May 18, 2023

# Advanced Mechanical Elements
## (Lecture 6)

***Kinetostatic analysis and motion control of underactuated mechanisms with elastic elements***

***- Motion control of underactuated mechanisms constrained by elastic elements -***

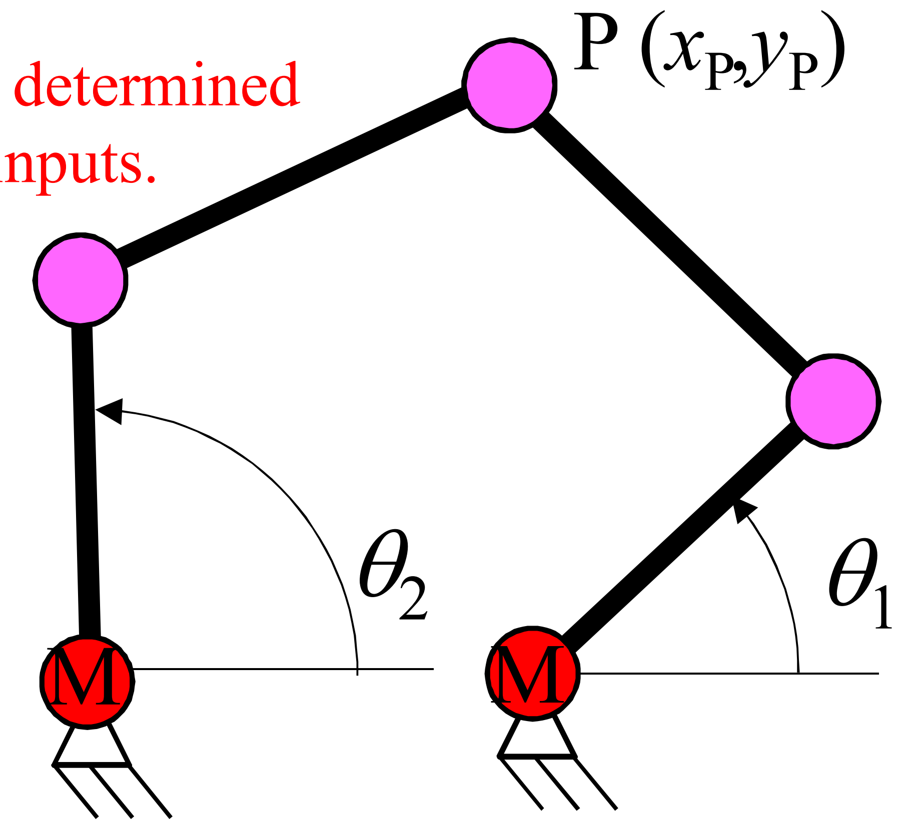
Tokyo Institute of Technology
Dept. of Mechanical Engineering
School of Engineering

**Prof. Nobuyuki Iwatsuki**

# 1. Underactuated Mechanisms

Ordinary mechanism in which a number of actuators is equal to DOF.

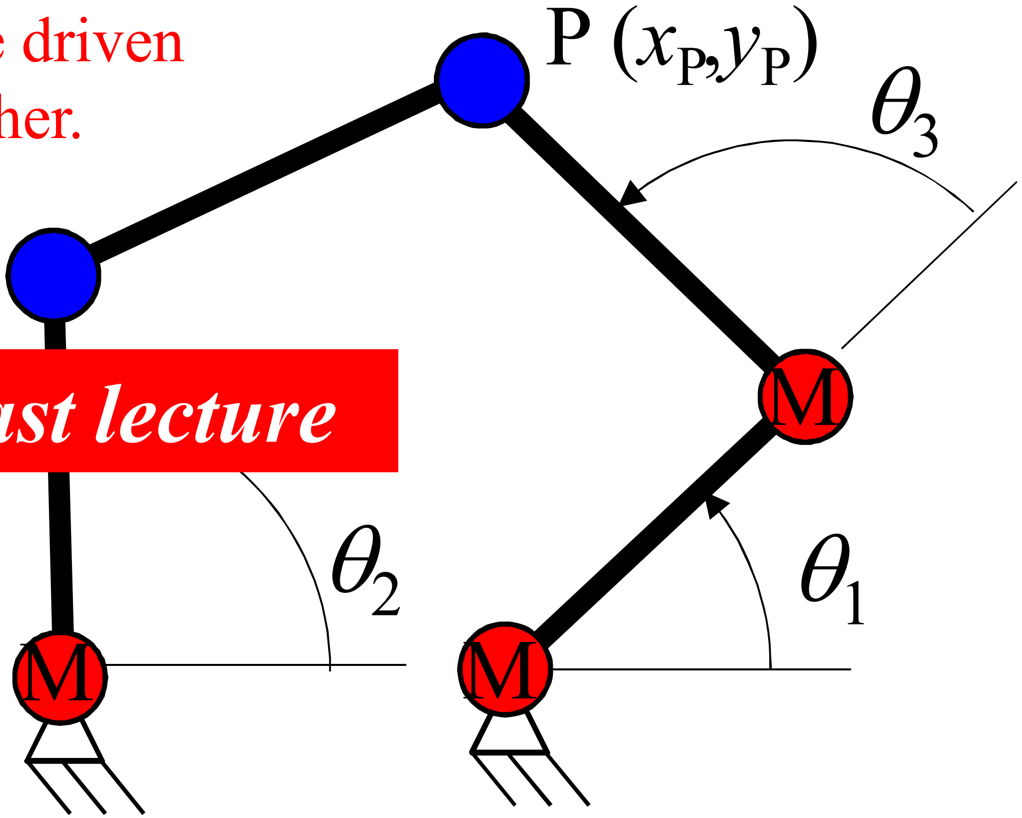
Output motion can be determined by giving two crank inputs.




