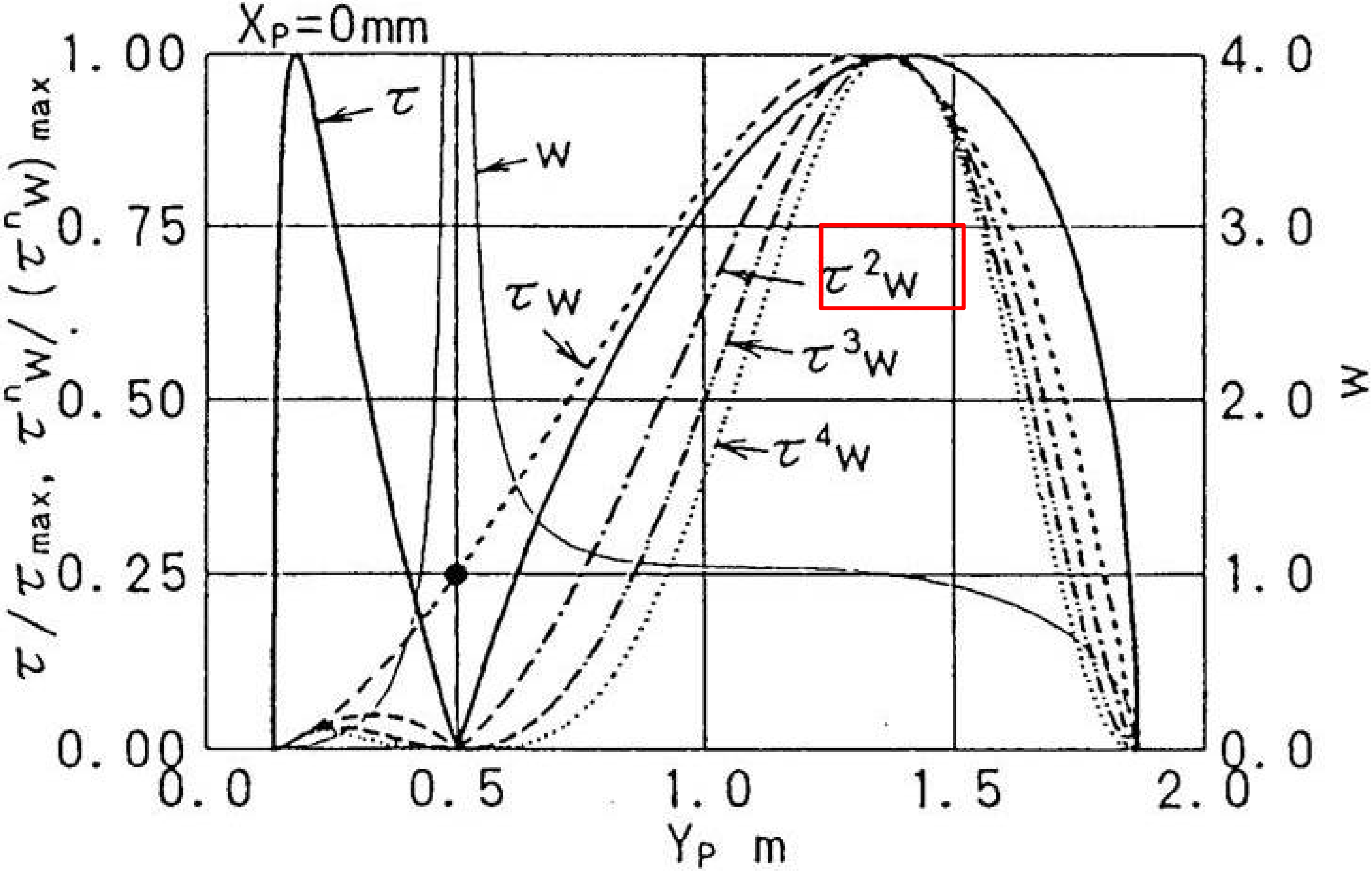
$$\begin{aligned}\det([U]) &= -l_2 \sin\gamma_1 \cdot (-l_2 \cos\gamma_2) - l_2 \cos\gamma_1 l_2 \sin\gamma_2 \\ &= l_2^2 (\sin\gamma_1 \cos\gamma_2 - \cos\gamma_1 \sin\gamma_2) \\ &= l_2^2 \sin(\gamma_1 - \gamma_2)\end{aligned}$$

$$\therefore \tau = \frac{\left|l_2^2 \sin(\gamma_1 - \gamma_2)\right|}{\max\left[\left|l_2^2 \sin(\gamma_1 - \gamma_2)\right|\right]} = \left|\sin(\gamma_1 - \gamma_2)\right|$$



Manipulability measure and Transmission index

Transmission index becomes zero when mechanism takes its singular configuration.



