May 11, 2023

Advanced Mechanical Elements
(Lecture 5)

Kinetostatic analysis and motion control of overactuator mechanisms

- Motion control of overactuator mechanisms using relaxation with elastic elements-

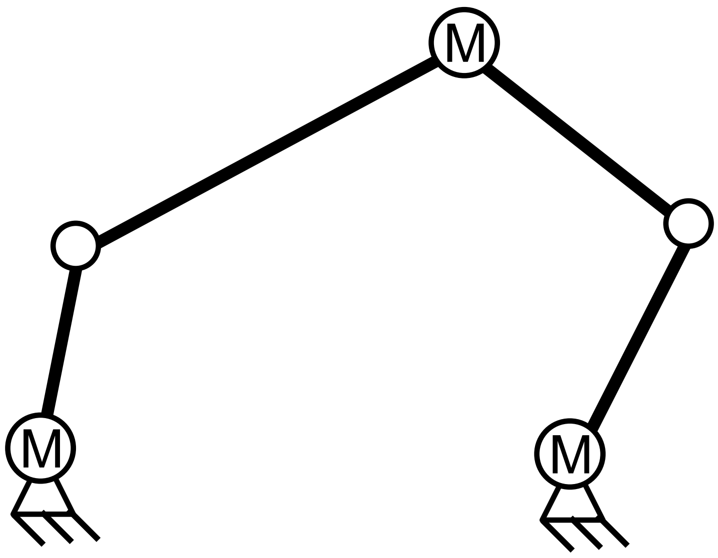
Tokyo Institute of Technology
Dept. of Mechanical Engineering
School of Engineering

Prof. Nobuyuki Iwatsuki

1. Motion Control of overactuator mechanisms with redundancy

1.1 Overactuator mechanisms with redundancy

There is a possibility to obtain overactuator mechanisms when we design hyper redundant multi-loop mechanisms.



Degrees-of-Freedom of mechanism:

$$F = 3(5-1) - 5(3-1) = 2$$

Number of Actuators:

$$N_A = 3$$

A simple example of overactuator mechanism

1. Motion Control of overactuator mechanisms with redundancy

1.1 Overactuator mechanisms with redundancy

There is a possibility to obtain overactuator mechanisms when we design hyper redundant multi-loop mechanisms.

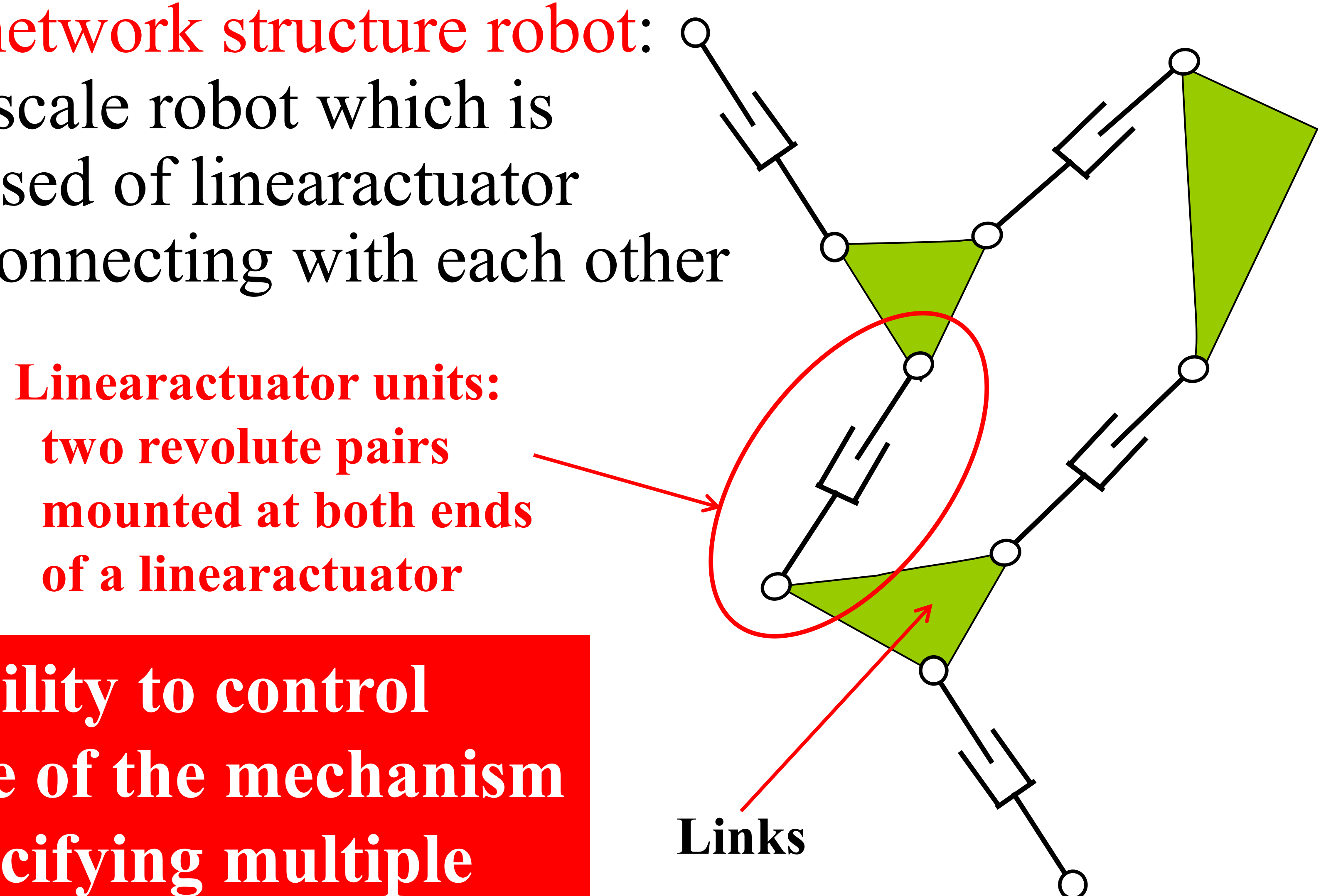
(1)Cooperation of small actuators to generate high power

(2) The robot can move while some actuators are broken down.

How to design and control the overactuator mechanisms especially taking account of interfere between actuators

1.2 Proposal of network-structure robot

Planar network structure robot:
Large scale robot which is composed of linearactuator units connecting with each other



Planar network structure robot

Possibility to control outline of the mechanism by specifying multiple output points

Planar network-structure robot composed of modules

Proposal: Network-structure module as a minimum unit of network structure robot

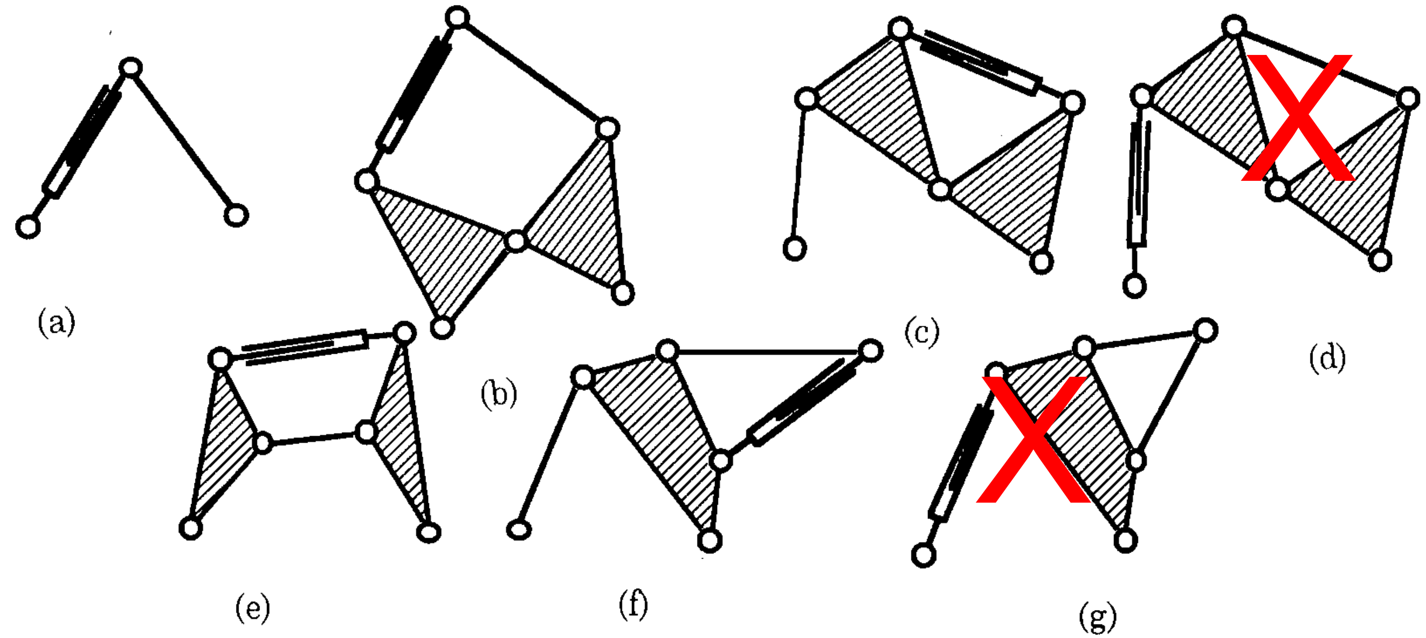
Definition of network structure module

(1)A planar link chain composed of linear actuators and links with multiplerevolute joints

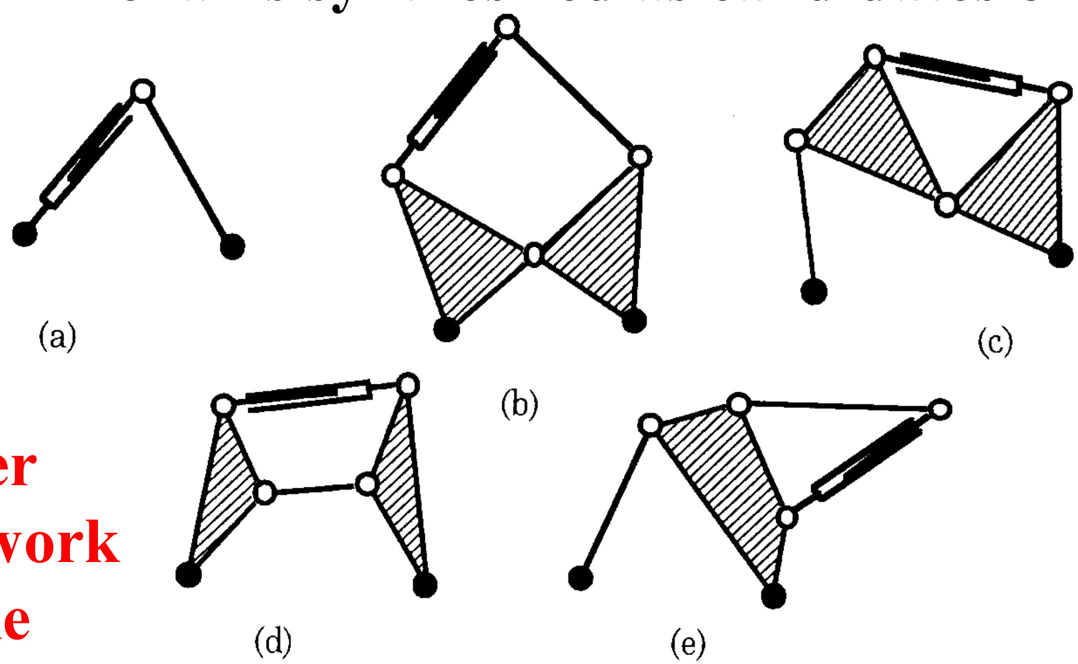
(2)Connected with a frame or other modules with connective joints

(3)A mechanism obtained by connecting all connective joints of the module with frame has more actuators than DOF.

(4)Not include other modules in itself



Examples of link chains synthesized as candidates of modules



Examples of modules

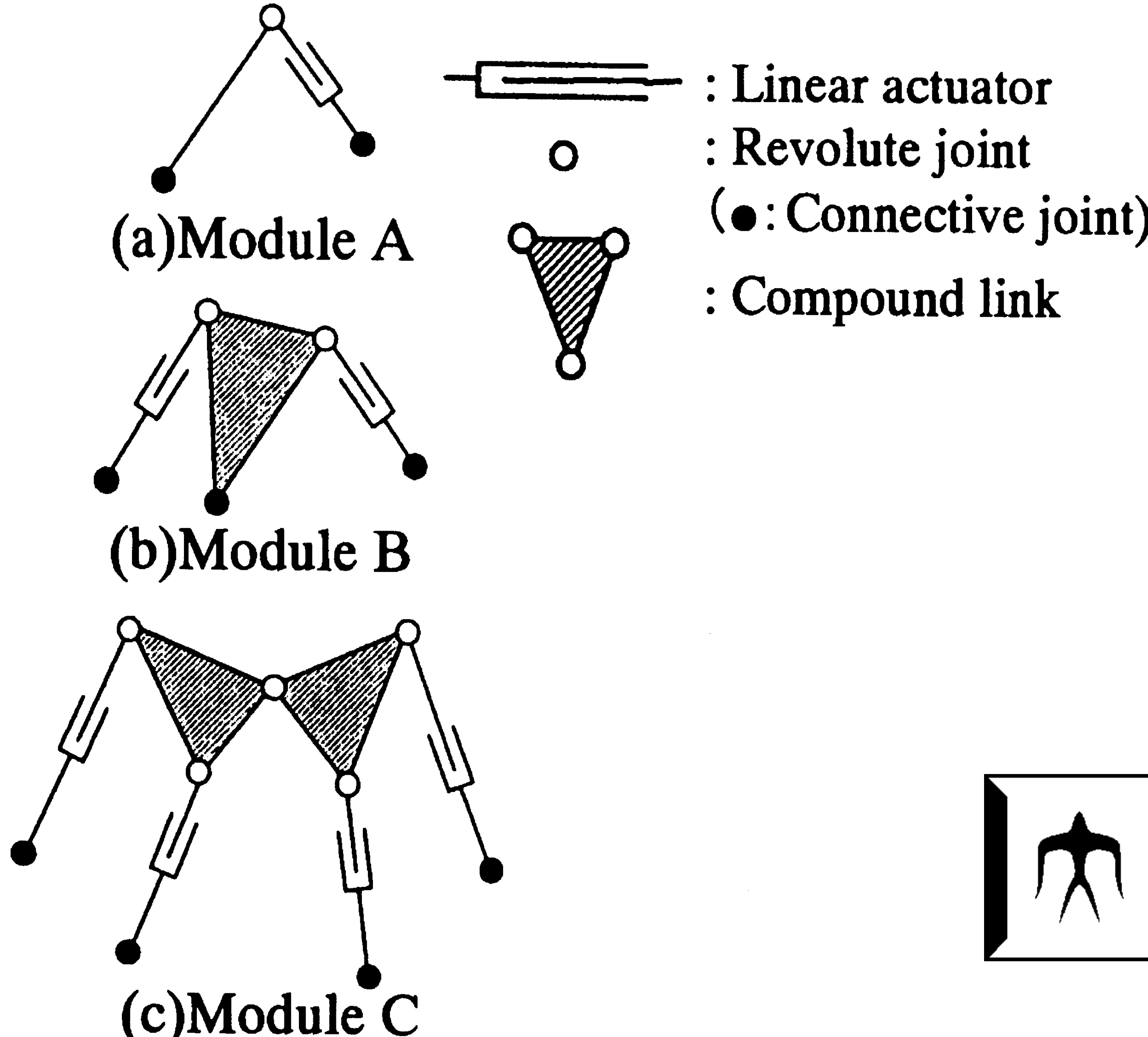
Result of number synthesis of network structure module

1DOF/1Act.

1DOF/2Act.

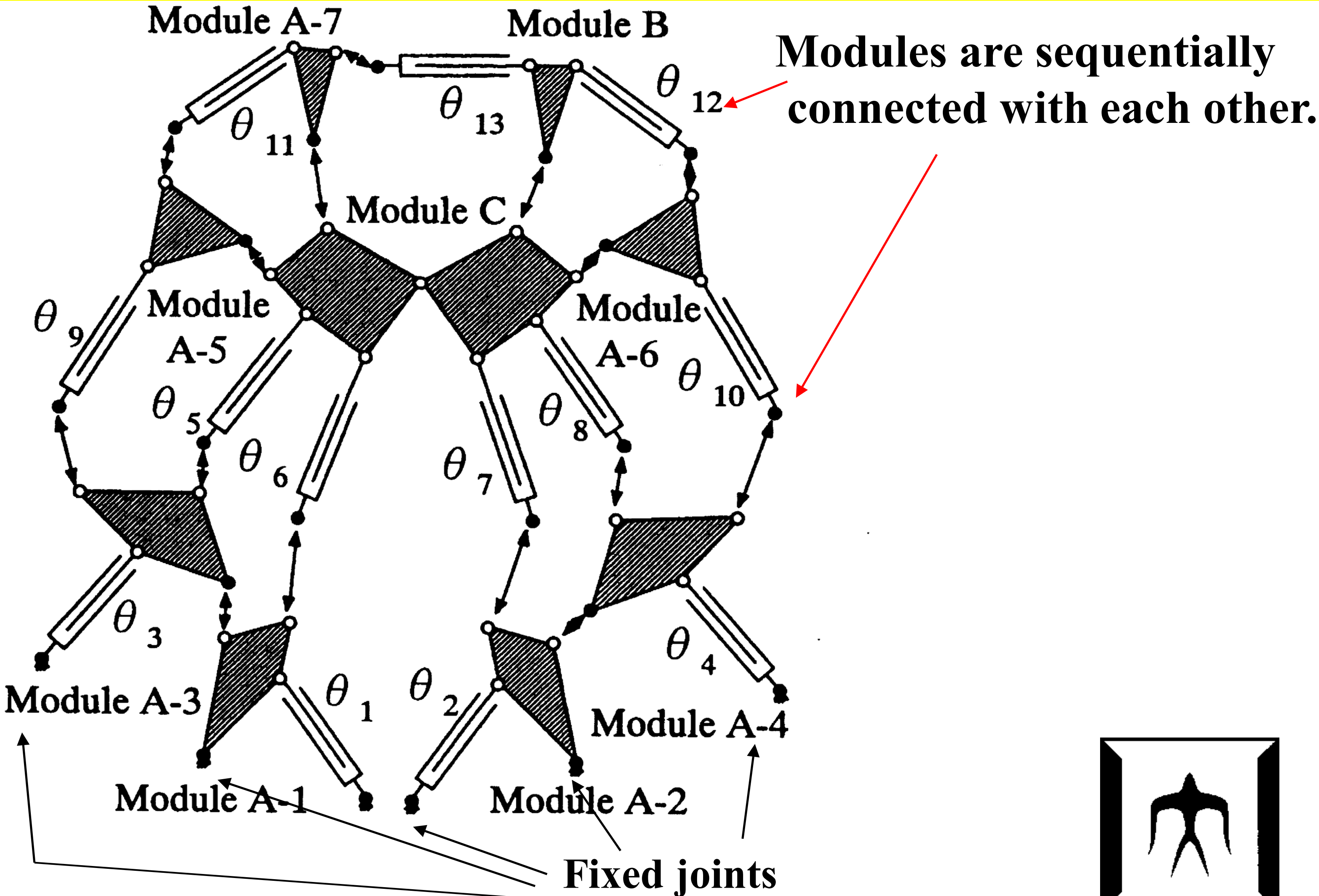
Overactuator

4DOF/4Act.