Planar 5-bar link mechanism with 2DOF and 2 actuators

Overactuated mechanism in which a number of actuators is more than to DOF.

All actuators should be driven dependently to each other.

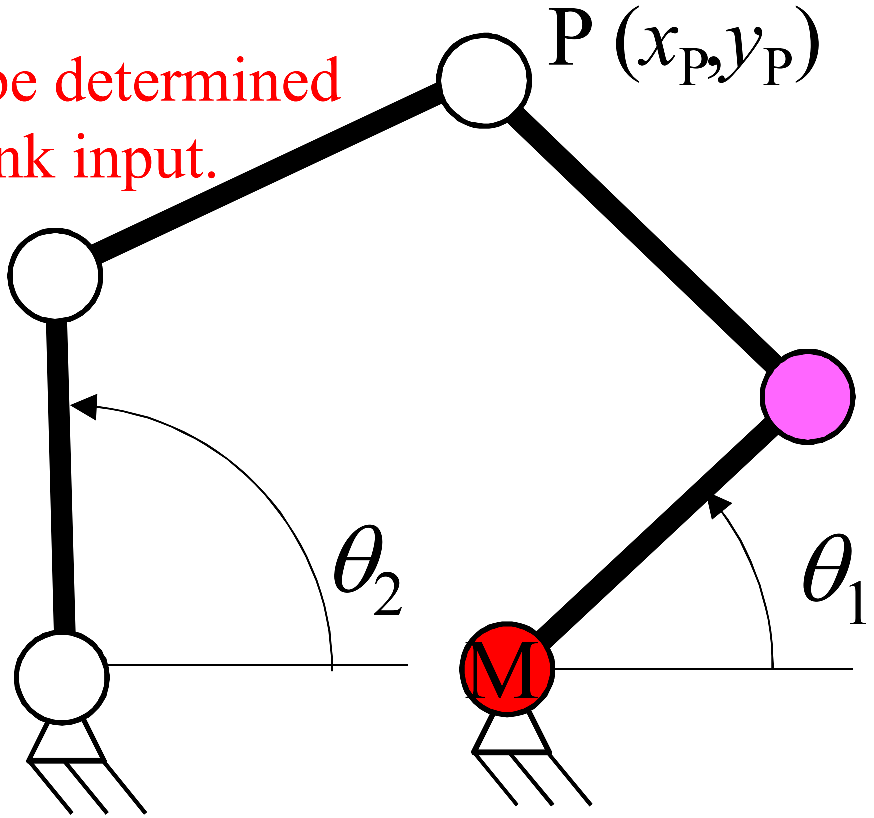



***Explained in last lecture***

Planar 5-bar link mechanism with 2DOF and 3 actuators

Underactuated mechanism in which a number of actuators is less than DOF.

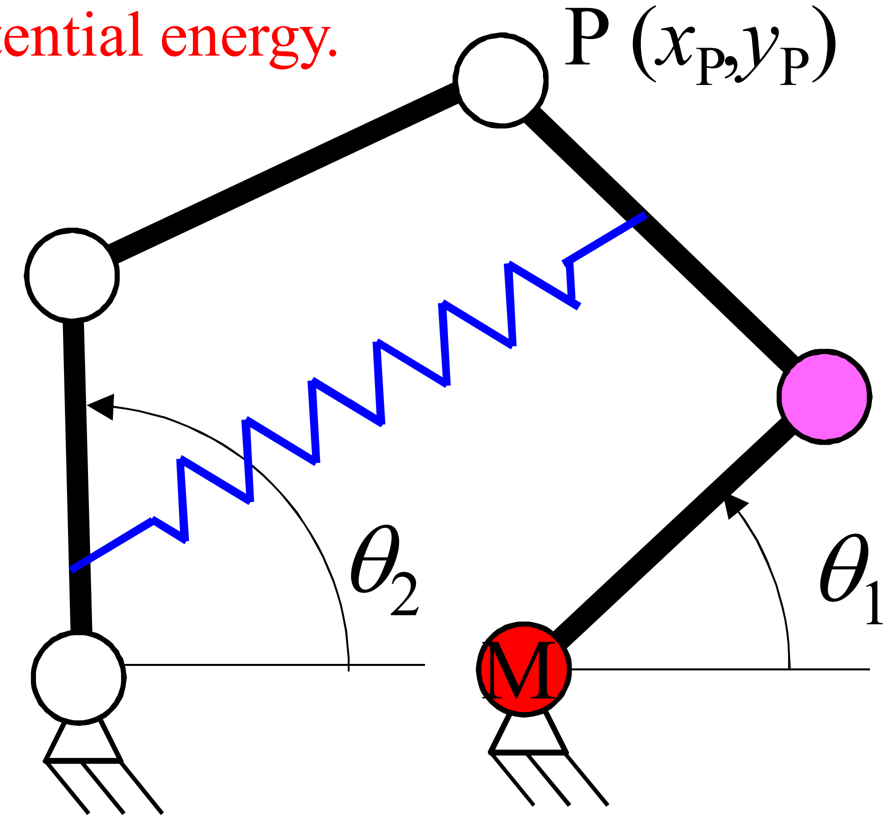
Output motion cannot be determined by giving only one crank input.