Indices for planar closed-loop manipulator with 2 DOF

$\tau^2 W$ is suitable to evaluate dexterity taking account of singular configuration.

Therefore a new objective function:

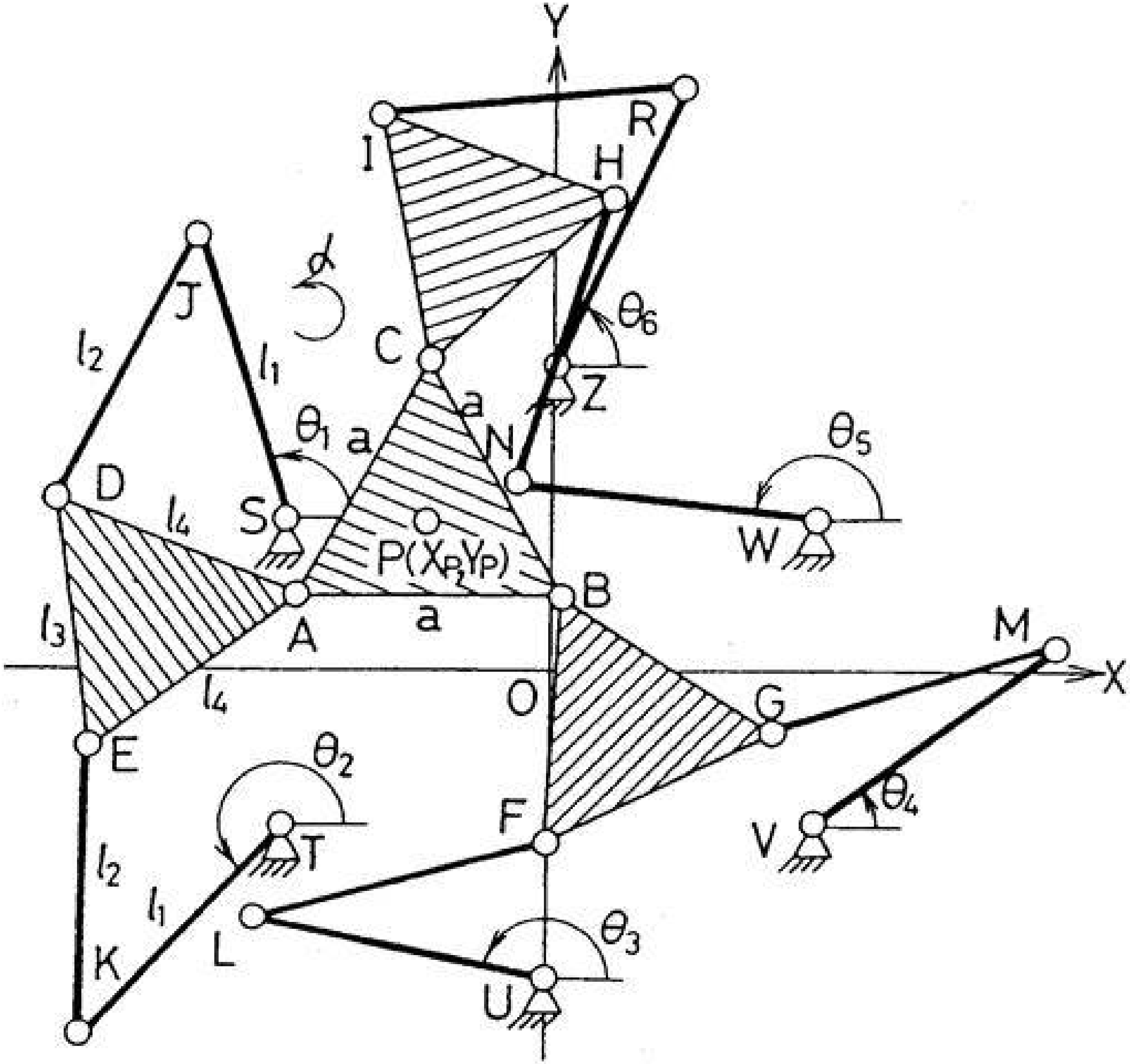
$$\Phi = \tau^n w$$

is adopted.