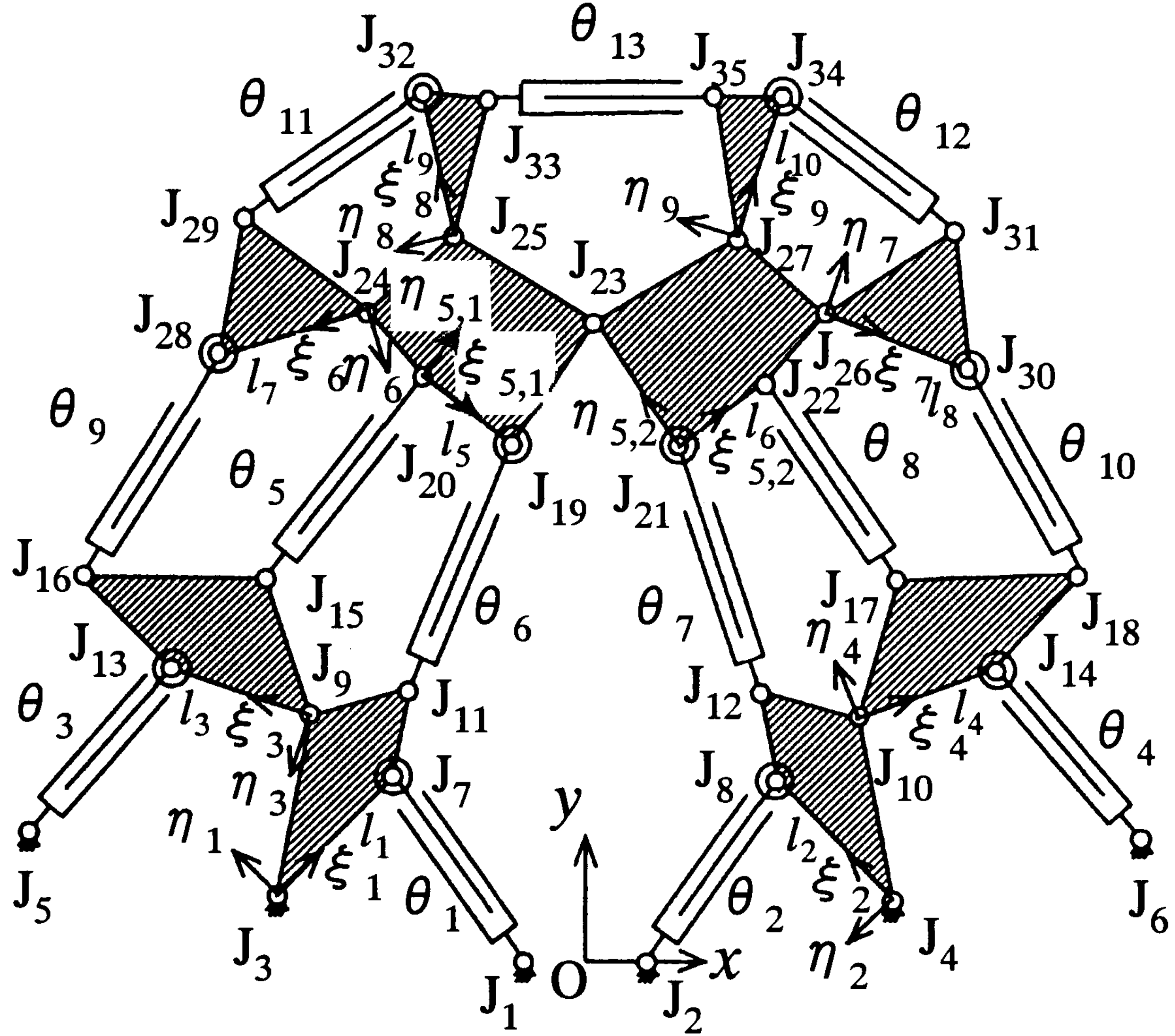


Examples of synthesized planar network structure modules

1.3 Synthesis of large scale network structure robot by connecting modules



Connection of network structure modules

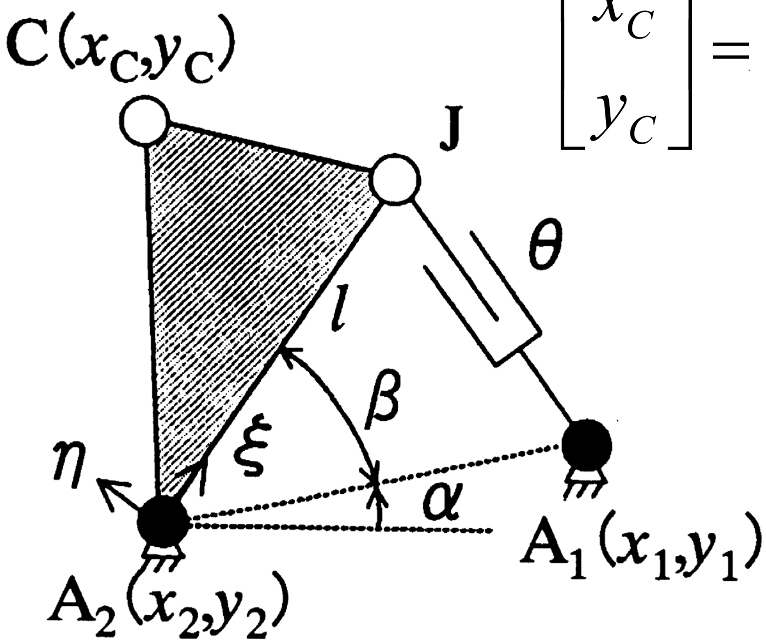


Synthesized network structure robot with 12 DOF 13 actuators

1.4 Forward kinematics of network-structure robot

Forward kinematics of network-structure modules

(a)Module A



Module A

$$\begin{bmatrix} x_C \\ y_C \end{bmatrix} = \begin{bmatrix} \xi\cos(\alpha\pm\beta)-\eta\sin(\alpha\pm\beta)+x_2 \\ \xi\sin(\alpha\pm\beta)-\eta\cos(\alpha\pm\beta)+y_2 \end{bmatrix}$$

where

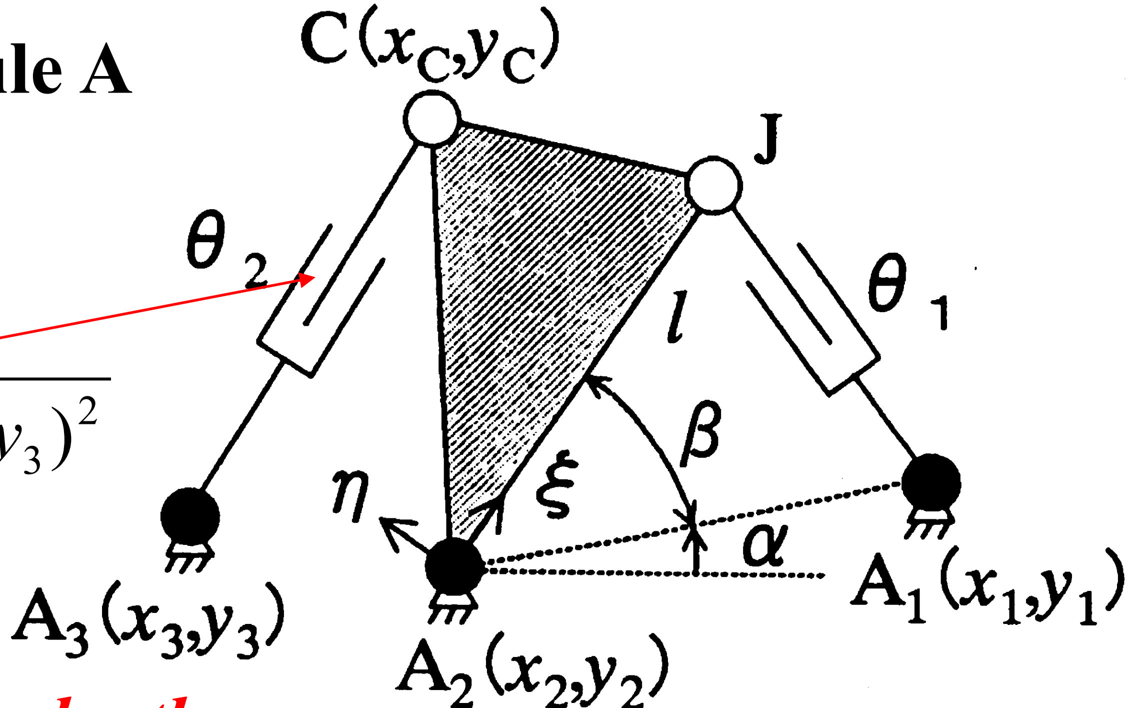
$$\left.\begin{aligned} \alpha &= \tan^{-1}\frac{y_1-y_2}{x_1-x_2} \\ \beta &= \cos^{-1}\frac{l^2+q^2-\theta^2}{2lq} \\ q &= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \end{aligned}\right\}$$

(b)Module B

Overactuator module

As same as module A

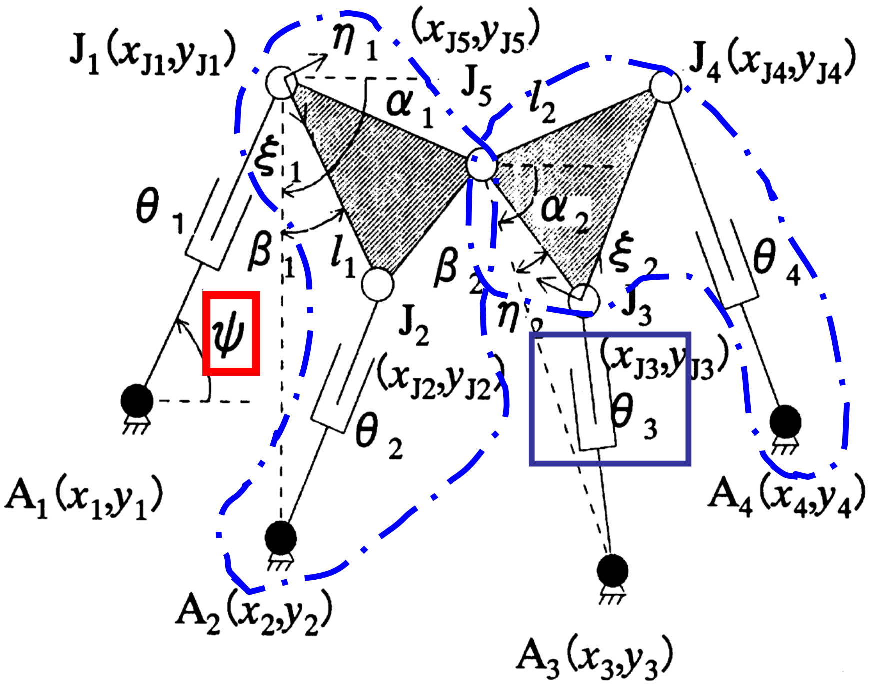
$$\theta_2 = \sqrt{(x_C - x_3)^2 + (y_C - y_3)^2}$$



To be determined dependently to another actuator input

Module B

(c)Module C



Module C

New variable ψ:

$$\begin{bmatrix} x_{J1} \\ y_{J1} \end{bmatrix} = \begin{bmatrix} \theta_1 \cos\psi + x_1 \\ \theta_1 \sin\psi + y_1 \end{bmatrix}$$

Forward kinematics of module A:

$J_1J_2A_2J_5$, $J_5J_4A_4J_3$

Calculation of position of J_3

Solve the nonlinear equation and obtain ψ

$$f(\psi) = [x_{J3}(\psi) - x_3]^2 + [y_{J3}(\psi) - y_3]^2 - \theta_3{}^2 = 0$$

Calculate all joint positions using ψ

Forward kinematics of robot

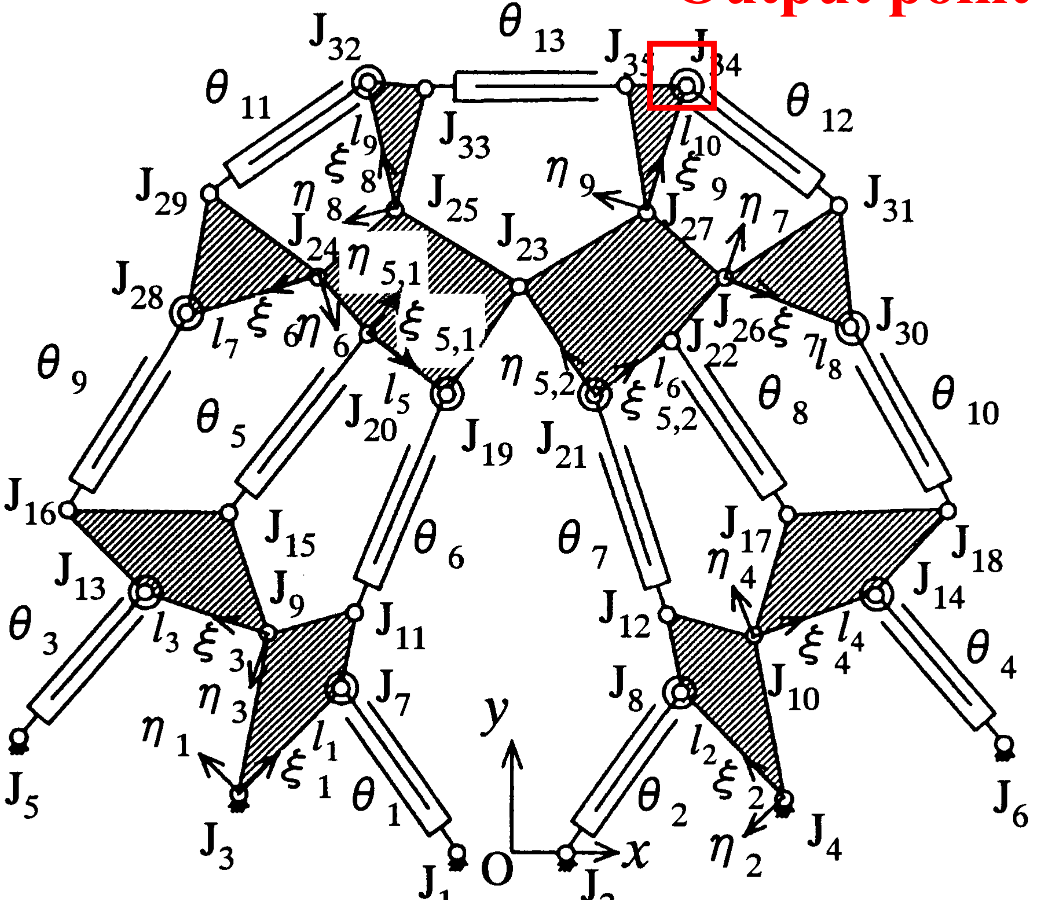
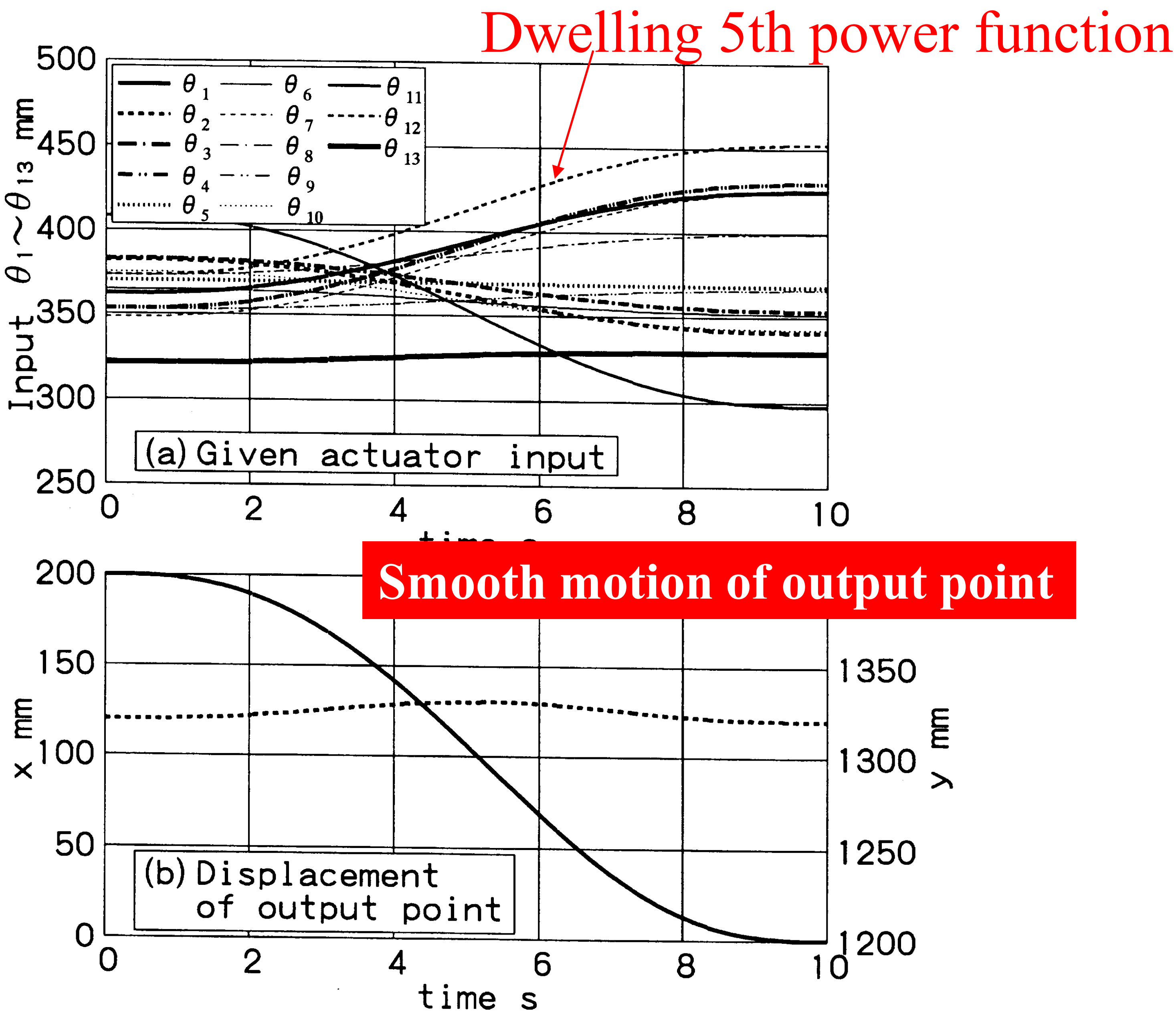


Table 1 Dimensions of a planar network-structure robot with 12 degrees-of-freedom (Unit: mm)

(x_1, y_1)	(-75.00, 0.00)	l_1	266.81
(x_2, y_2)	(-450.00, 75.00)	l_2	266.81
(x_3, y_3)	(-830.00, 115.00)	l_3	234.01
(x_4, y_4)	(75.00, 0.00)	l_4	234.01
(x_5, y_5)	(450.00, 75.00)	l_5	175.92
(x_6, y_6)	(830.00, 115.00)	l_6	175.92
		l_7	248.20
		l_8	248.20
		l_9	233.08
		l_{10}	233.08
(ξ_9, η_9)	(246.29, 164.19)	$J_2-\xi_1, \eta_1$	
(ξ_{10}, η_{10})	(246.29, 164.19)	$J_5-\xi_2, \eta_2$	
(ξ_{11}, η_{11})	(389.95, 82.10)	$J_2-\xi_1, \eta_1$	
(ξ_{12}, η_{12})	(389.95, -82.10)	$J_5-\xi_2, \eta_2$	
(ξ_{15}, η_{15})	(140.41, -187.21)	$J_9-\xi_3, \eta_3$	
(ξ_{16}, η_{16})	(421.22, -93.30)	$J_9-\xi_3, \eta_3$	
(ξ_{17}, η_{17})	(140.41, 187.21)	$J_{10}-\xi_4, \eta_4$	
(ξ_{18}, η_{18})	(421.12, 93.60)	$J_{10}-\xi_4, \eta_4$	
(ξ_{23}, η_{23})	(171.67, 236.81)	$J_{20}-\xi_{5,1}, \eta_{5,1}$	
	(4.25, 236.81)	$J_{21}-\xi_{5,2}, \eta_{5,2}$	
(ξ_{24}, η_{24})	(-136.70, 29.17)	$J_{20}-\xi_{5,1}, \eta_{5,1}$	
(ξ_{25}, η_{25})	(-93.93, 209.49)	$J_{20}-\xi_{5,1}, \eta_{5,1}$	
(ξ_{26}, η_{26})	(312.63, 29.17)	$J_{21}-\xi_{5,2}, \eta_{5,2}$	
(ξ_{27}, η_{27})	(269.85, 209.49)	$J_{21}-\xi_{5,2}, \eta_{5,2}$	
		$J_{24}-\xi_6, \eta_6$	
		$J_{25}-\xi_7, \eta_7$	
		$J_{26}-\xi_8, \eta_8$	
		$J_{27}-\xi_9, \eta_9$	

Can be calculated with sequential calculation of forward kinematics of modules

The planar network-structure robot with 12 DOF and 13 actuators



One example of forward kinematics of robot

1.5 Inverse kinematics using representative points on module

Inverse kinematics of module

Desired displacement of module to generate desired position of output point

⬇

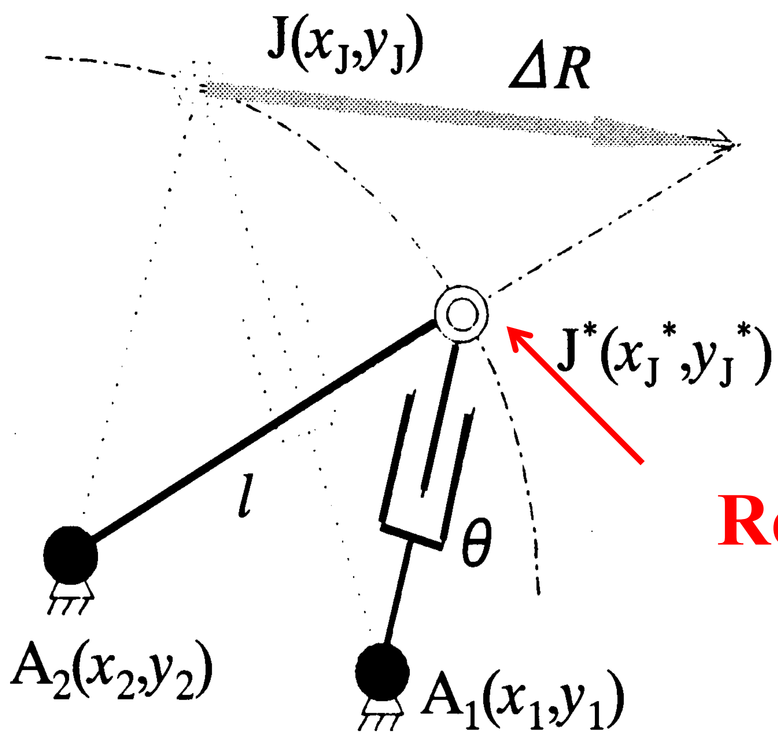
Describe the displacement of module with motion of one or a few *representative points* on the module

⬇

For Less DOF : The nearest motion for the desired displacement

For more DOF : Increase representative points

(1) Module A and B



Desired displacement for representative point

Representative point

Solution as the nearest displacement:

$$\theta = \sqrt{(x_J^* - x_1)^2 - (y_J^* - y_1)^2}$$

where

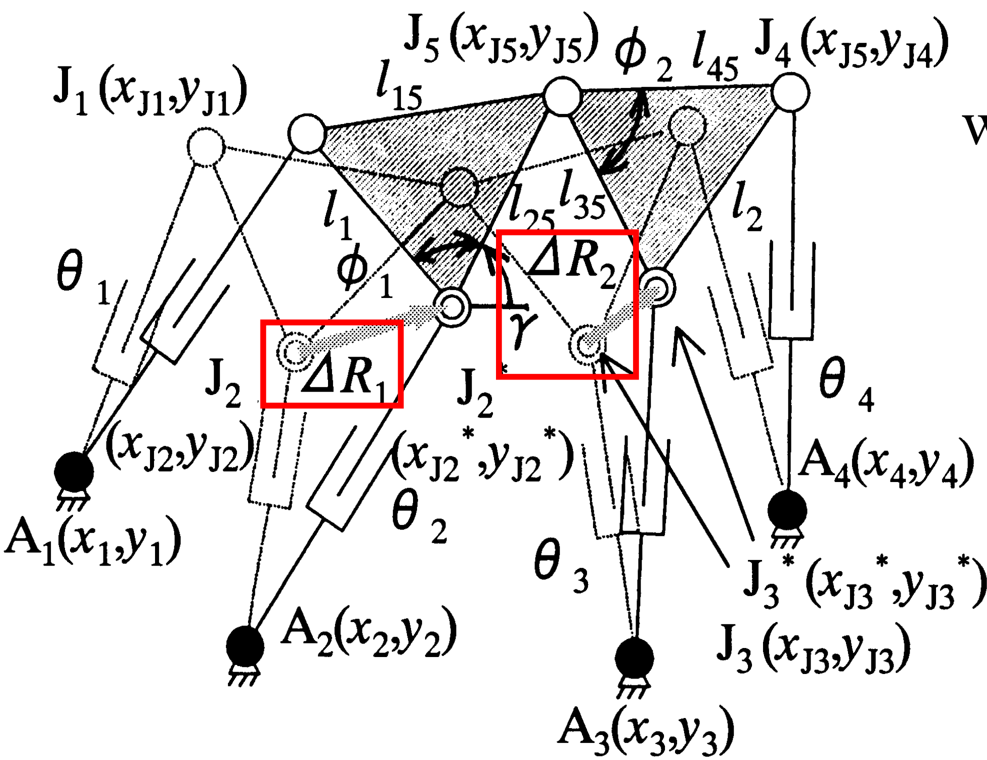
$$\begin{bmatrix} x_J^* \\ y_J^* \end{bmatrix} = \frac{l}{\sqrt{(x_J - x_2 + \Delta x_J)^2 + (y_J - y_2 + \Delta y_J)^2}} \begin{bmatrix} x_J - x_2 + \Delta x_J \\ y_J - y_2 + \Delta y_J \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

(2)Module C

"4 DOF $\rightarrow$ 2 Representative points"

$$\theta_k = \sqrt{(x^*_{Jk} - x_k)^2 + (y^*_{Jk} - y_k)^2}$$

(k=1～4)



where

$$\begin{bmatrix} x_{J5} \\ y_{J5} \end{bmatrix} = \begin{bmatrix} l_{25}\cos\gamma + x^*_{J2} \\ l_{25}\sin\gamma + y^*_{J2} \end{bmatrix}$$

$$\gamma = \tan^{-1}\frac{y^*_{J3} - y^*_{J2}}{x^*_{J3} - x^*_{J2}} \pm \cos^{-1}\frac{l_{25}^2 + l_{23}^2 - l_{35}^2}{2 l_{25} l_{23}}$$

$$l_{23} = \sqrt{(x^*_{J2} - x^*_{J3})^2 + (y^*_{J2} - y^*_{J3})^2}$$

$$\left.\begin{aligned} \begin{bmatrix} x_{J1} \\ y_{J1} \end{bmatrix} &= \begin{bmatrix} l_1\cos\phi_1\cos\phi_{25} - l_1\sin\phi_1\sin\phi_{25} + x^*_{J2} \\ l_1\cos\phi_1\sin\phi_{25} + l_1\sin\phi_1\cos\phi_{25} + y^*_{J2} \end{bmatrix} \\ \begin{bmatrix} x_{J4} \\ y_{J4} \end{bmatrix} &= \begin{bmatrix} l_{45}\cos\phi_2\cos\phi_{53} - l_{45}\sin\phi_2\sin\phi_{53} + x_{J5} \\ l_{45}\cos\phi_2\sin\phi_{53} + l_{45}\sin\phi_2\cos\phi_{53} + y_{J5} \end{bmatrix} \end{aligned}\right\}$$

1.6 Inverse kinematics of robot using representative points on modules

Inverse kinematics of Robot

Inverse kinematics calculation of modules for given desired displacement of representative points on modules

It is difficult to obtain precise desired output motion of robot.