Planar 5-bar link mechanism with 2DOF and 1 actuator

Underactuated mechanism constrained with an elastic element

Dependent output motions are determined so as to minimize potential energy.

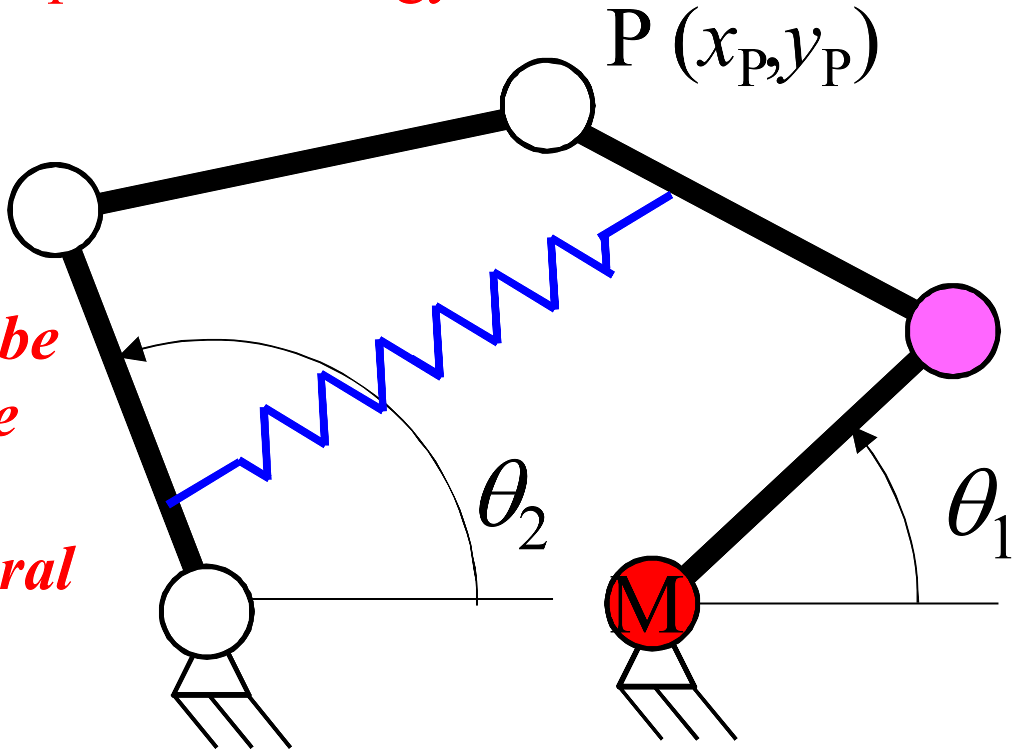



Planar 5-bar link mechanism with 2DOF and 1 actuator

Underactuated mechanism constrained with an elastic element

Dependent output motions are determined so as to minimize potential energy.