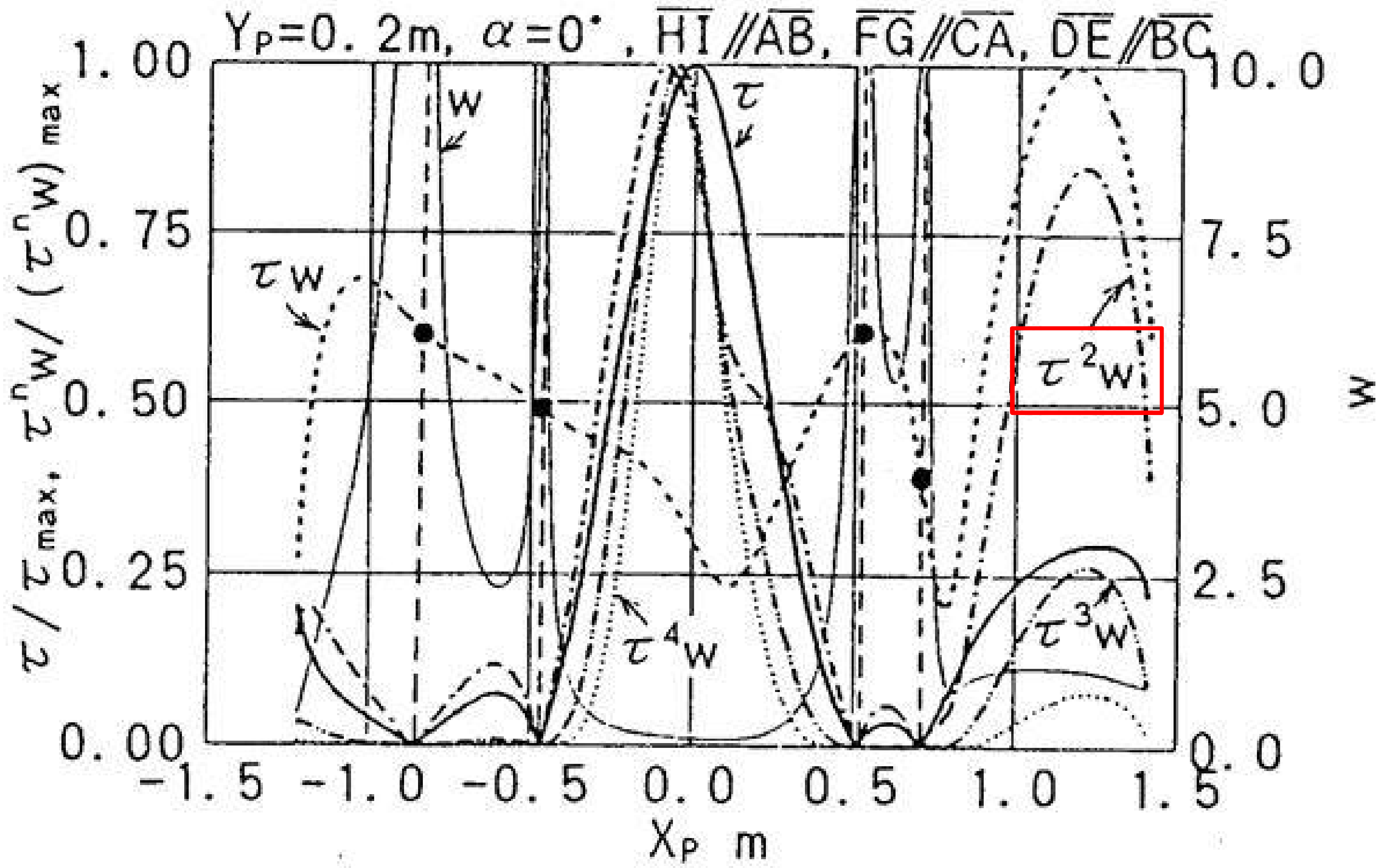
This is because

Dexterous → Manipulability measure becomes large

Approach to singular configuration → Transmission index becomes zero



A Redundant closed-loop planar manipulator with 6 DOF



Indices for closed-loop planar manipulator with 6 DOF

6. Concluding remarks

Optimum motion control of redundant link mechanism based on dexterity was carried out.