⬇

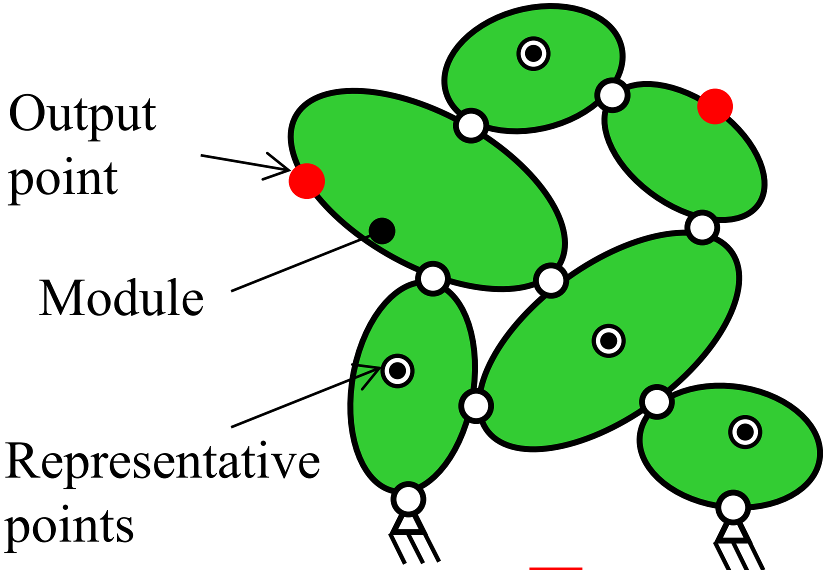
Forward kinematics calculation of robot to obtain actual output of robot.

⬇

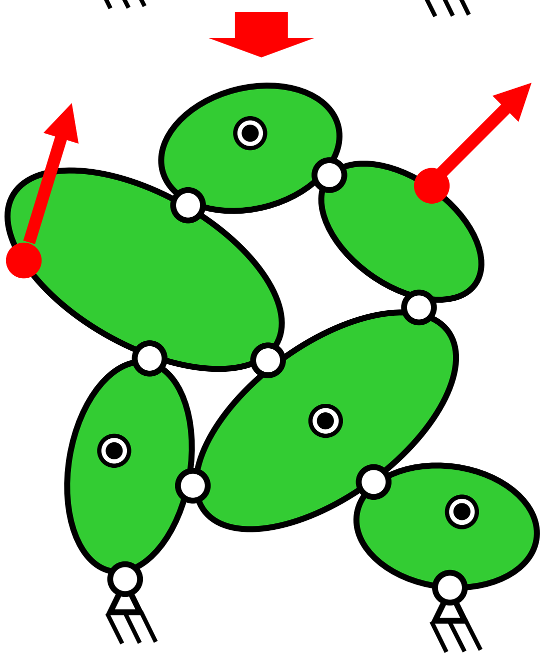
Calculate output motion error and set the error as the next desired displacement of robot.

⬇

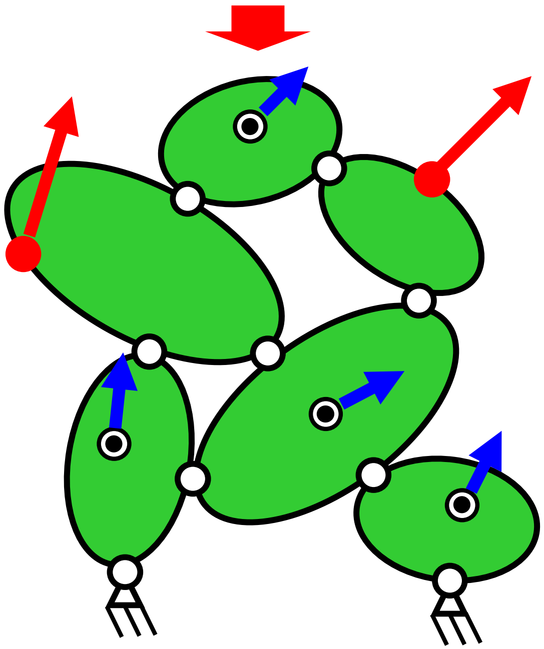
Iterative calculation to make the output error converged



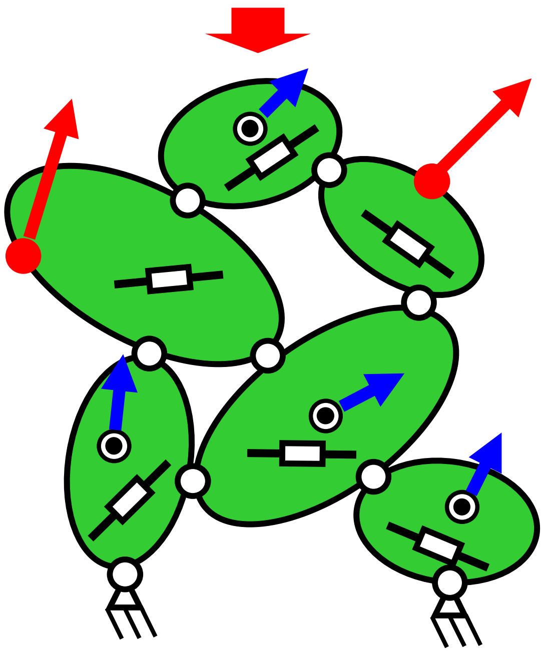
(1)Set representative points on module



(2)Designate the desired position of output point

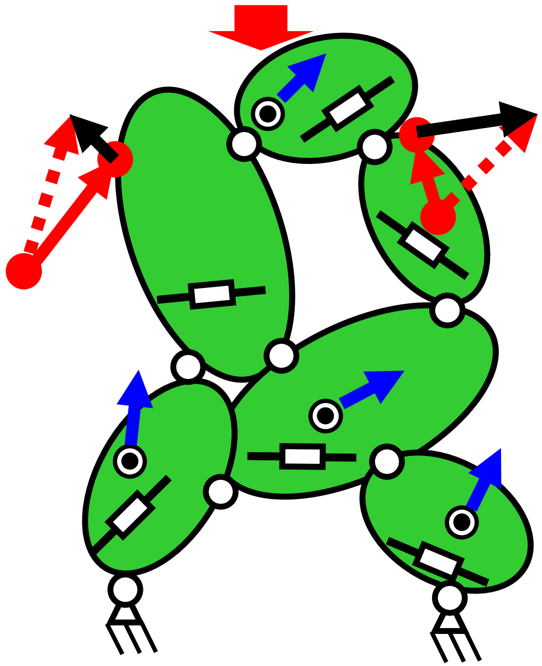


(3)Calculate desired position of representative points



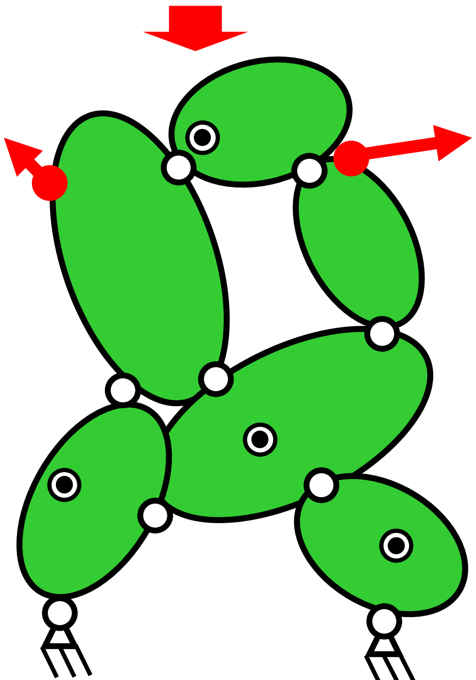
(4)Inverse kinematics calculation of each module for the representative point

Note: Each module cannot generate precise motion because it has only 1 DOF.

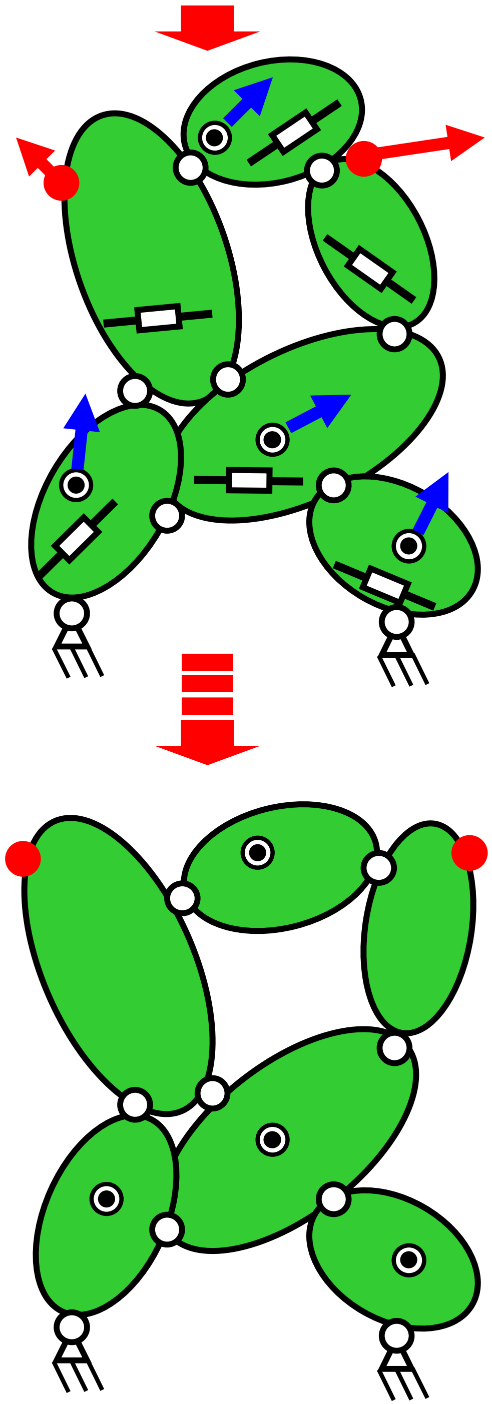


(5)Forward kinematics calculation based on inverse kinematics of modules

Note: Output errors occur.



(6)Set output errors as new desired positions of output points



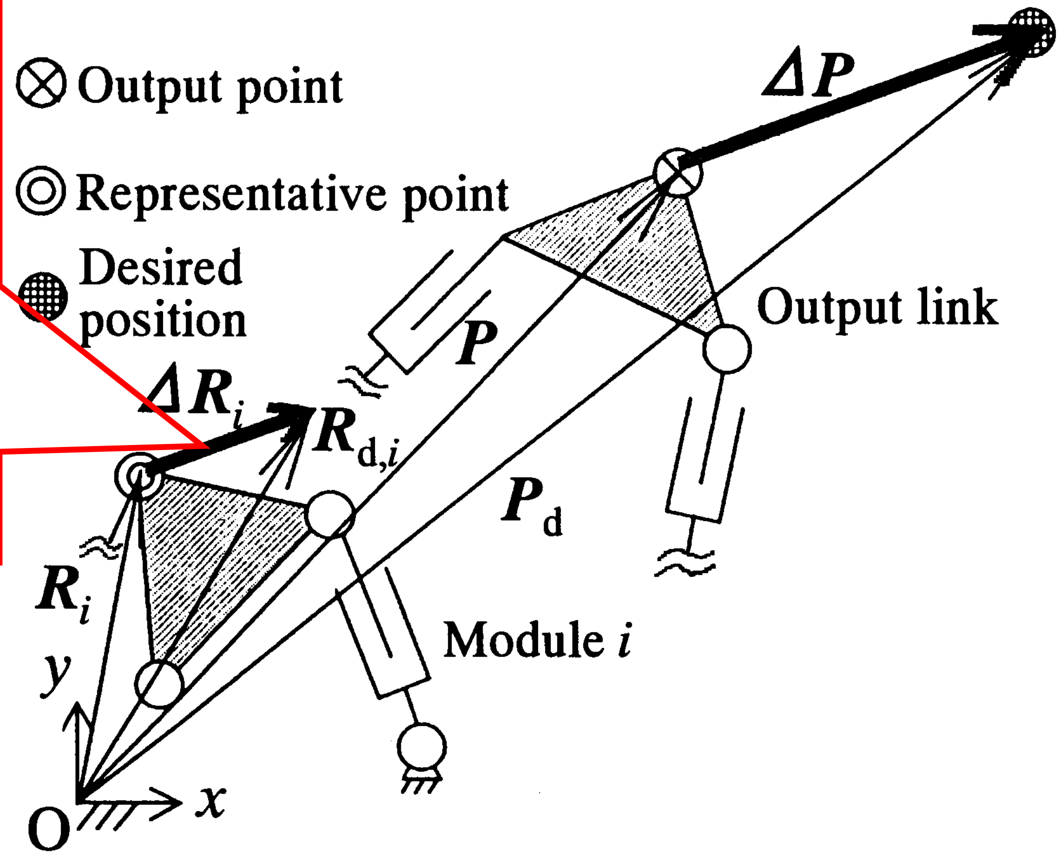
(7)Repeat the inverse kinematics of modules and forward kinematics of the robot until the output errors are converged.

(8)Output points will reach their desired position.

Calculation of desired displacement of representative point

Direction:
As same as the desired displacement of the output point

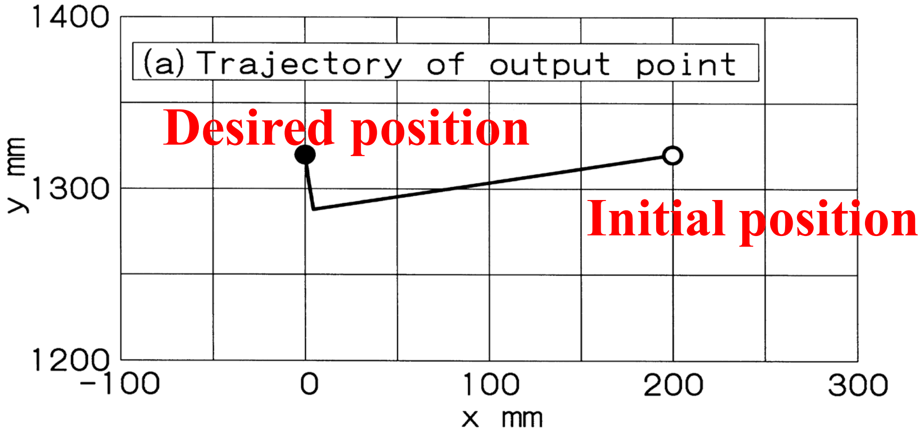
Amplitude:
In proportion to the distance from origin

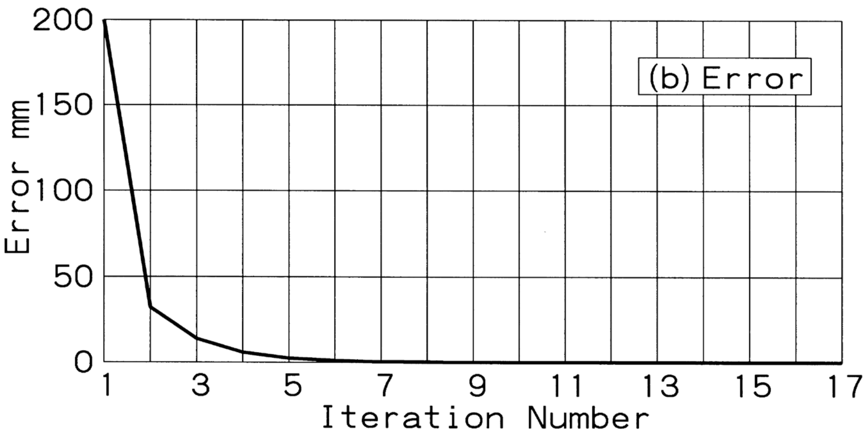


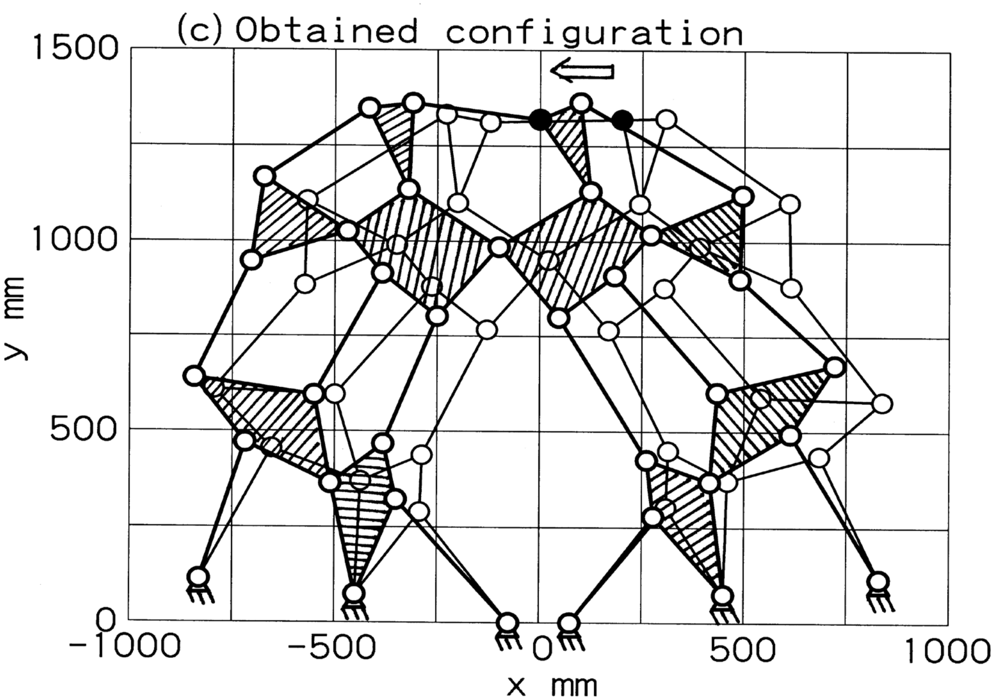
$$\Delta \boldsymbol{R}_i = \frac{|\boldsymbol{R}_i|}{|\boldsymbol{P}|}\Delta \boldsymbol{P} = \frac{\sqrt{x_{R,i}^2 + y_{R,i}^2}}{\sqrt{x_P^2 + y_P^2}}\begin{bmatrix}\Delta x_P \\ \Delta y_P\end{bmatrix}$$

Example of inverse kinematics of robot

Convergence process



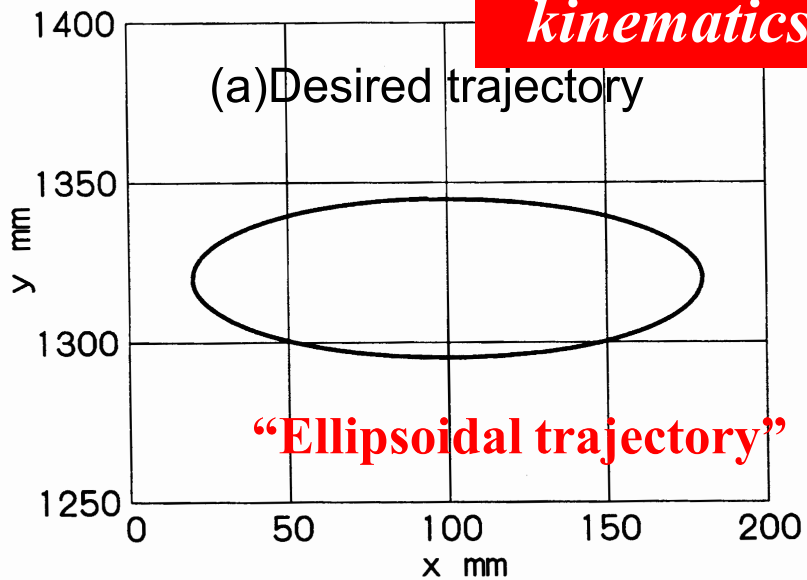




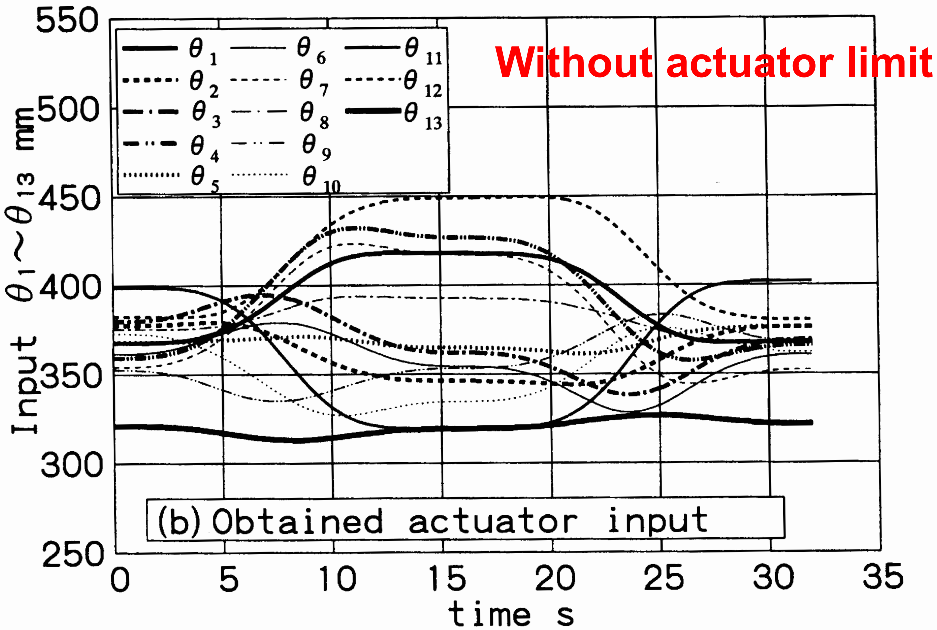
Positioning error was converged with several iterations.