***Mechanism will be driven so that the added spring becomes its natural length.***

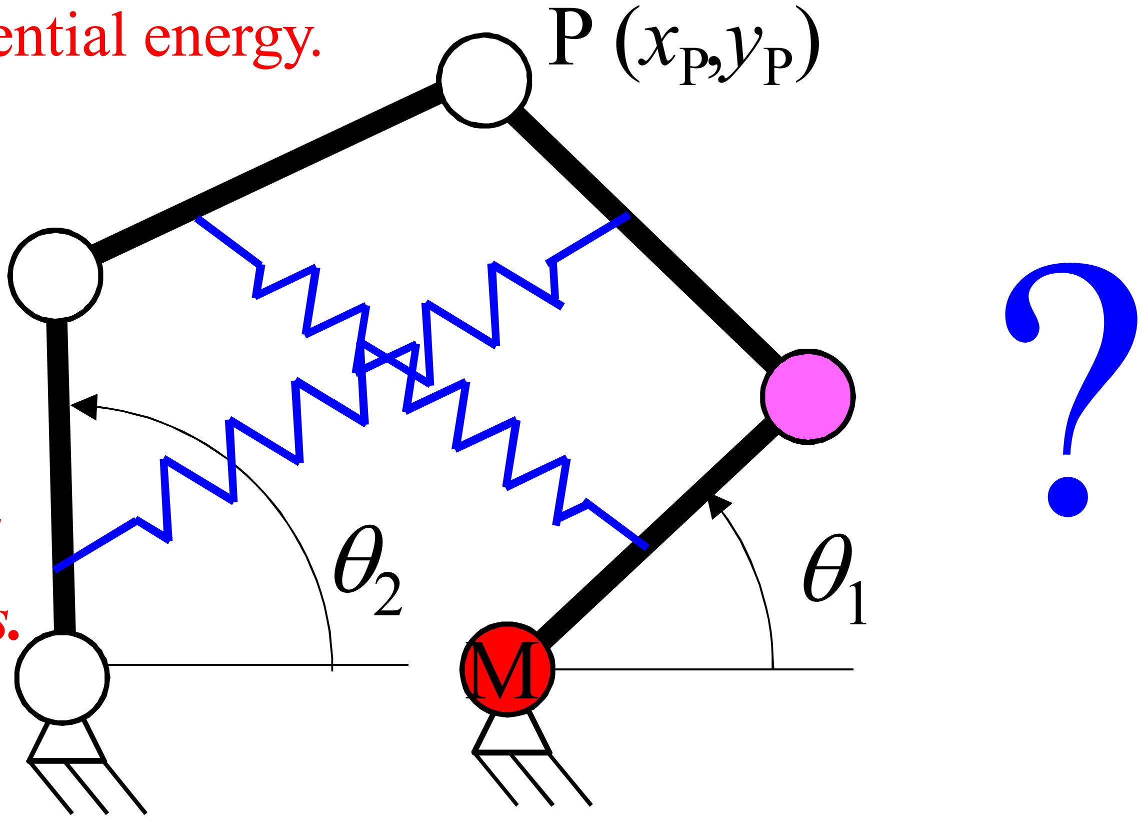



Planar 5-bar link mechanism with 2DOF and 1 actuator

# Underactuated mechanism constrained with an elastic element

Dependent output motions are determined so as to minimize potential energy.

*In case where the mechanism is constrained with more complicated combination of springs.*