(1)Inverse kinematics can be calculated for displacement /velocity inputs by specifying redundant DOF .

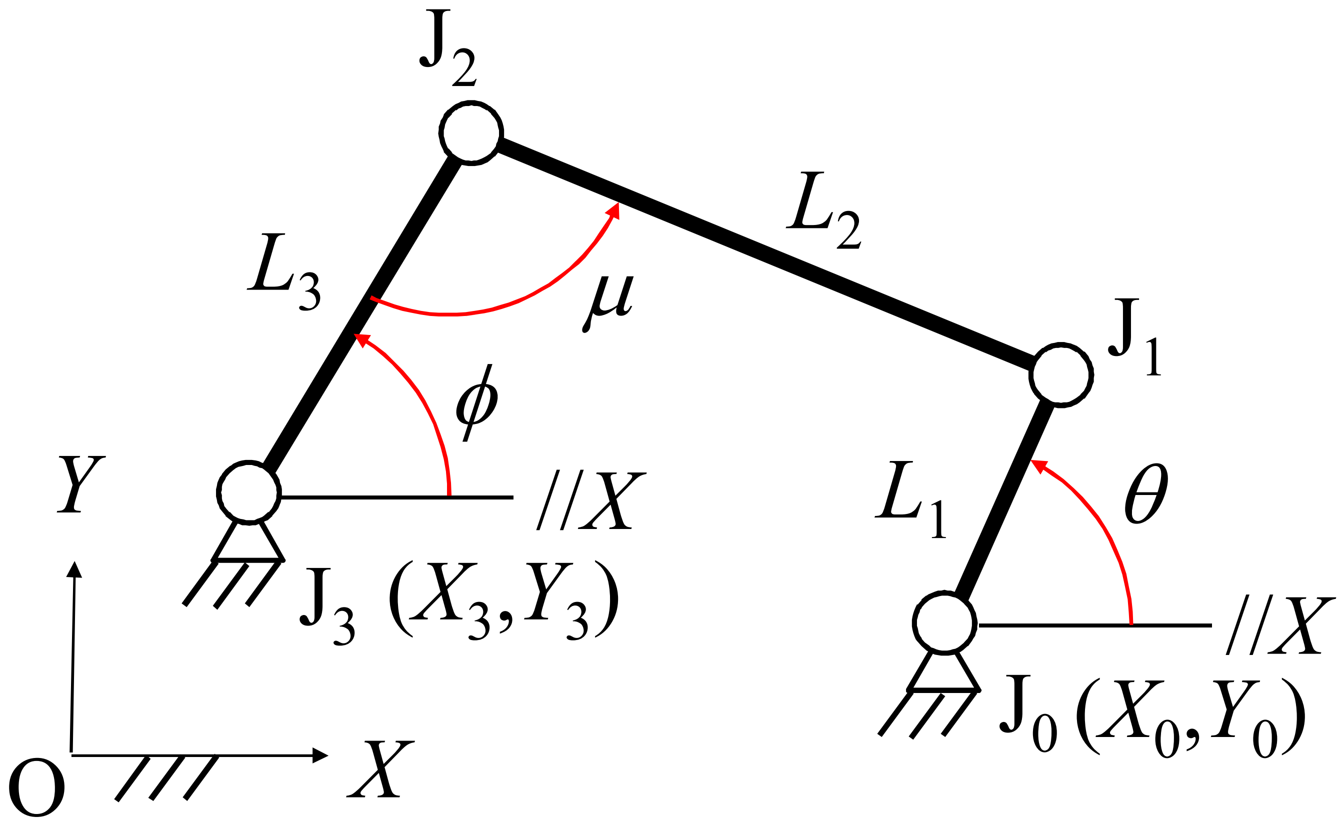
(2)Manipulability measure taking account of posture angle of end-effector is suitable to evaluate dexterous manipulation.

(3)Optimum motion control to maximize the proposed dexterity measure was achieved.

(4)New objective function with transmission index was adopted to avoid fatal singular configuration of closed-loop manipulator in dexterous control.

Homework 4

Derive the transmission index τ of a planar 4-bar link mechanism shown below.



The result will be summarized in A4 size PDF with less than 3 pages and sent to Prof. Iwatsuki via T2SCHOLA by May 15, 2023.