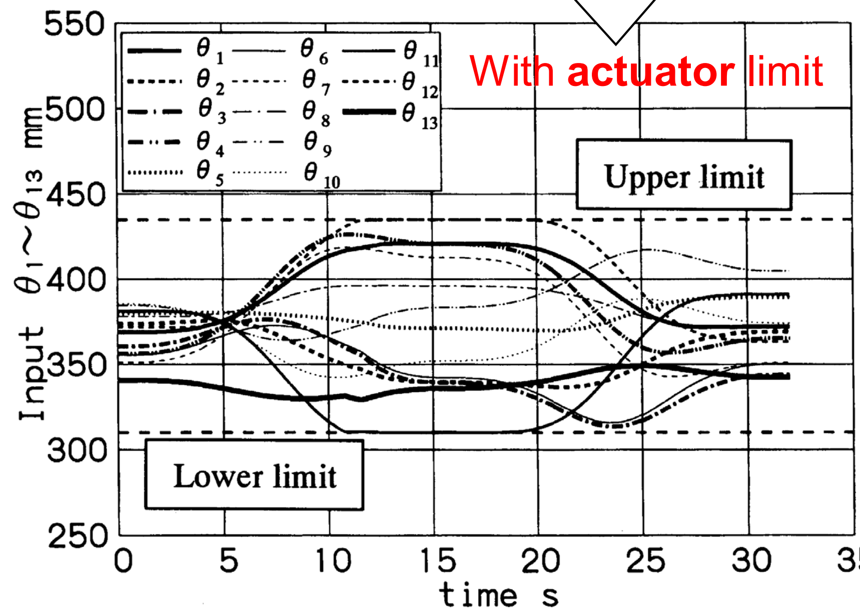
CP control

It is possible to continuously analyze the inverse kinematics by taking account of actuator limit.



Actuator inputs are fixed at the limit values if the solution of inverse kinematics excess the limit value.





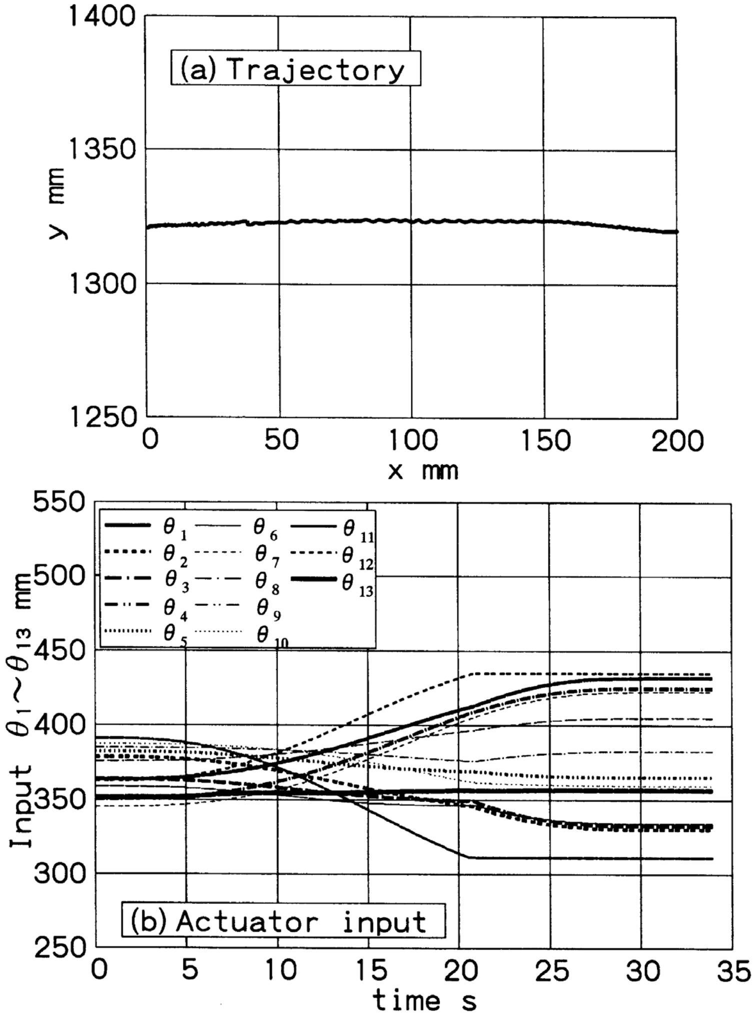
(b)Results of inverse kinematics (Actuator inputs)

1.7 Experiments of motion control

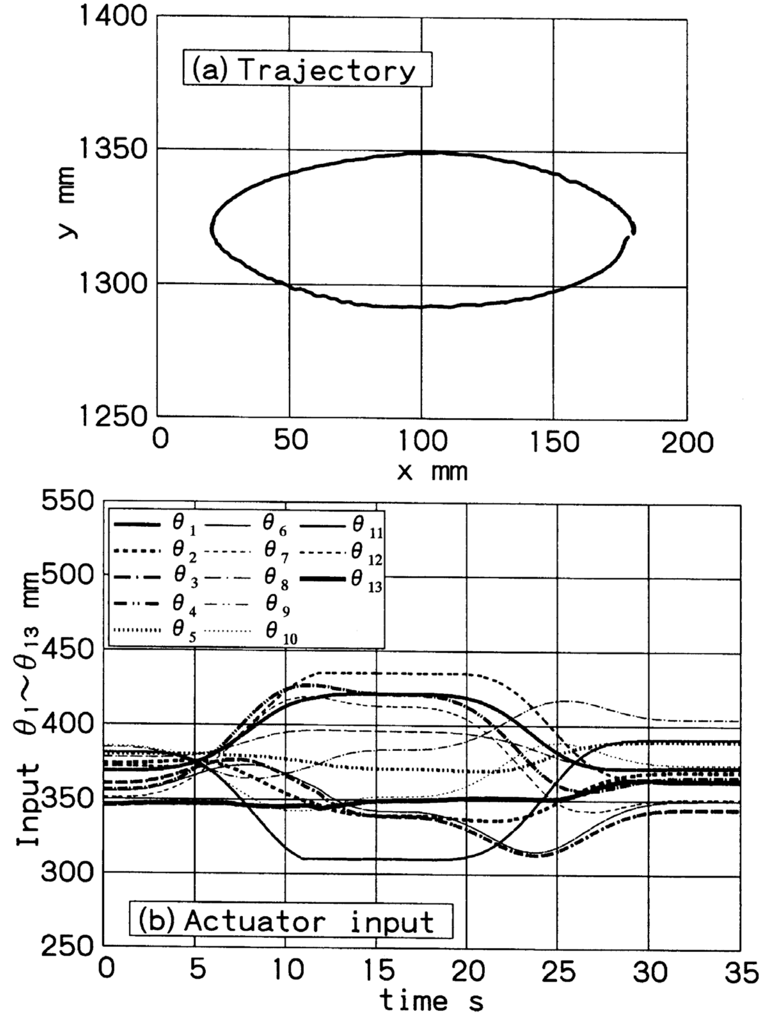
Prototype

Photograph of a prototype

Results of CP control



(1)Straight line trajectory



(2)Ellipsoidal trajectory

By utilizing representative points on modules,
CP control of network-structure robot was achieved.

2. Motion control of overactuator mechanisms with elastic elements

2.1 Issues to be solved in overactuator mechanism

(1)Cooperative control of overactuators to avoid the interference is not easy because of servo error.

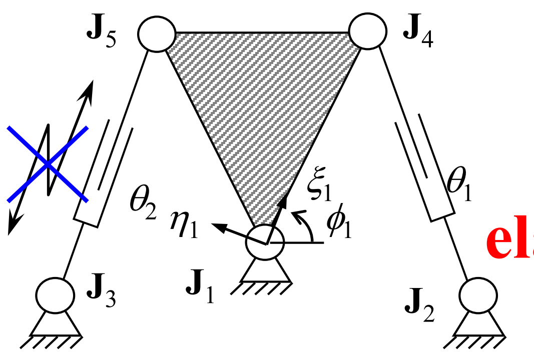
(2)Flexibility should be installed.

By adding elastic elements to relax the interference between overactuators, flexible motion control can be achieved.

2.2 Planar network structure robot with elastic elements

Proposal : Relaxation of the interference by adding elastic elements

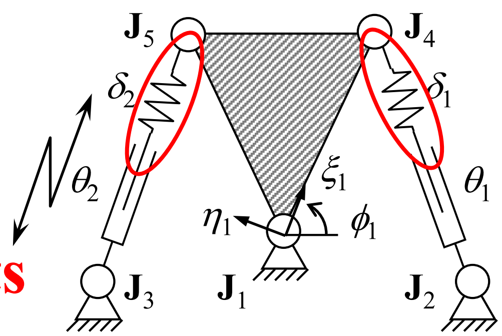
Both actuators should be driven cooperatively.



Overactuator mechanism

Adding elastic elements

Each actuator can be driven independently.



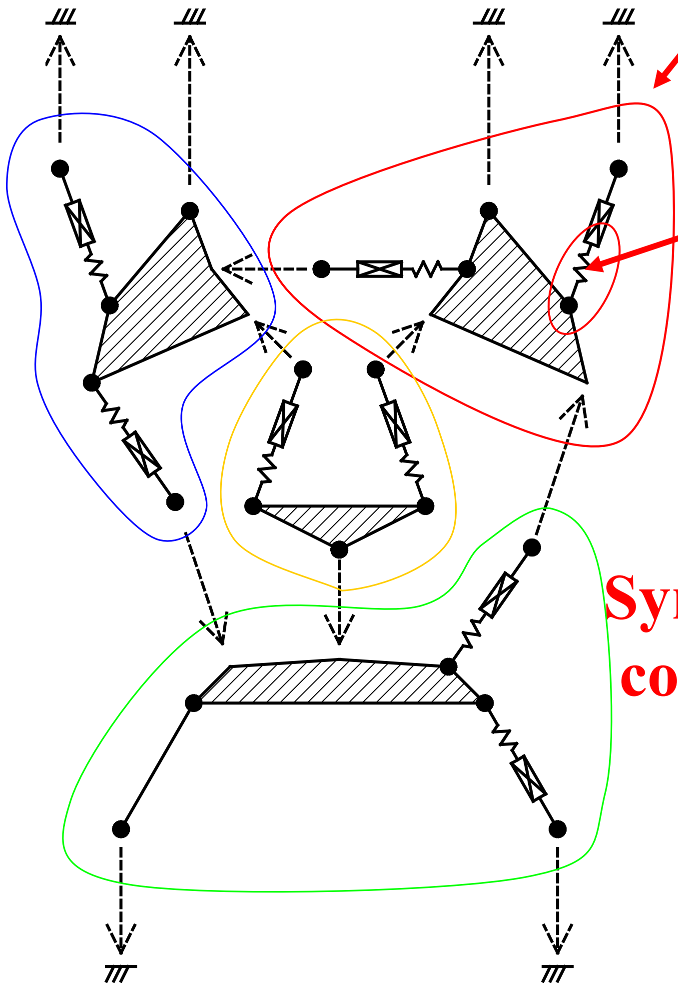
Overactuator mechanism with elastic element

To analyze a motion

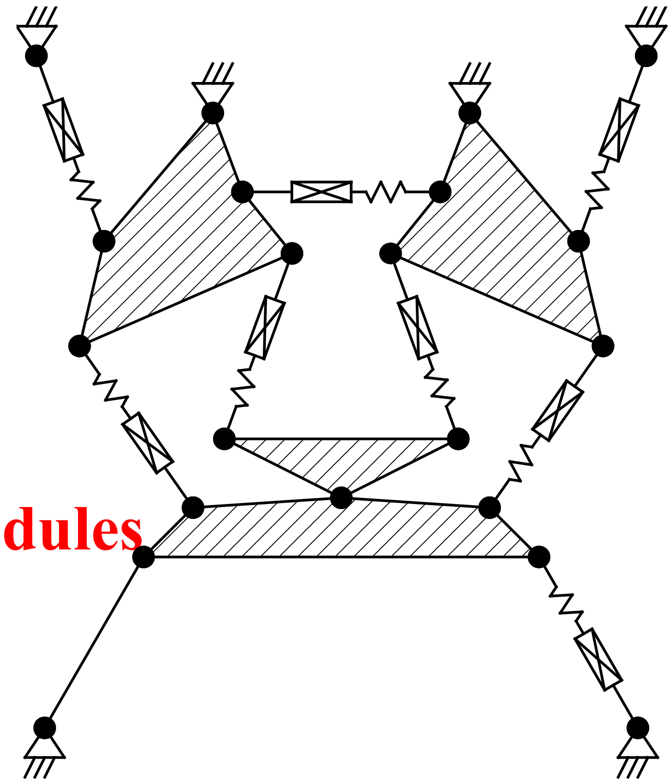
Not only kinematic relations, but also force balance should be taken into account.

Module: minimum unit of Network

A coil spring is added serially to each actuator



Synthesized by connecting modules



Number of actuator > DOF

2.3 Forward kinematics

Output Degrees-of-Freedom

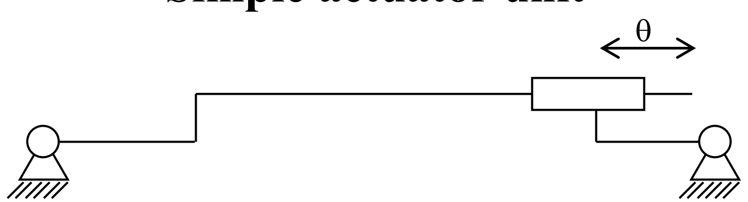
Configuration of the robot can be determined with distance between revolute joints of lineractuator unit.

Output DOF

DOF in case additional coil springs are ignored.

Ex. Linearactuator unit

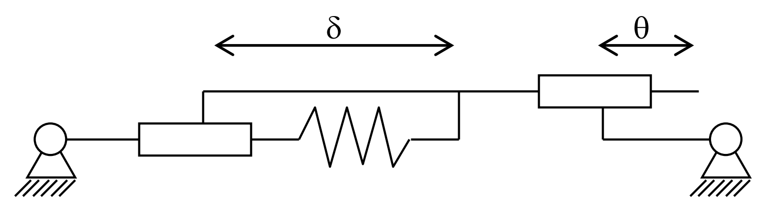
Simple actuator unit



$F = 3 \times 2 - 2 \times 3 = \mathbf{0}$

$<$

Elastic actuator unit



$F = 3 \times 3 - 2 \times 4 = \mathbf{1}$

Elastic actuator units have one more DOF because of coil spring.

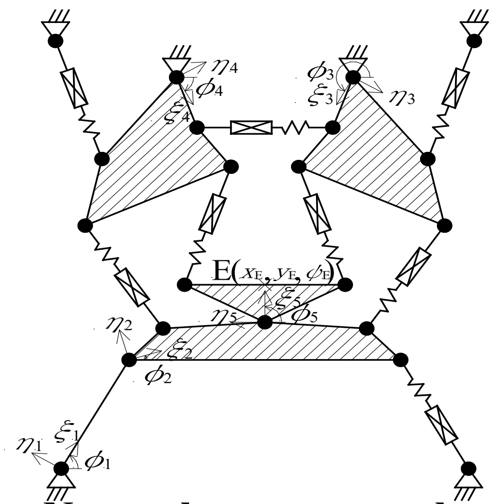
When coil spring is ignored

Output DOF of elastic actuator unit becomes zero.

Configuration-Determining Parameter

Output DOF of actuator unit = 0

⇨ Actuator units don't affect the output DOF of mechanism.

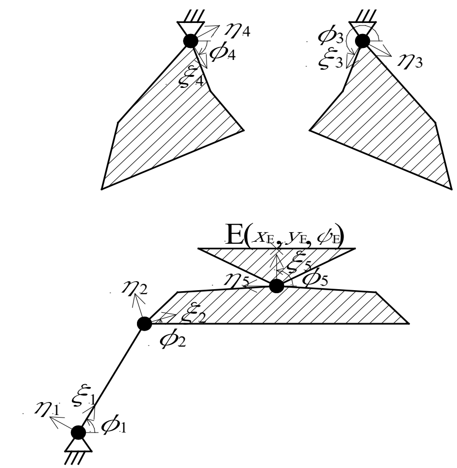


Network-structure robot with elastic elements

Output DOF mechanism

Remove

actuator units



Kinematic chains obtained from robot

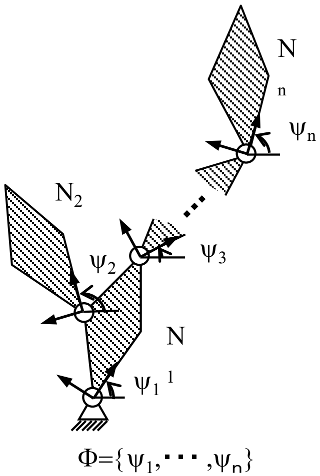
DOF of kinematic chains

The configuration of mechanism can be represented by the chains.

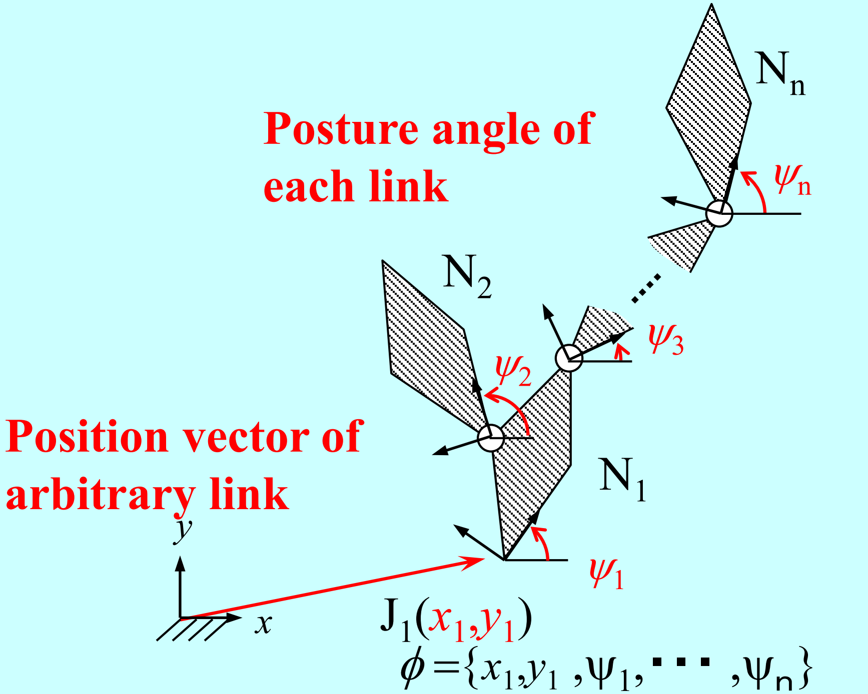
Configuration determining parameter(C.D.P.)

Necessary and sufficient parameter(s) to represent positions and postures of all links.

The kinematic chains can be classified to 4 cases.
In each case configuration-determining parameters can be set.



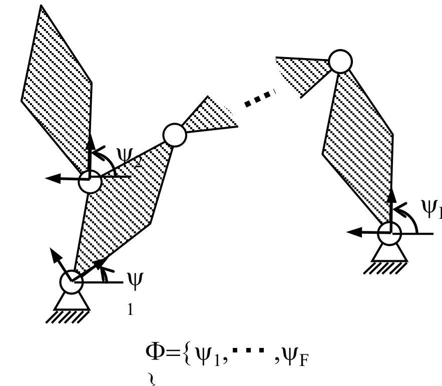
(a) Open chain connected with frame



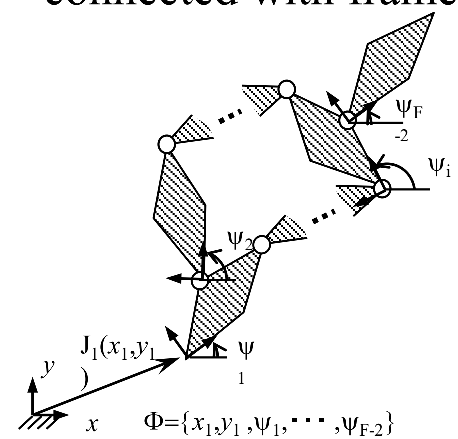
$$F = 3n - 2(n-1) = n + 2$$

The number of C.D.P. should be equal to DOF.