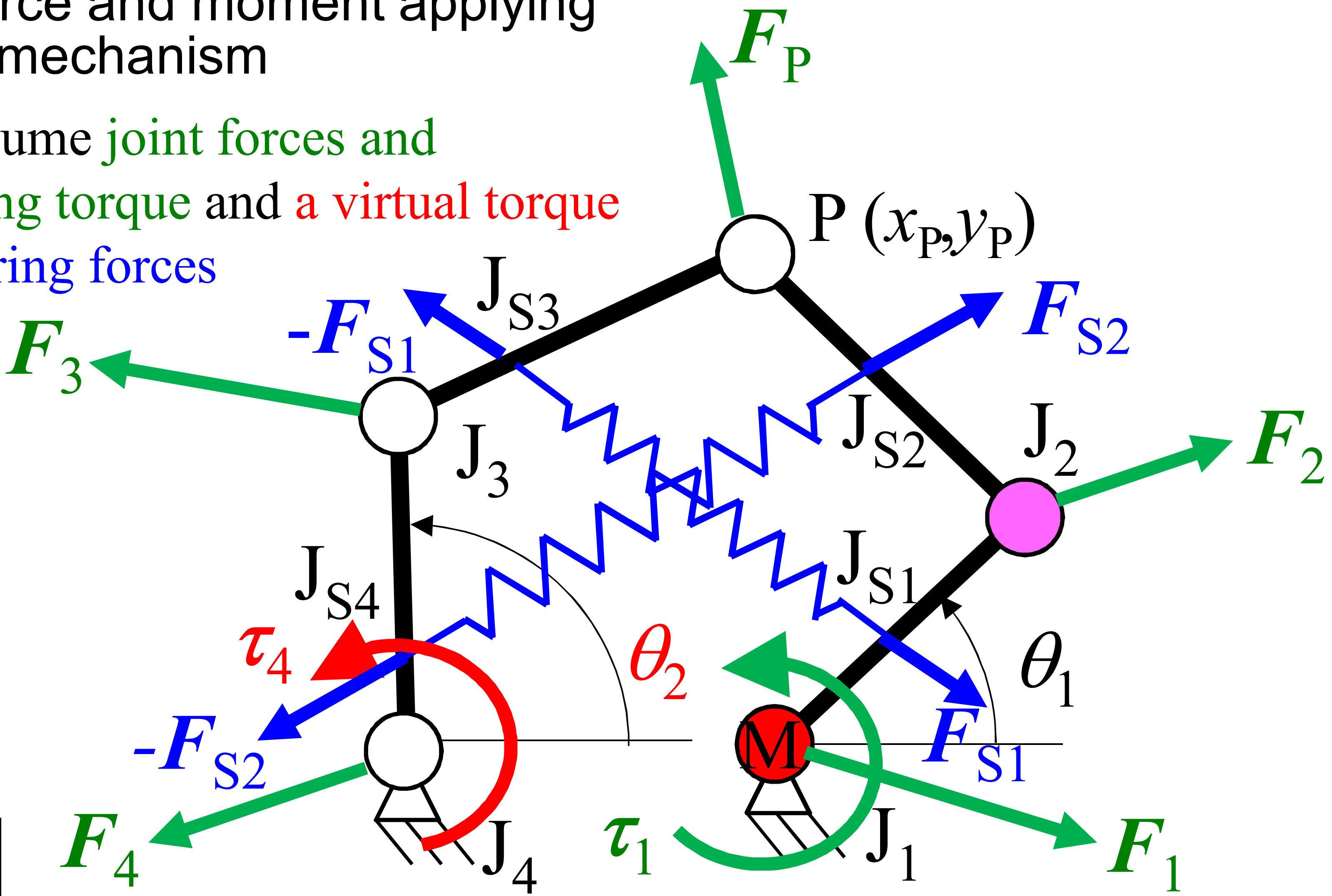



Planar 5-bar link mechanism with 2DOF and 1 actuator

# 2. Kinetostatics Analysis of Underactuated Mechanisms

## 2.1 Force and moment applying to mechanism

Let assume joint forces and a driving torque and a virtual torque and spring forces




Planar 5-bar link mechanism with 2DOFand 1 actuator

## 2.2 Statics equation

***Statics equation for each link***

$$\boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_{S1} = \boldsymbol{0}$$

$$\boldsymbol{J}_1 \times \boldsymbol{F}_1 + \boldsymbol{J}_2 \times \boldsymbol{F}_2 + \boldsymbol{J}_{\mathrm{S1}} \times \boldsymbol{F}_{S1} + \tau_1 = 0$$

$$-\boldsymbol{F}_2 + \boldsymbol{F}_P + \boldsymbol{F}_{S2} = \boldsymbol{0}$$

$$-\boldsymbol{J}_2 \times \boldsymbol{F}_2 + \boldsymbol{P} \times \boldsymbol{F}_P + \boldsymbol{J}_{\mathrm{S2}} \times \boldsymbol{F}_{S2} = 0$$

$$-\boldsymbol{F}_P + \boldsymbol{F}_3 - \boldsymbol{F}_{S2} = \boldsymbol{0}$$

$$-\boldsymbol{P} \times \boldsymbol{F}_P + \boldsymbol{J}_3 \times \boldsymbol{F}_3 - \boldsymbol{J}_{\mathrm{S3}} \times \boldsymbol{F}_{S2} = 0$$

$$-\boldsymbol{F}_3 + \boldsymbol{F}_4 - \boldsymbol{F}_{S1} = \boldsymbol{0}$$

$$-\boldsymbol{J}_3 \times \boldsymbol{F}_3 + \boldsymbol{J}_4 \times \boldsymbol{F}_4 - \boldsymbol{J}_{\mathrm{S4}} \times \boldsymbol{F}_{S1} + \tau_4 = 0$$

where position vectors, $\boldsymbol{P}$, $\boldsymbol{J}_3$, $\boldsymbol{J}_4$, $\boldsymbol{J}_{\mathrm{S3}}$, $\boldsymbol{J}_{\mathrm{S4}}$ and spring forces, $\boldsymbol{F}_{\mathrm{S1}}$, $\boldsymbol{F}_{\mathrm{S2}}$ are given as functions with respect to joint angle $\theta_2$

## 2.3 Forward analysis

$$[A(\theta_2)]\begin{bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \\ \boldsymbol{F}_3 \\ \boldsymbol{F}_4 \\ \boldsymbol{F}_P \\ \tau_1 \\ \tau_4 \end{bmatrix} = [\boldsymbol{b}(\theta_2)] \Rightarrow \begin{bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \\ \boldsymbol{F}_3 \\ \boldsymbol{F}_4 \\ \boldsymbol{F}_P \\ \tau_1 \\ \tau_4 \end{bmatrix} = [\boldsymbol{A}(\theta_2)]^{-1}[\boldsymbol{b}(\theta_2)]$$

Inverse statics analysis

**The assumed virtual torque $\tau_4$ should be zero.**

$\Downarrow$

**Nonlinear equation with respect to $\theta_2$, $\tau_4(\theta_2)=0$, can be solved with numerical method.**

## 2.4 Inverse analysis

After specifying desired output motion, $\theta_2$, position vectors, $\boldsymbol{P}$, $\boldsymbol{J}_3$, $\boldsymbol{J}_4$, $\boldsymbol{J}_{S3}$, $\boldsymbol{J}_{S4}$ and spring forces, $\boldsymbol{F}_{S1}$, $\boldsymbol{F}_{S2}$ are given as functions with respect to joint angle $\theta_1$