(b) Open loop chain disconnected with frame



(c) Closed-loop chain connected with frame



(d) Closed-loop Chain disconnected with frame

Positions of all joint in mechanism can be calculated as a function of C.D.P. $\Phi = \{\phi_1, \cdots, \phi_c\}$.

Elastic Input/output Equation

Joint positions are given as functions of $\boldsymbol{\Phi}$.

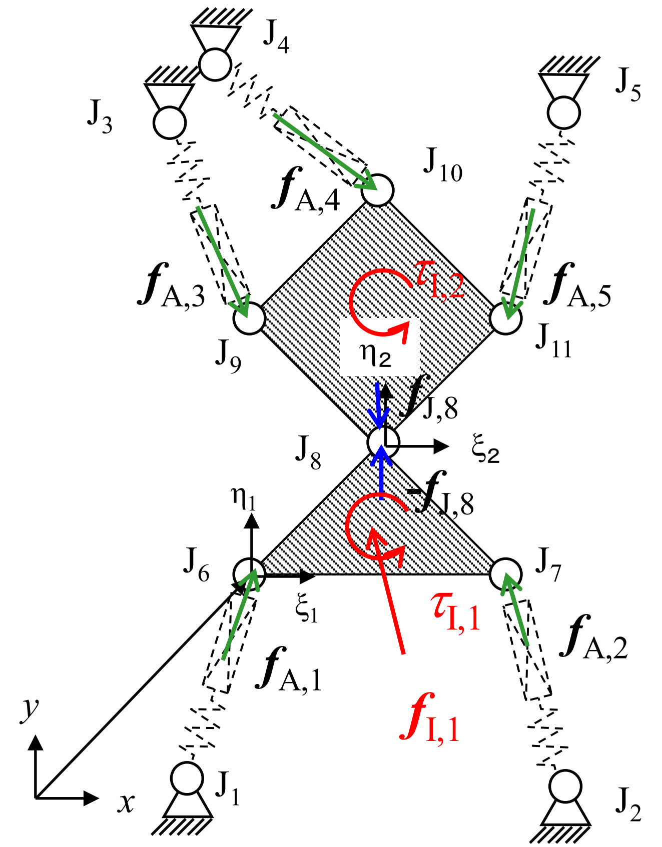
⇩

Spring force can be calculated as

$$\boldsymbol{f}_{A,i}(\theta_i,\boldsymbol{\Phi}) = k_i\left[\left|\boldsymbol{J}_m(\boldsymbol{\Phi}) - \boldsymbol{J}_n(\boldsymbol{\Phi})\right| - (L_i - \theta_i)\right]\cdot \boldsymbol{a}_i$$

Joint forces, $\boldsymbol{f}_{\mathrm{J}}$, are set on revolute joints between compound links.

Imaginary forces, $\boldsymbol{f}_{\mathrm{I}}$, and moments, τ_{I} are assumed on C.D.P. to balance.



A system of force balance equations

$$\begin{cases}
\boldsymbol{f}_{\mathrm{J},8} + \boldsymbol{f}_{\mathrm{I},1} + \boldsymbol{f}_{\mathrm{A},1} + \boldsymbol{f}_{\mathrm{A},2} = \boldsymbol{0} \\
-\boldsymbol{f}_{\mathrm{J},8} + \boldsymbol{f}_{\mathrm{A},3} + \boldsymbol{f}_{\mathrm{A},4} + \boldsymbol{f}_{\mathrm{A},5} = \boldsymbol{0} \\
\boldsymbol{J}_8 \times \boldsymbol{f}_{\mathrm{J},8} + \tau_{\mathrm{I},1} + \boldsymbol{J}_6 \times \boldsymbol{f}_{\mathrm{A},1} + \boldsymbol{J}_7 \times \boldsymbol{f}_{\mathrm{A},2} = 0 \\
-\boldsymbol{J}_8 \times \boldsymbol{f}_{\mathrm{J},8} + \tau_{\mathrm{I},2} + \boldsymbol{J}_9 \times \boldsymbol{f}_{\mathrm{A},3} + \boldsymbol{J}_{10} \times \boldsymbol{f}_{\mathrm{A},4} + \boldsymbol{J}_{11} \times \boldsymbol{f}_{\mathrm{A},5} = 0
\end{cases}$$

Unknown variables (the terms $\boldsymbol{f}_{\mathrm{J},8}$, $\boldsymbol{f}_{\mathrm{I},1}$, $\tau_{\mathrm{I},1}$, $\tau_{\mathrm{I},2}$)

Force balance equations can be summarized as

$$3n'\text{ rows}\left\{\begin{array}{c}\left[\begin{array}{c|c} \boldsymbol{C}_1(\boldsymbol{\Phi}) & \begin{matrix}1 & & 0\\ & \ddots & \\ 0 & & 1\end{matrix} \\ \hline \boldsymbol{C}_2(\boldsymbol{\Phi}) & \boldsymbol{0} \end{array}\right]\end{array}\right. \cdot \left\{\begin{array}{c} \boldsymbol{f}_{J,1}\\ \vdots \\ \boldsymbol{f}_{J,J_R} \\ \hline \boldsymbol{f}_I \\ \boldsymbol{\tau}_I \end{array}\right\} \begin{array}{l} \left.\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right\} 2J_R \\ \left.\vphantom{\begin{matrix}a\\a\end{matrix}}\right\} F' \end{array} + \{\boldsymbol{D}(\boldsymbol{\theta},\boldsymbol{\Phi})\} = \boldsymbol{0}$$

$$\underbrace{}_{2J_R} + \underbrace{}_{F'} \text{ columns} = 3n' \quad \Longleftrightarrow \quad \begin{array}{l}\text{Eq. of DOF}\\ F' = 3n' - 2J_{\mathrm{R}}\end{array}$$

$\Rightarrow$ These equations can be analytically solved.

Thus Imaginary force $\boldsymbol{F}_{\mathbf{I}} = (\boldsymbol{f}_{\mathbf{I}}, \boldsymbol{\tau}_{\mathbf{I}})^{\mathrm{T}}$ is derived as

$$\Rightarrow \boldsymbol{F}_I(\boldsymbol{\theta},\boldsymbol{\Phi}) = [\boldsymbol{A}(\boldsymbol{\Phi})]\cdot\boldsymbol{\theta} + \{\boldsymbol{B}(\boldsymbol{\Phi})\}$$

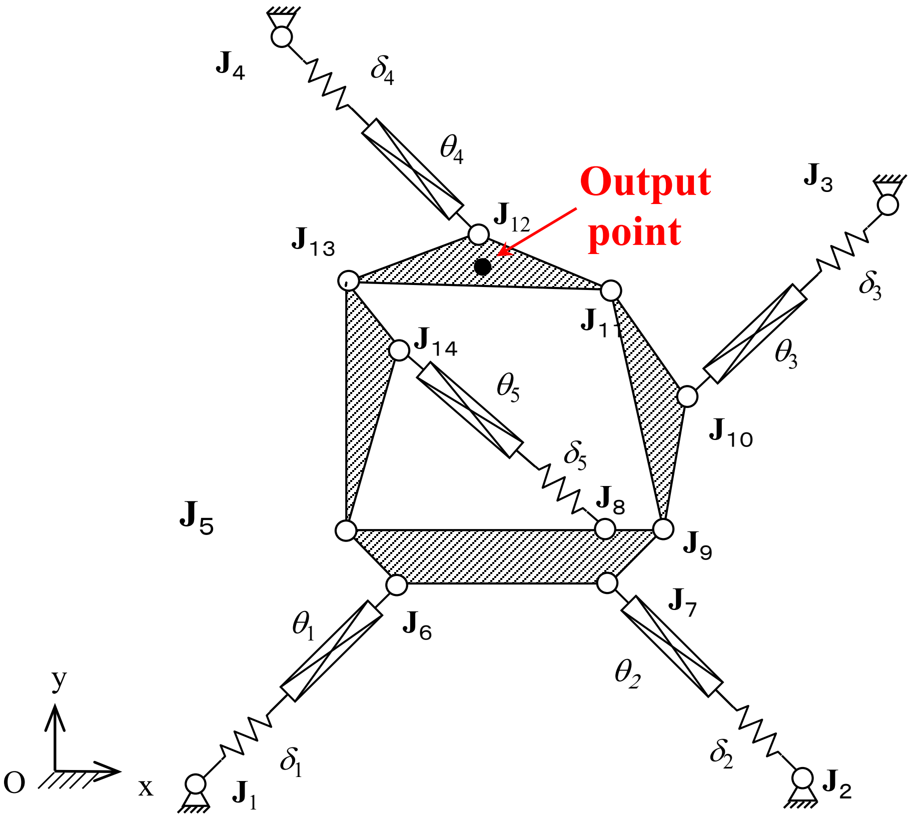
Then, set imaginary forces as zero

$$[\boldsymbol{A}(\boldsymbol{\Phi})]\cdot\boldsymbol{\theta} + [\boldsymbol{B}(\boldsymbol{\Phi})] = \boldsymbol{0}$$

This equation is called as ***'Elastic Input/Output Equation'.***

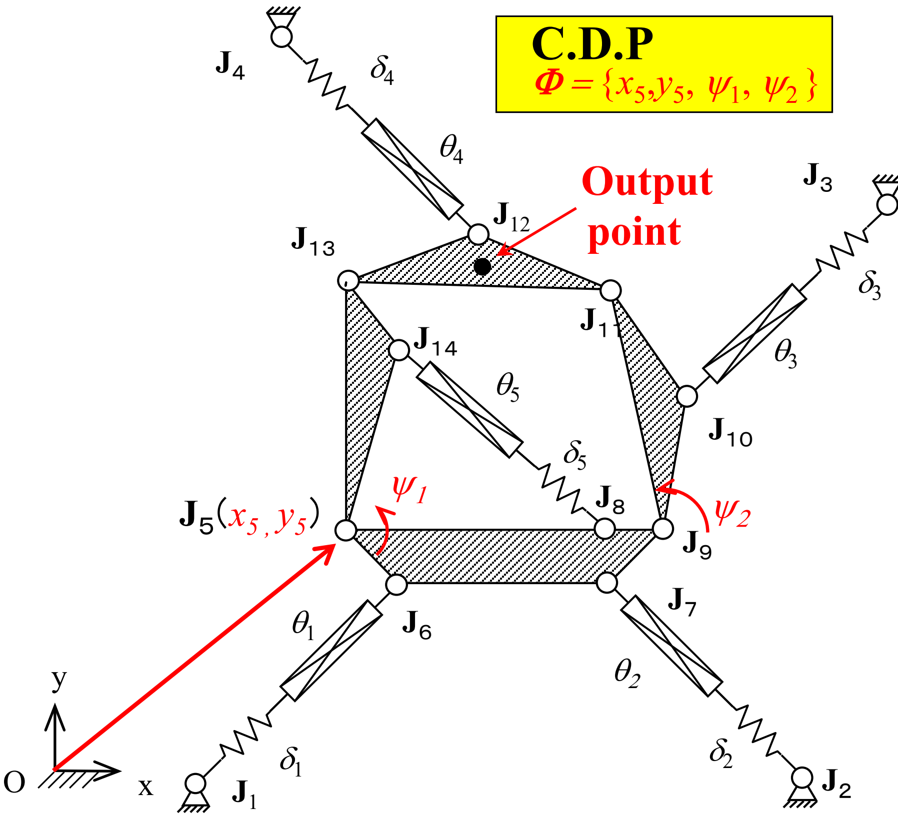
By solving elastic input/output equation for C.D.P. with Newton-Raphson method, forward kinematics can be achieved.

Example of analysis



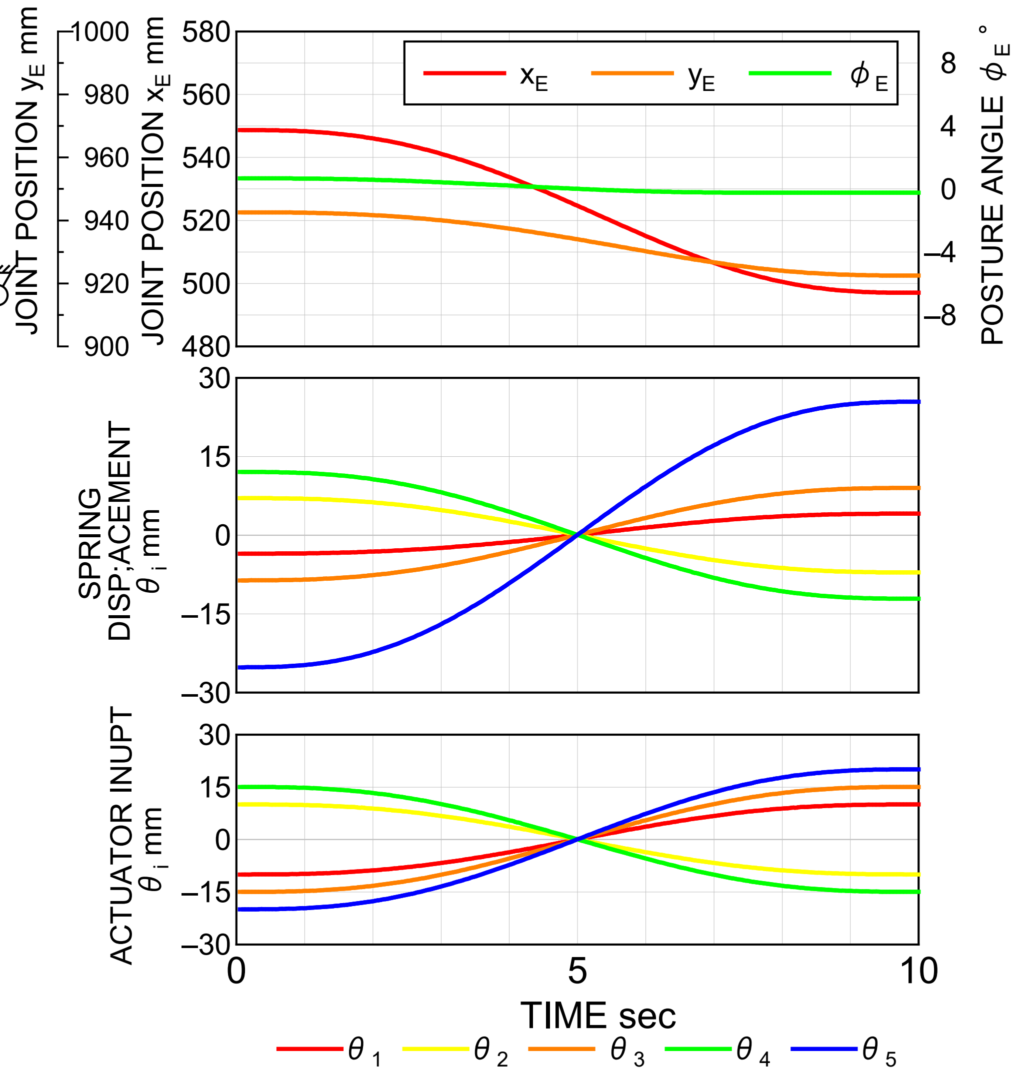
A planar network-structure robot
with 4 D.O.F and 5 elastic actuator units

Example of analysis



A planar network-structure robot

with 4 D.O.F and 5 elastic actuator units



Input motions are given as 5th power dwelling function.

Because output displacement varied smoothly as input, it was confirmed that the proposed forward kinematics was correct and effective.

2.4 Inverse Kinematics

Iterative Calculation to Minimize Output Compliance

Redundant mechanism ⇨ **To obtain an optimum solution based on a criterion**

Gradient Projection Method :

Forward kinematics: $\boldsymbol{r} = \boldsymbol{g}(\boldsymbol{\theta})$

⇨ $\Delta \boldsymbol{r} = [\boldsymbol{J}(\boldsymbol{\theta})] \cdot \Delta \boldsymbol{\theta}$

⇩ General solution of inverse kinematics

$$\Delta \boldsymbol{\theta} = \underline{\boldsymbol{J}^{\#} \Delta \boldsymbol{r}} - [\boldsymbol{I} - \boldsymbol{J}^{\#} \boldsymbol{J}] \cdot k \left(\frac{\partial \boldsymbol{\Phi}}{\partial x} \right)$$

Solution for minimum norm of input increment

$$\boldsymbol{J}^{\#} = \boldsymbol{J}^{\mathrm{T}} \cdot \left(\boldsymbol{J} \cdot \boldsymbol{J}^{\mathrm{T}} \right)^{-1}$$

Pseudo inverse of Jacobian matrix

Objective function :

Compliance of output link

"To keep output stiffness"

By adding external forces in the force balance equations

$$\boldsymbol{f}_J + \boldsymbol{f}_I + \boldsymbol{f}_A + \boldsymbol{f}_e = \boldsymbol{0}$$

⇩ Forward kinematics $\boldsymbol{r} = \boldsymbol{g}(\boldsymbol{\theta}, \boldsymbol{f}_e)$

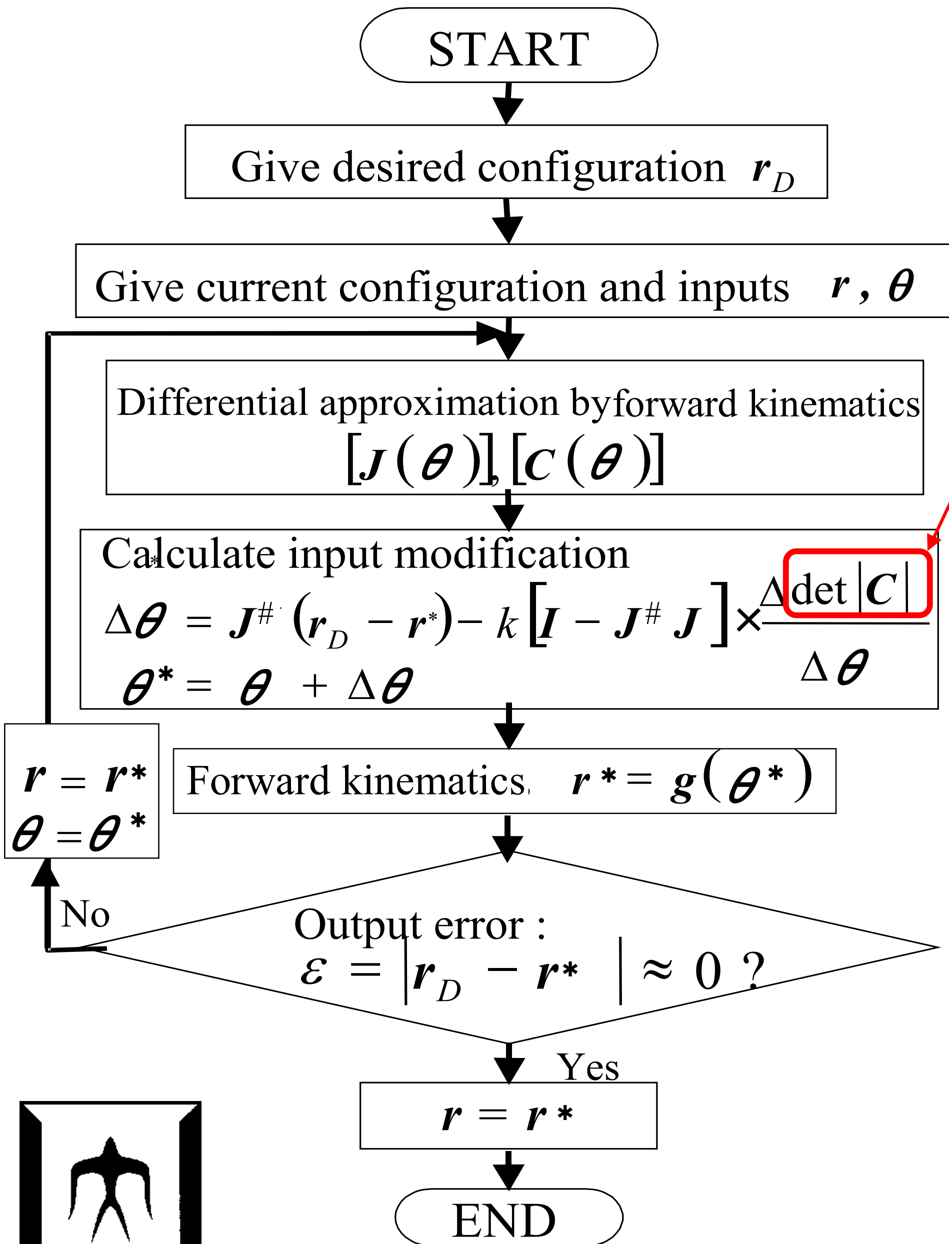
&

Differential approximation

Compliance matrix of output link :

$$[\boldsymbol{C}(\boldsymbol{\theta})] = \begin{bmatrix} \dfrac{\Delta g_1}{\Delta f_{e,x}} & \dfrac{\Delta g_1}{\Delta f_{e,y}} & \dfrac{\Delta g_1}{\Delta \tau_e} \\ \dfrac{\Delta g_2}{\Delta f_{e,x}} & \dfrac{\Delta g_2}{\Delta f_{e,y}} & \dfrac{\Delta g_2}{\Delta \tau_e} \\ \dfrac{\Delta g_3}{\Delta f_{e,x}} & \dfrac{\Delta g_3}{\Delta f_{e,y}} & \dfrac{\Delta g_3}{\Delta \tau_e} \end{bmatrix}$$

Flowchart of inverse kinematics



Determinant of compliance matrix

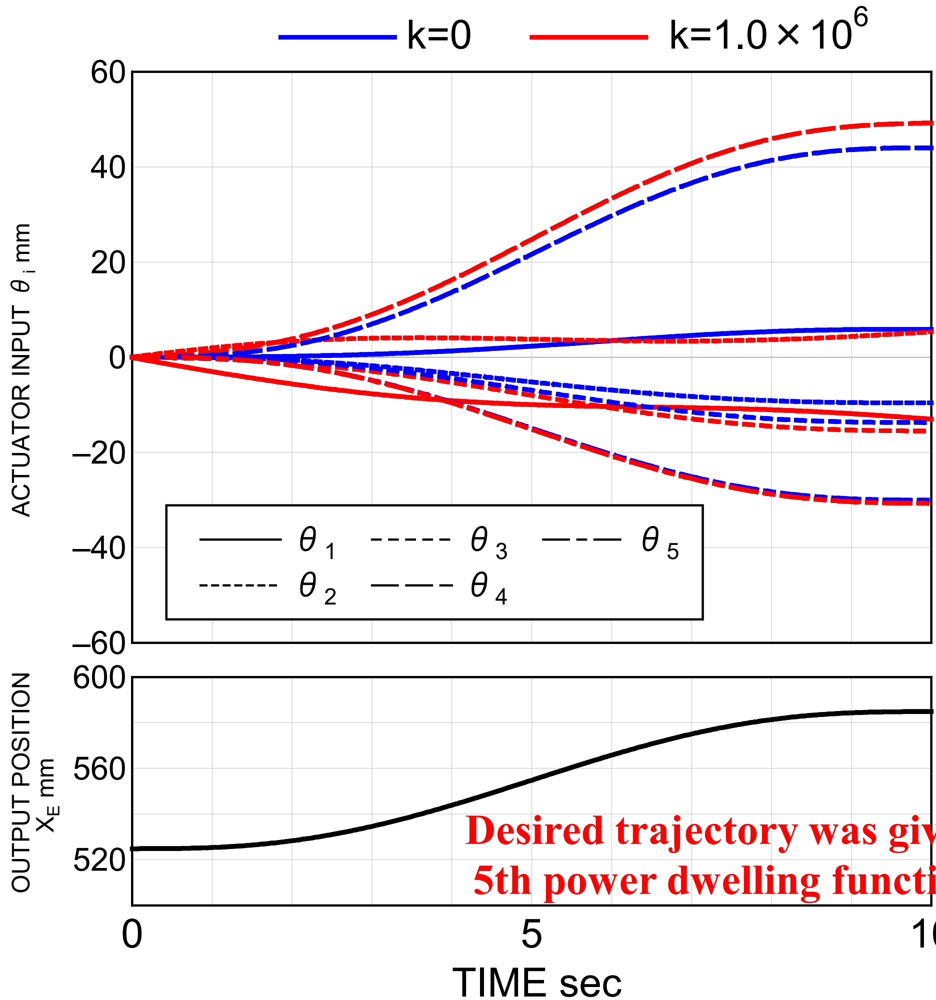
⇨ **To evaluate compliance in all direction (x,y,φ)**

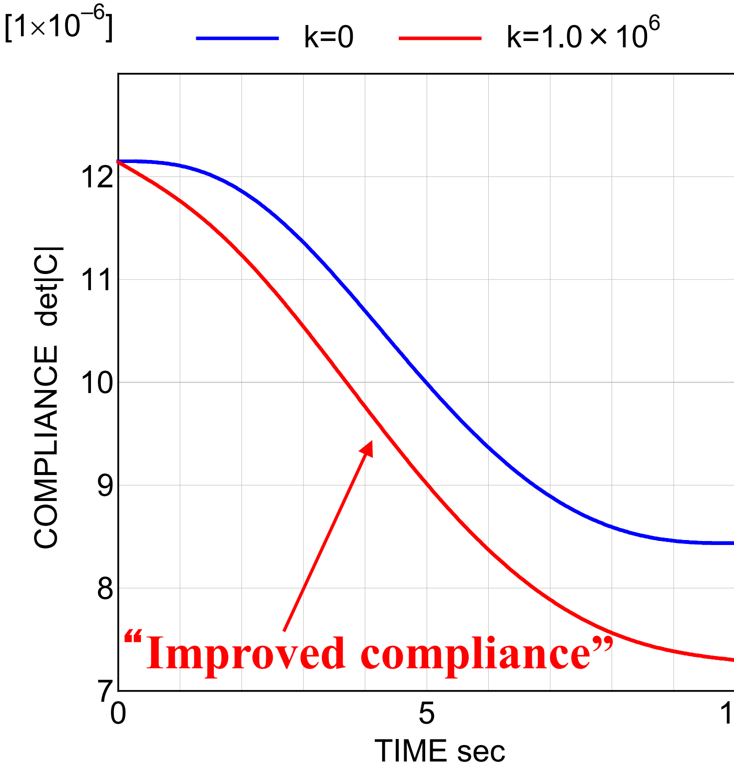
Iterative calculation till output error decreases adequately

⇩

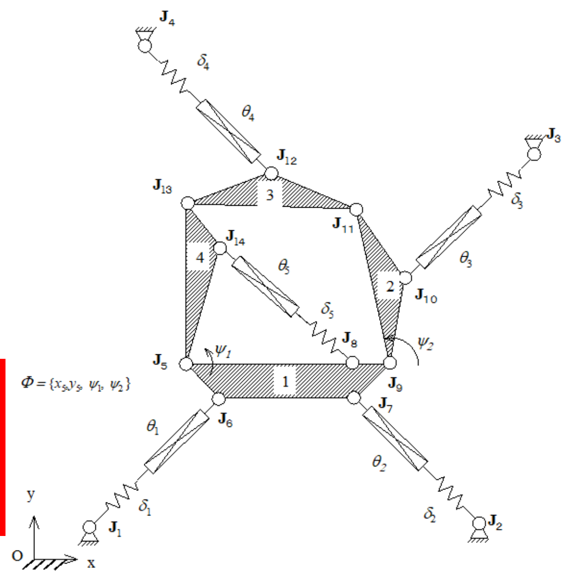
Solution of inverse kinematics with the minimum objective function can be obtained.

Example of inverse kinematics





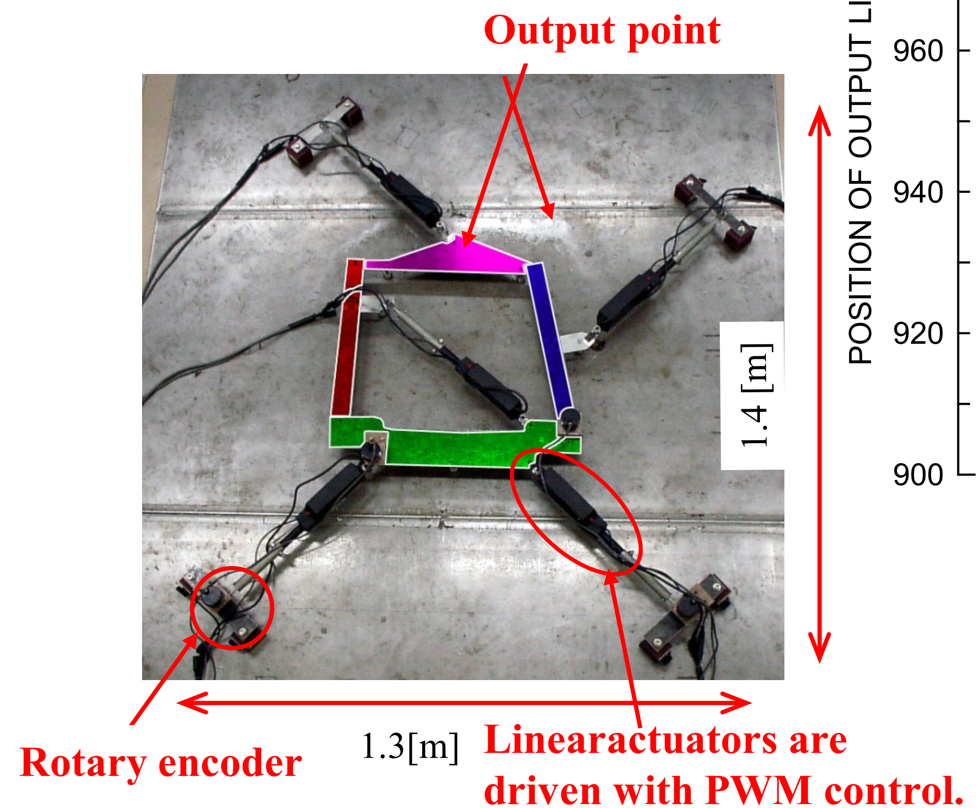
Desired trajectory was given as 5th power dwelling function.



The robot can achieve the specified trajectory while output compliance is improved.

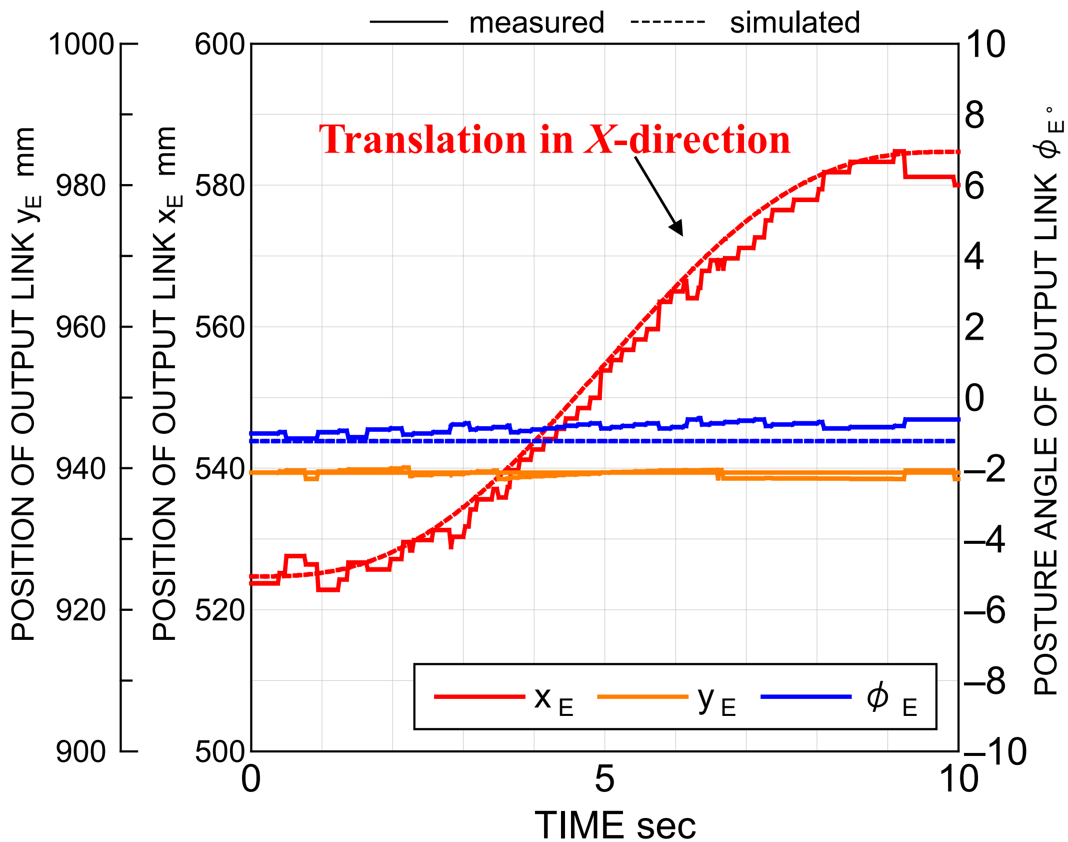
2.5 Motion Control Experiment

Linearspring unit



A prototype of planar network-structure robot with 4 DOF and 5 actuators

CP control based on the proposed inverse kinematics



The output link could be driven along the desired trajectory.

It was confirmed that the proposed inverse kinematics was effective and useful.

3. Position and stiffness control of elastic parallel manipulators with redundancy

3. 1 Elastic parallel manipulators with redundancy

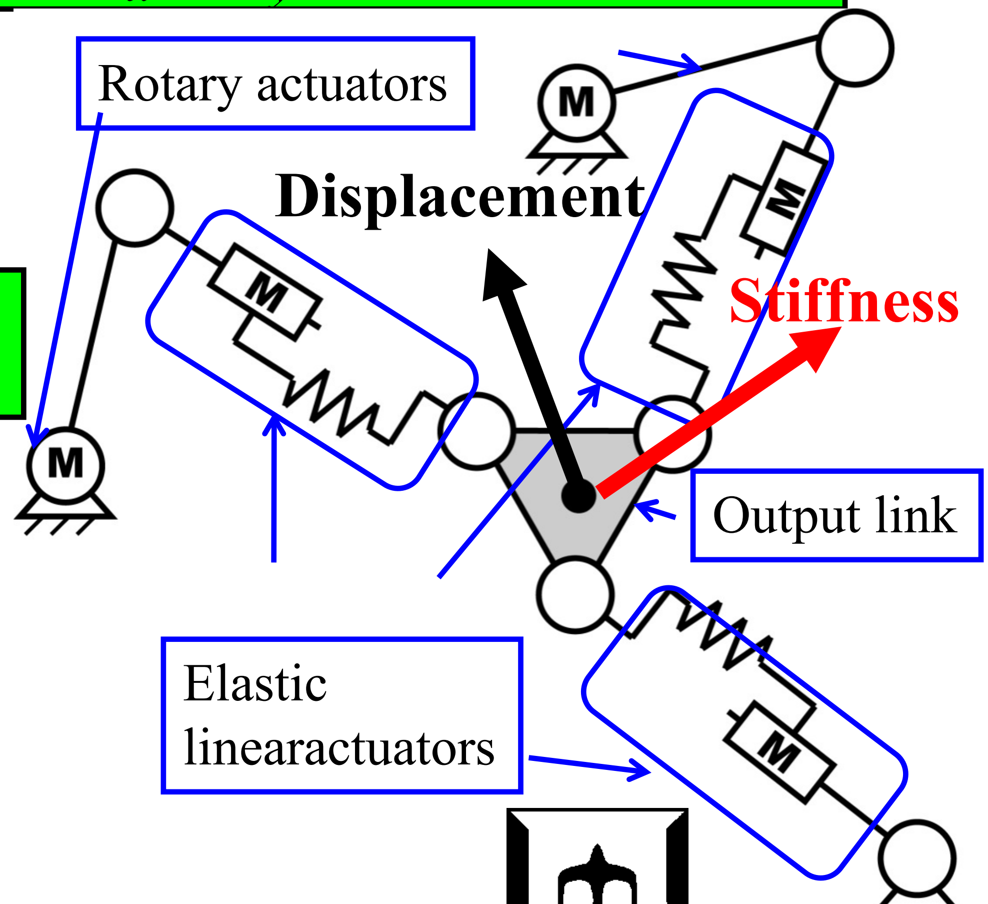
Planar redundant mechanism which can control translational displacement, x_P , y_P, and stiffness, K_x , K_y, of an output link

Redundant mechanism with 4 outputs and 5 actuators

3 elastic elements for 2D-stiffness to utilize nonlinearity

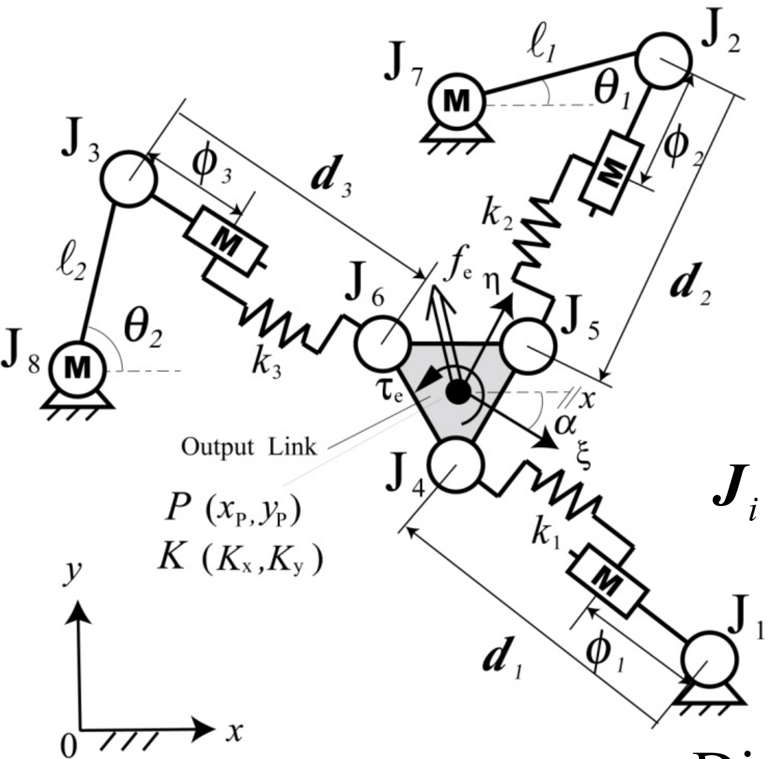
Elastic linearactuators are connected with moving links to magnify workspace

A planar parallel manipulator with 5DOF of 2 rotary actuators and 3 elastic linearactuators



3. 2 Kineto-statics analysis

Basic kinematics



By using rotary actuator inputs and position and posture of output link, joint positions are calculated as

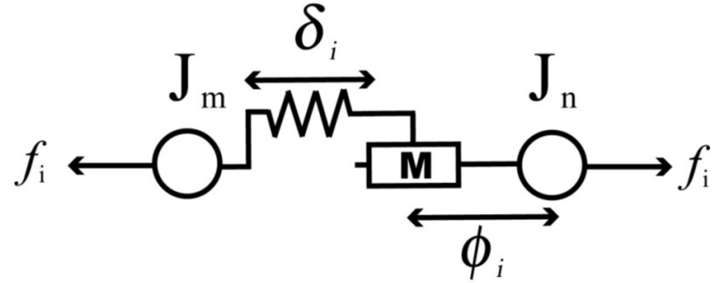
$$
\boldsymbol{J}_i = \begin{cases} \begin{bmatrix} \ell_{i-1} \cos\theta_{i-1} + x_{i+5} \\ \ell_{i-1} \sin\theta_{i-1} + y_{i+5} \end{bmatrix}, & (i = 2,3) \\ \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \xi_{i-3} \\ \eta_{i-3} \end{bmatrix} + \begin{bmatrix} x_p \\ y_p \end{bmatrix}, & (i = 4,5,6) \end{cases}
$$

Directional vectors of elastic linearactuators are calculated as

$$
\boldsymbol{d}_i = \begin{bmatrix} d_{ix} \\ d_{iy} \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \xi_{i-3} \\ \eta_{i-3} \end{bmatrix} + \begin{bmatrix} x_p - x_i \\ y_p - y_i \end{bmatrix}
$$

Force balance equation

Elastic linearactuator:



Configuration determining parameter
$\boldsymbol{\Psi}(x_p, y_p, \alpha, \theta_1, \theta_2)$

$$\boldsymbol{f}_i(\phi_i, \boldsymbol{\Psi}) = -k_i[|\boldsymbol{d}_i| - (\phi_i + \delta_i)]\frac{\boldsymbol{d}_i}{|\boldsymbol{d}_i|}$$

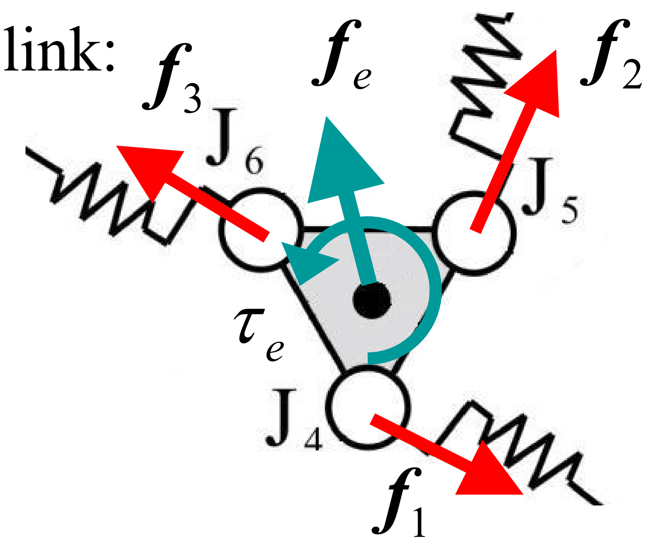
Spring coefficient

Deformation of spring

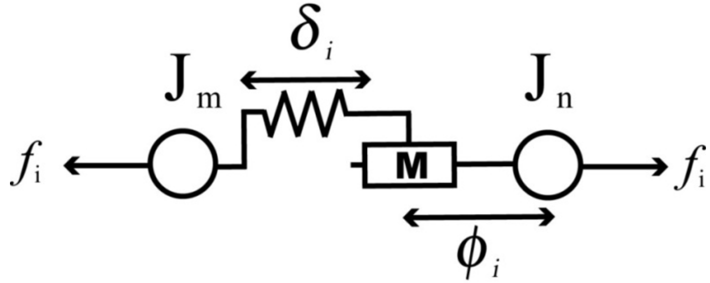
Unit directional vector of elastic linearactuator

Force and torque balance equation at output link:

$$\begin{cases} \displaystyle\sum_{i=1}^{3} \boldsymbol{f}_i + \boldsymbol{f}_e = \boldsymbol{0} \\ \displaystyle\sum_{i=1}^{3} (\boldsymbol{J}_{i+3} - \boldsymbol{P}) \times \boldsymbol{f}_i + \tau_e = 0 \end{cases}$$



Elastic linearactuator:



Configuration determining parameter
$\boldsymbol{\Psi}(x_p, y_p, \alpha, \theta_1, \theta_2)$

$$\boldsymbol{f}_i(\phi_i, \boldsymbol{\Psi}) = -k_i\left[\left|\boldsymbol{d}_i\right| - (\phi_i + \delta_i)\right]\frac{\boldsymbol{d}_i}{\left|\boldsymbol{d}_i\right|}$$

Spring coefficient

Deformation of spring

Unit directional vector of elastic linearactuator

Force and torque balance equation at output link:

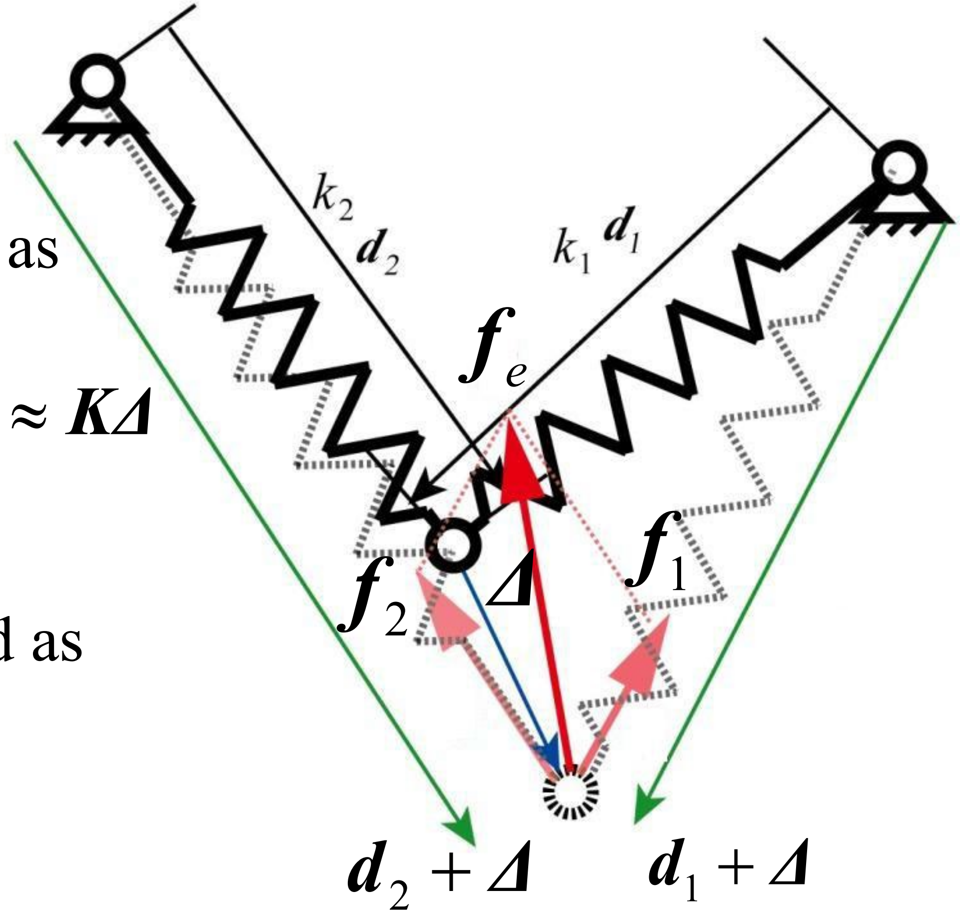
$$\begin{cases} \sum_{i=1}^{3} \boldsymbol{f}_i + \boldsymbol{f}_e = \boldsymbol{0} \\ \sum_{i=1}^{3} (\boldsymbol{J}_{i+3} - \boldsymbol{P}) \times \boldsymbol{f}_i + \tau_e = 0 \end{cases}$$

By giving $\theta_1, \theta_2, \phi_1, \phi_2, \phi_3$
and setting $\boldsymbol{f}_e = \boldsymbol{0}, \tau_e = 0$
x_p, y_p, α can be calculated with
Newton-Raphson Method

Output stiffness

Reaction force can be calculated as

$$\boldsymbol{f}_e = -\sum_{i=1}^{3} k_i \left(1 - \frac{|\boldsymbol{d}_i|}{|\boldsymbol{d}_i + \boldsymbol{\Delta}|} \right) (\boldsymbol{d}_i + \boldsymbol{\Delta}) \approx \boldsymbol{K\Delta}$$



Output stiffness can be calculated as

$$\boldsymbol{K} = \begin{bmatrix} \underline{K_x} & K_{xy} \\ K_{yx} & \underline{K_y} \end{bmatrix},$$

$$K_x = -\sum_{i=1}^{3} k_i \frac{d_{ix}^2}{|\boldsymbol{d}_i|^2}, \qquad K_y = -\sum_{i=1}^{3} k_i \frac{d_{iy}^2}{|\boldsymbol{d}_i|^2},$$

Two-diemensional stiffness K_x, K_y can be calculated.