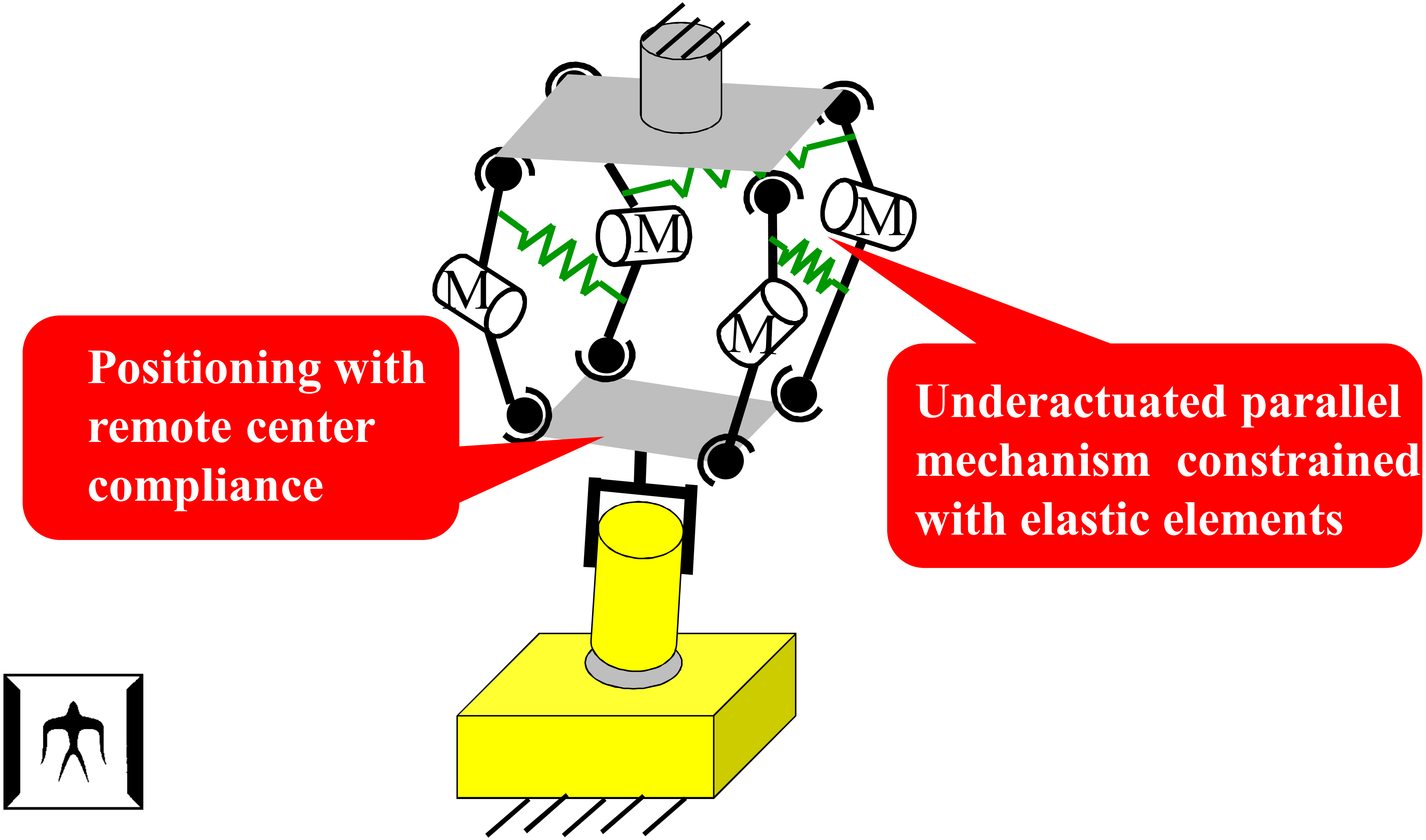
$$[A(\theta_1)]\begin{bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \\ \boldsymbol{F}_3 \\ \boldsymbol{F}_4 \\ \boldsymbol{F}_P \\ \tau_1 \\ \tau_4 \end{bmatrix} = [\boldsymbol{b}(\theta_1)] \quad \Rightarrow \quad \begin{bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \\ \boldsymbol{F}_3 \\ \boldsymbol{F}_4 \\ \boldsymbol{F}_P \\ \tau_1 \\ \tau_4 \end{bmatrix} = [A(\theta_1)]^{-1}[\boldsymbol{b}(\theta_1)]$$

Inverse statics analysis

**Nonlinear equation with respect to $\theta_1$, $\tau_4(\theta_1)=0$, can be solved with numerical method.**

# 3. Application to Spatial Parallel Mechanisms

## 3.1 Expected application utilizing flexibility



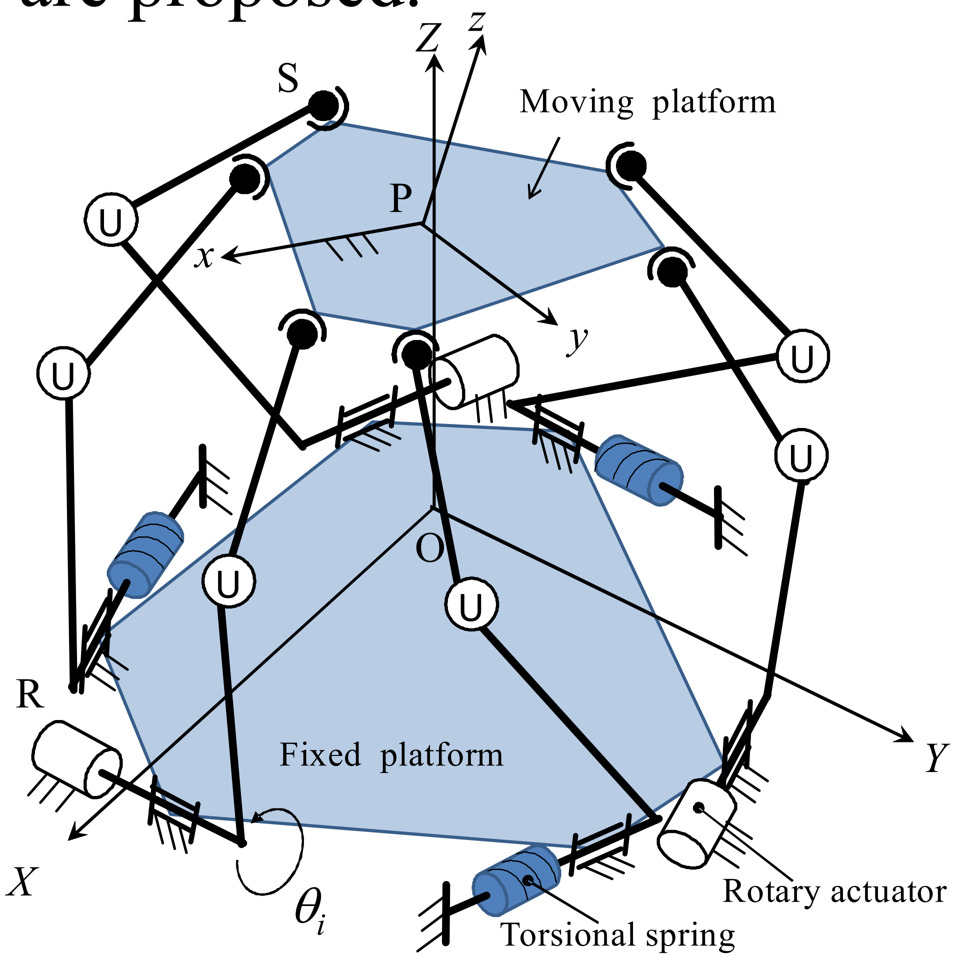

**Parallel manipulator with Remote Center Compliance**

# 3.2 Example

Aiming to develop a novel flexible joints with multi-DOF, underactuated spatial parallel mechanisms constrained with several elastic elements are proposed.

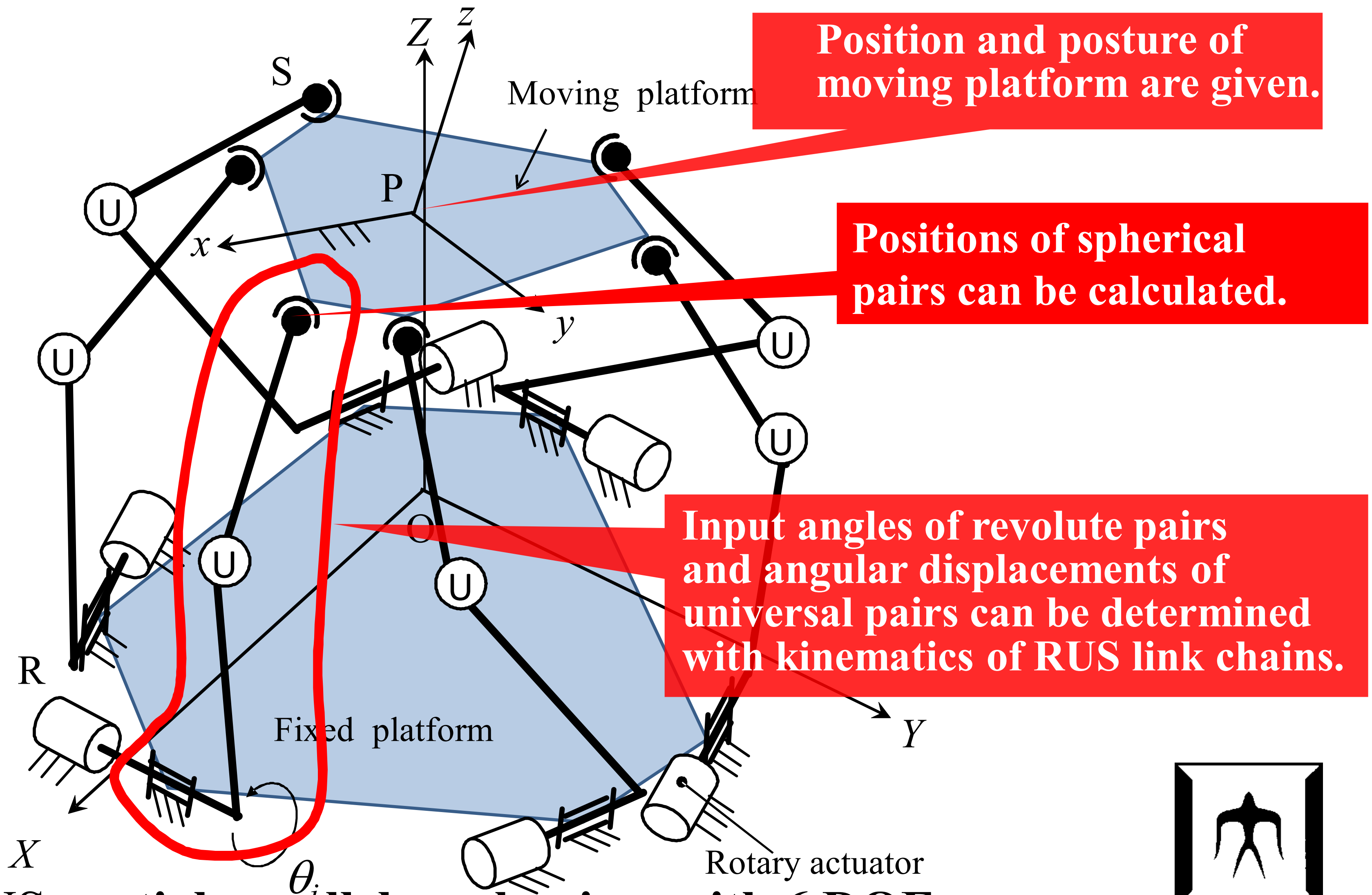
(1)Inverse kinematics analysis based on statics analysis with virtual forces is proposed and formulated.