3.3 Inverse analysis

Condition on output stiffness

At first $\alpha, \theta_1, \theta_2$ are determined to give the desired output stiffness.

Linear approximation of the relation:

$$\begin{bmatrix} \Delta K_x \\ \Delta K_y \end{bmatrix} = \boldsymbol{C} \begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \Delta\alpha \end{bmatrix}, \; \square \; \boldsymbol{C} = \begin{bmatrix} \dfrac{\partial K_x}{\partial \theta_1} & \dfrac{\partial K_x}{\partial \theta_2} & \dfrac{\partial K_x}{\partial \alpha} \\ \dfrac{\partial K_y}{\partial \theta_1} & \dfrac{\partial K_y}{\partial \theta_2} & \dfrac{\partial K_y}{\partial \alpha} \end{bmatrix}$$

Solution by gradient projection method:

Redundant

$$\begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \Delta\alpha \end{bmatrix} = \boldsymbol{C}^{\#} \begin{bmatrix} \Delta K_x \\ \Delta K_y \end{bmatrix} + (\boldsymbol{I} - \boldsymbol{C}^{\#}\boldsymbol{C})\boldsymbol{\varepsilon}$$

$\boldsymbol{C}^{\#} = \boldsymbol{C}^T(\boldsymbol{C}\boldsymbol{C}^T)^{-1}$: Pseudo inverse matrix

$\boldsymbol{\varepsilon}$: Derivative of objective function

Examples of objective function:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \sum_{i=1}^{3} \dfrac{\partial\left[\left|\boldsymbol{d}_i\right| - (\phi_i + \delta_i)\right]^2}{\partial\theta_1} \\ \sum_{i=1}^{3} \dfrac{\partial\left[\left|\boldsymbol{d}_i\right| - (\phi_i + \delta_i)\right]^2}{\partial\theta_2} \\ \sum_{i=1}^{3} \dfrac{\partial\left[\left|\boldsymbol{d}_i\right| - (\phi_i + \delta_i)\right]^2}{\partial\alpha} \end{bmatrix}$$

linearactuators keep located around its neutral position

Condition on output displacement

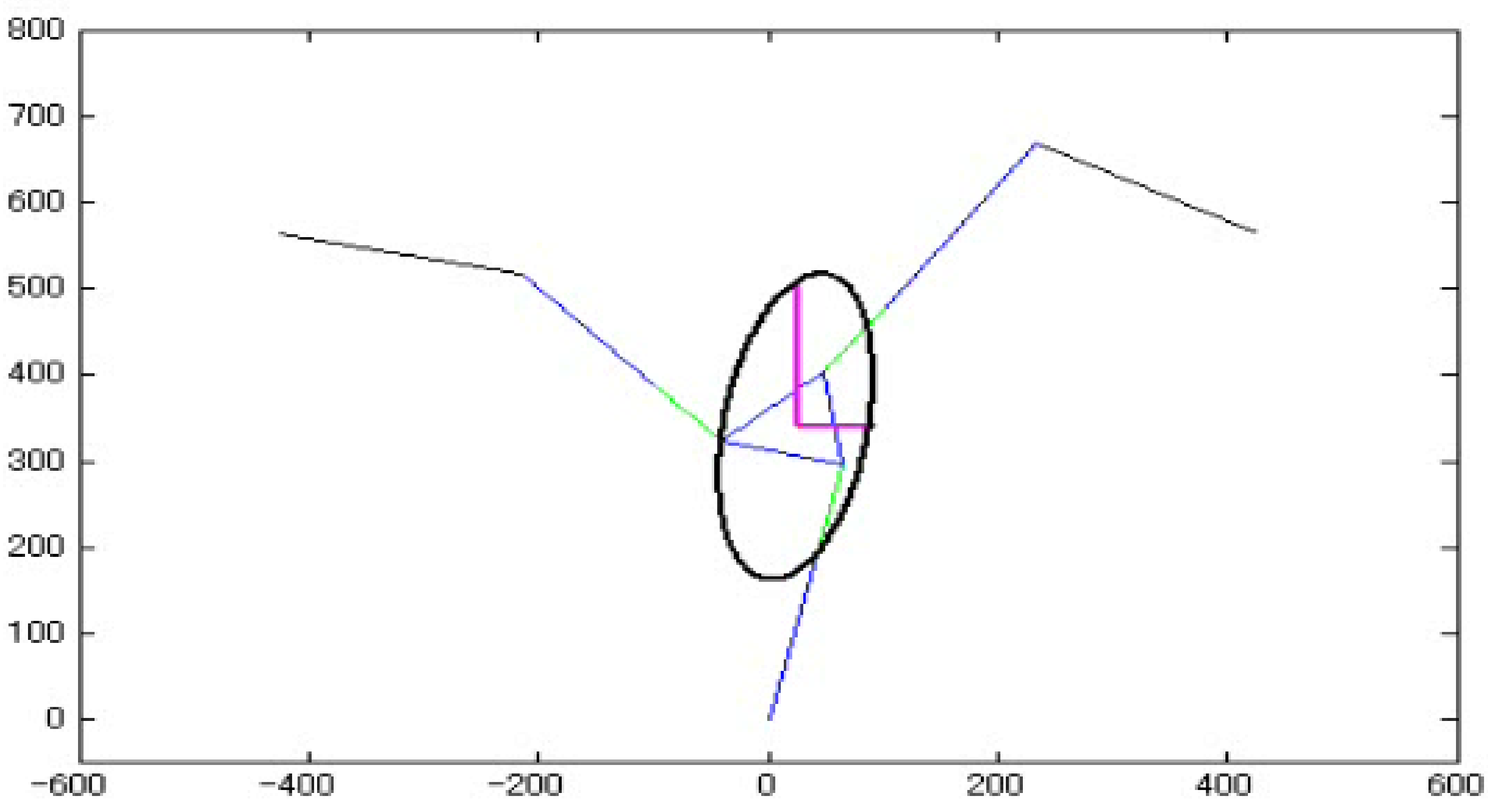
Then linearactuator inputs, ϕ_i, are calculated by solving force balance equation with Newton-Raphson method.

Inverse kineto-static analysis can be achieved.

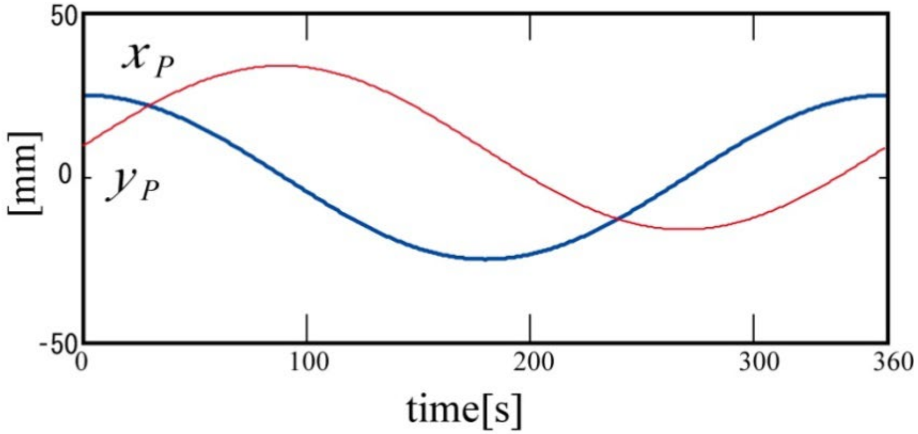
3.4 Simulations of position and stiffness control

Circular displacement of 25mm in radius while keeping stiffness as

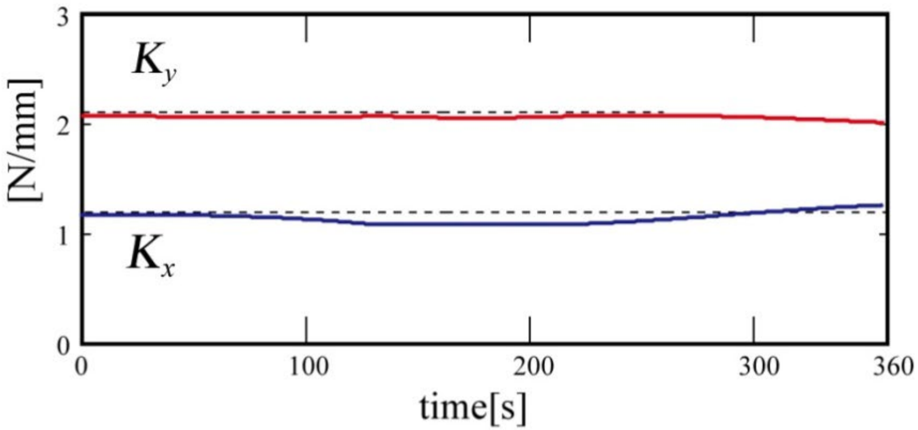
$K_x = 1.2[\mathrm{N/mm}], K_y = 2.1[\mathrm{N/mm}]$



Output values

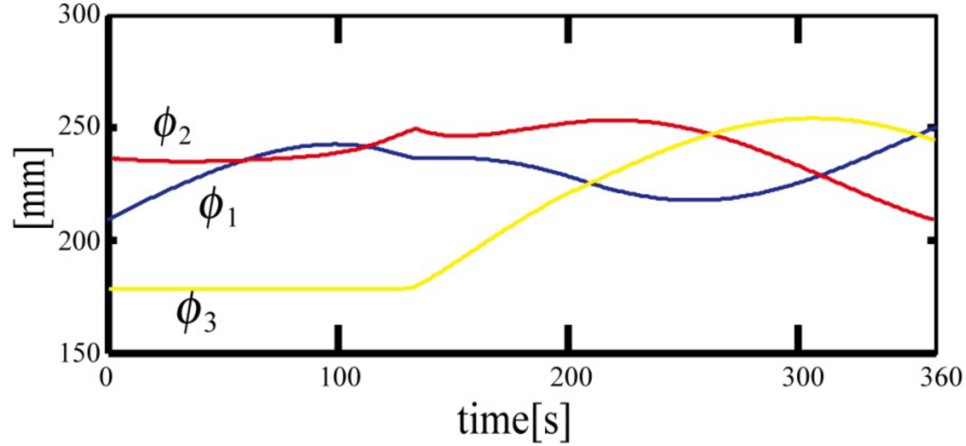


Output displacement

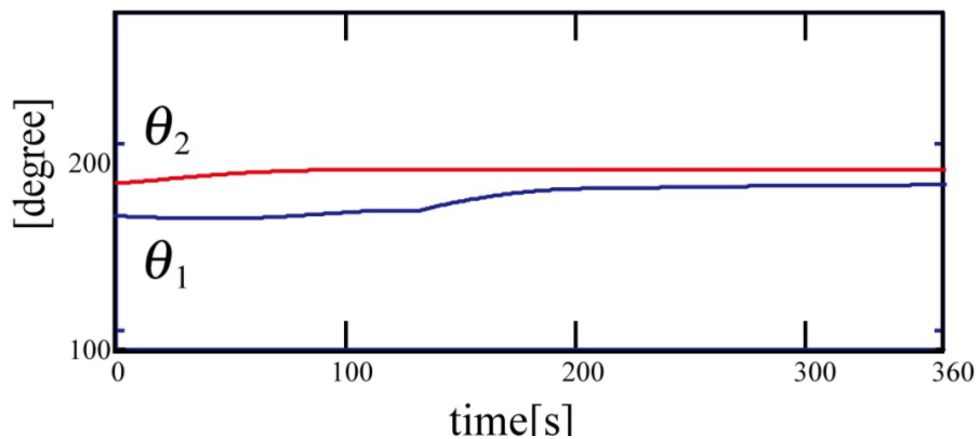


(b Output stiffness

Actuator inputs



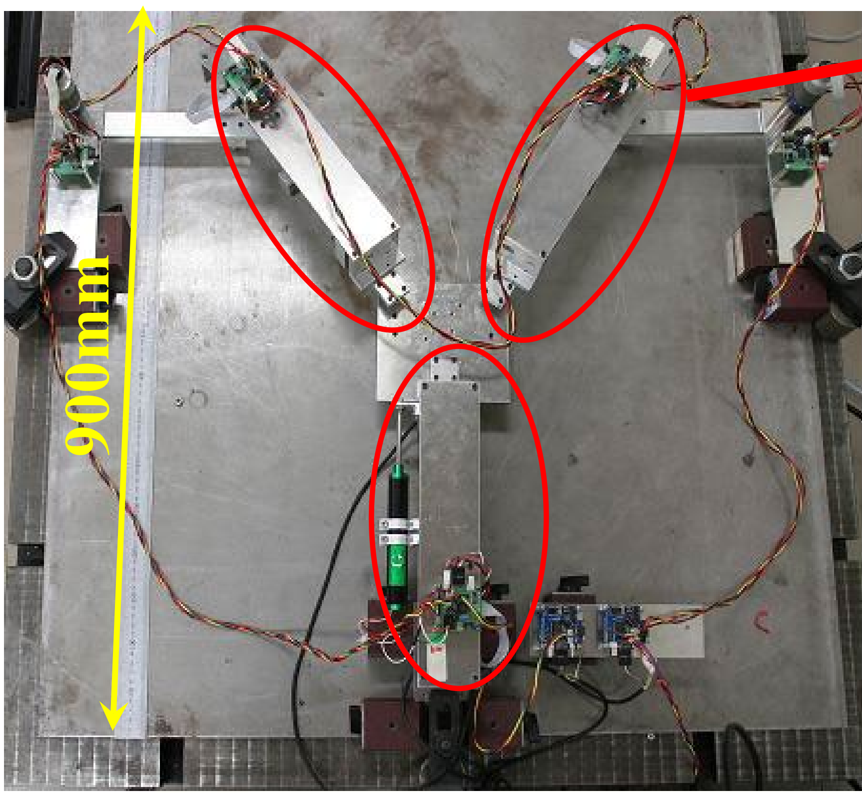
Linearactuator inputs



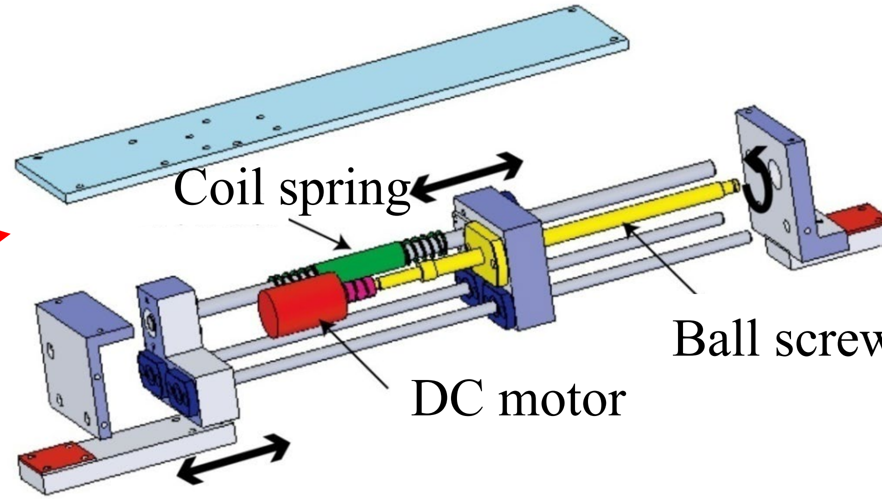
Rotary actuator inputs

Continuous desired displacement and stiffness were obtained with the proposed objective function.

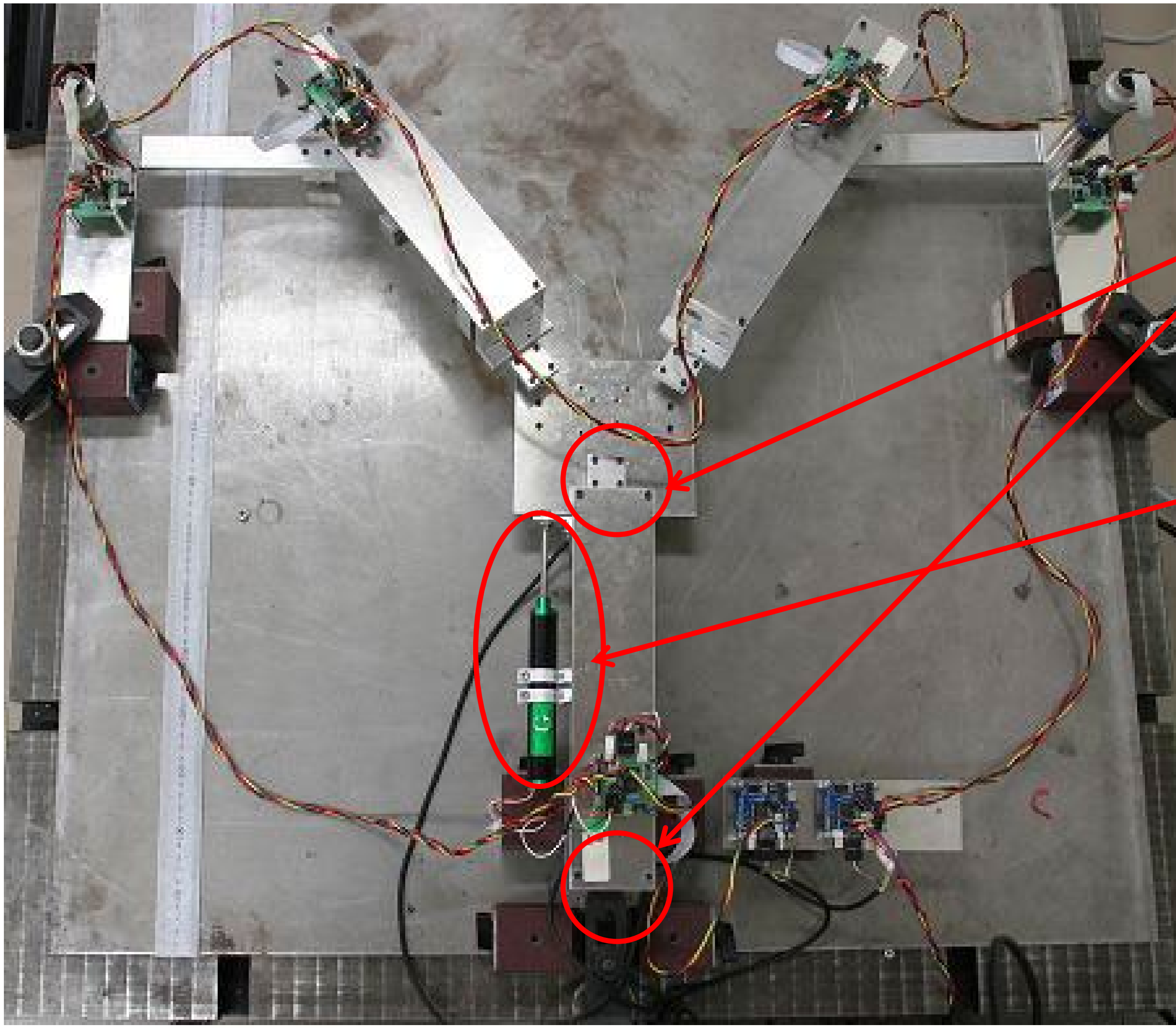
3.5 Experiments on position/stiffness control



A prototype with 5DOF



An elastic linearactuator



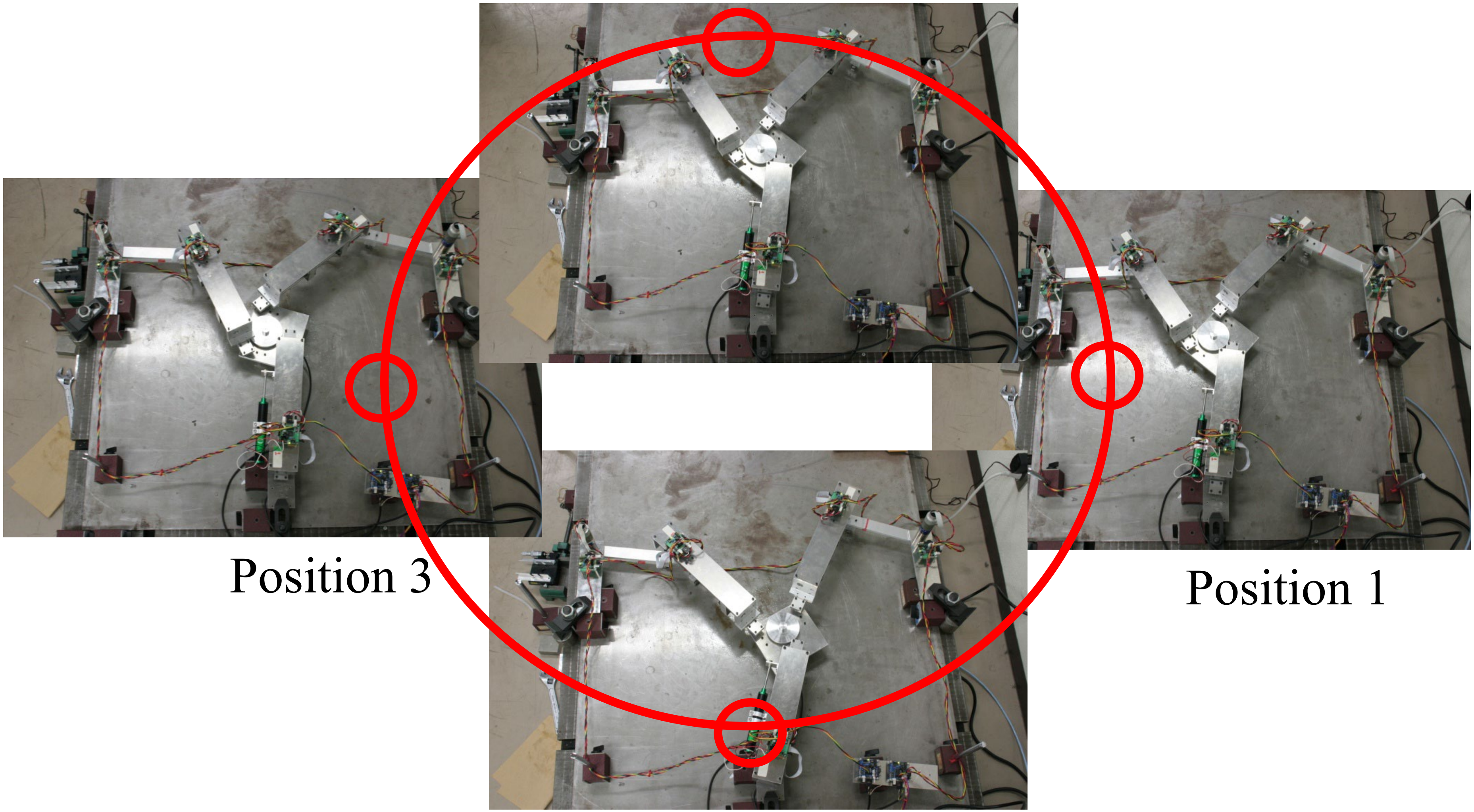
Rotary encoders to measure joint angles

Linearpotentiometers to measure spring deformation

Calculate position and posture of output link

(1)Constant stiffness on the circular trajectory

Position 2

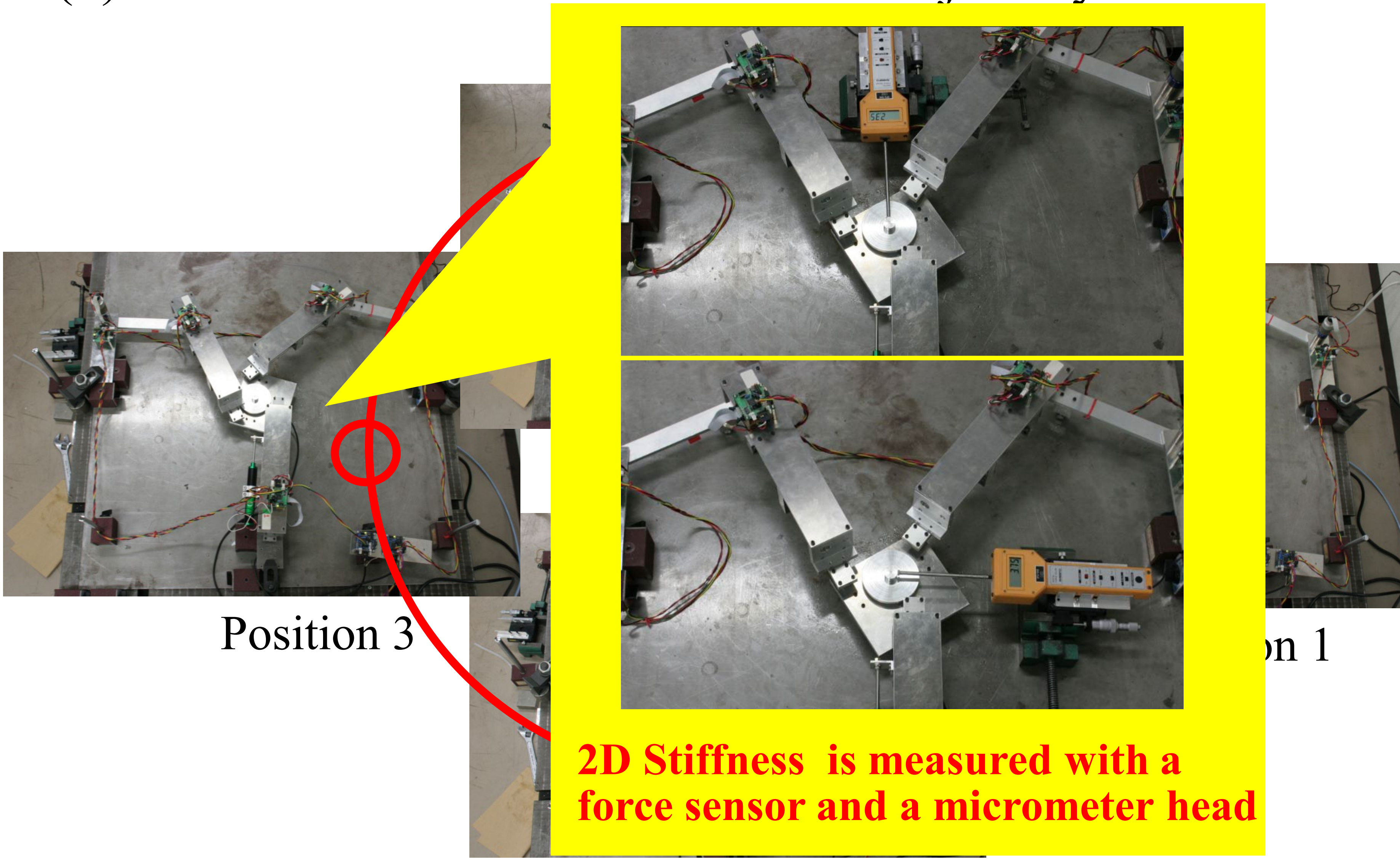


Position 3

Position 1

Position 4

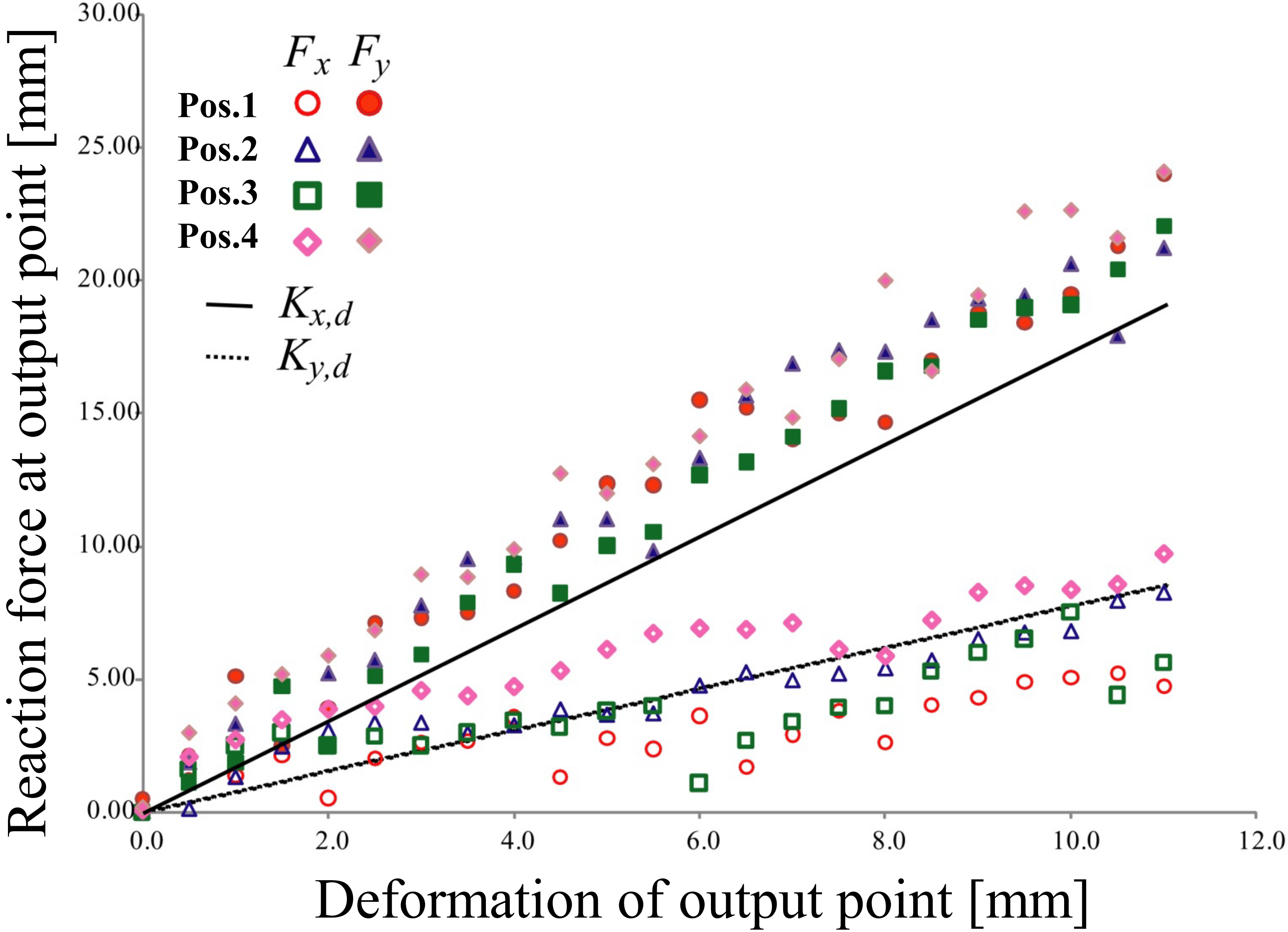
(1)Constant stiffness on the circular trajectory



Position 3

on 1

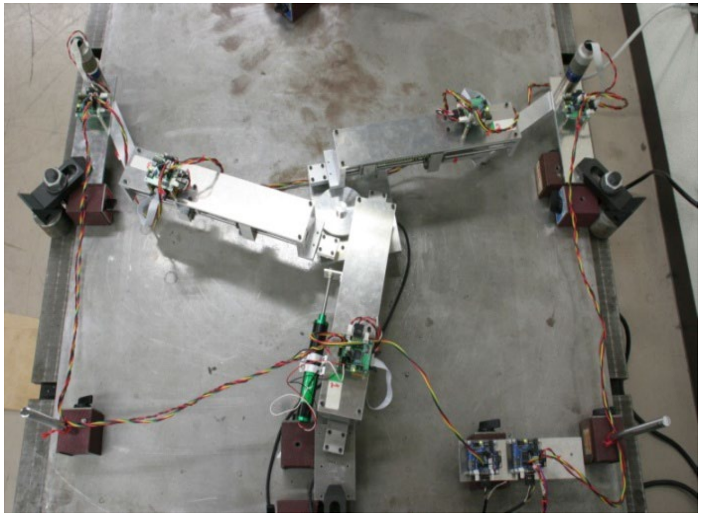
Position 4



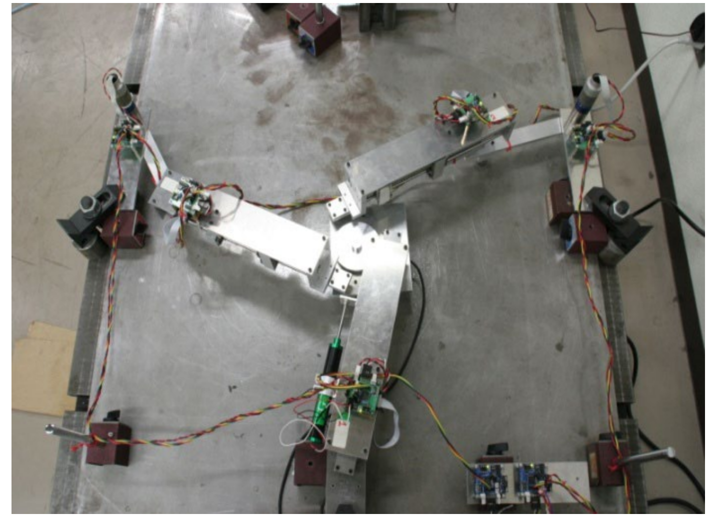
Desired stiffness is almost generated.

(2)Change of stiffness at the same output displacement

$$x_{P,d} = -20[\mathrm{mm}], y_{P,d} = 350[\mathrm{mm}]$$

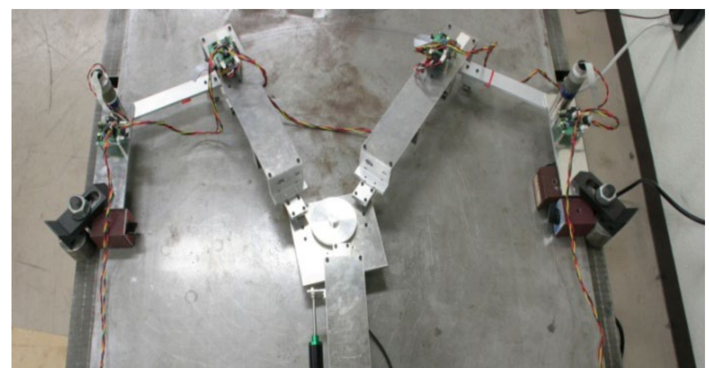


$K_x = 1.6, K_y = 0.8$



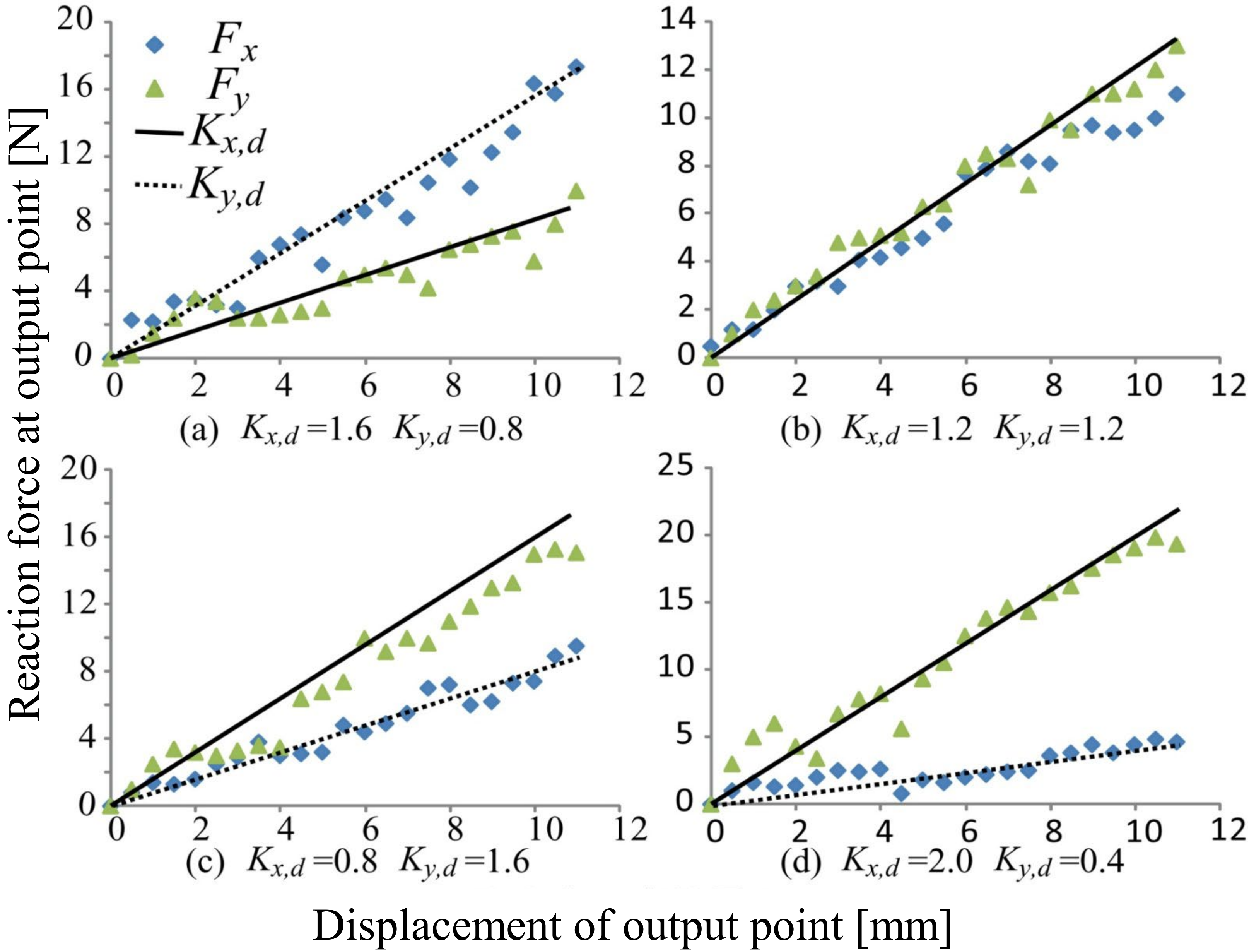
$K_x = 1.2, K_y = 1.2$

$K_x = 0.8, K_y = 1.6$



$K_x = 0.4, K_y = 2.0$

The manipulator change its configuration to control stiffness vector.



Manipulator can control stiffness at the same output position.

4. Concluding remarks

Syntheses and Forward/inverse kinematics analysis of overactuator mechanisms were carried out.

(1)Network structure robots composed of network modules are proposed and synthesized.

(2)Iterative calculation based on representative points can achieve inverse kinematics and motion control of the network structure robots.

Motion control of redundant overactuator mechanisms with elastic elements was also realized.

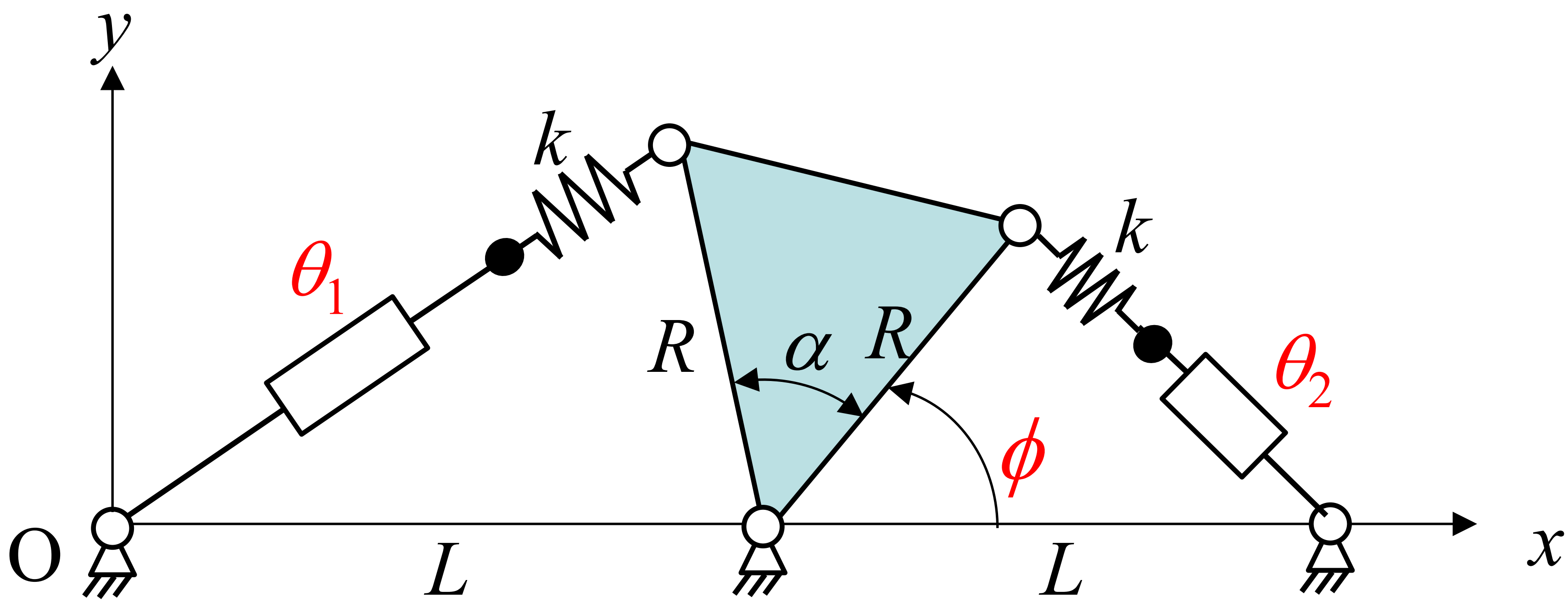
(3) Interference between overactuators can be relaxed with elastic elements on actuators.

(4)Optimum motion control of network structure robot with elastic elements was achieved.

(5)Position and stiffness control of a redundant elastic robot was achieved.

Homework 5

Derive the output motion ϕ with respect to inputs θ_1 and θ_2 of the following overactuator mechanism with two springs with stiffness k and natural length l.



The result will be summarized in A4 size PDF with less than 4 pages and sent to Prof. Iwatsuki via T2SCHOLAR by May 18, 2023.