(2)For an example, 6-RUS spatial parallel mechanism with 3 rotary actuators and 3 torsional springs is analyzed.

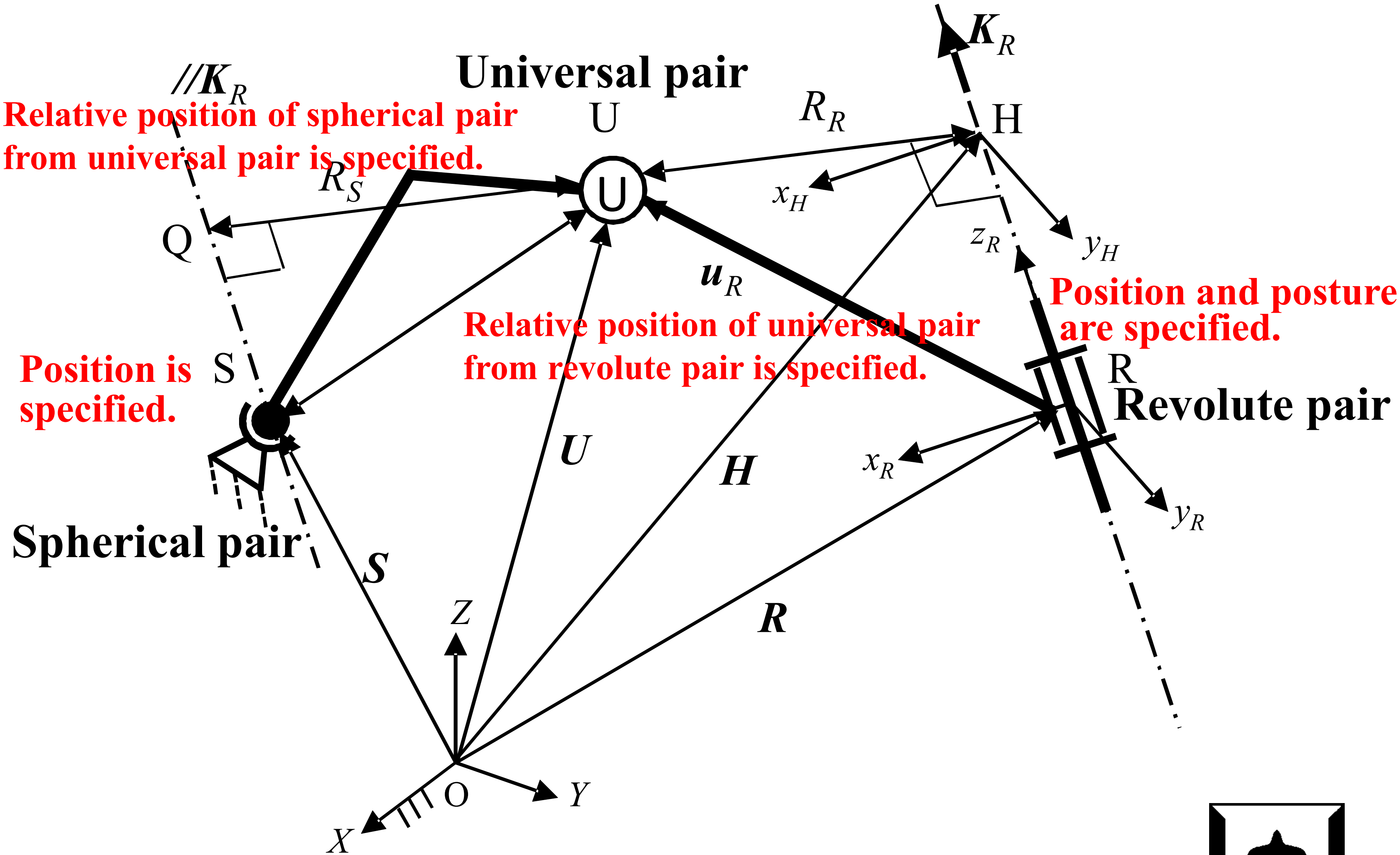


# 3.3 Inverse Kinetostatics analysis of 6-RUS spatial parallel mechanism

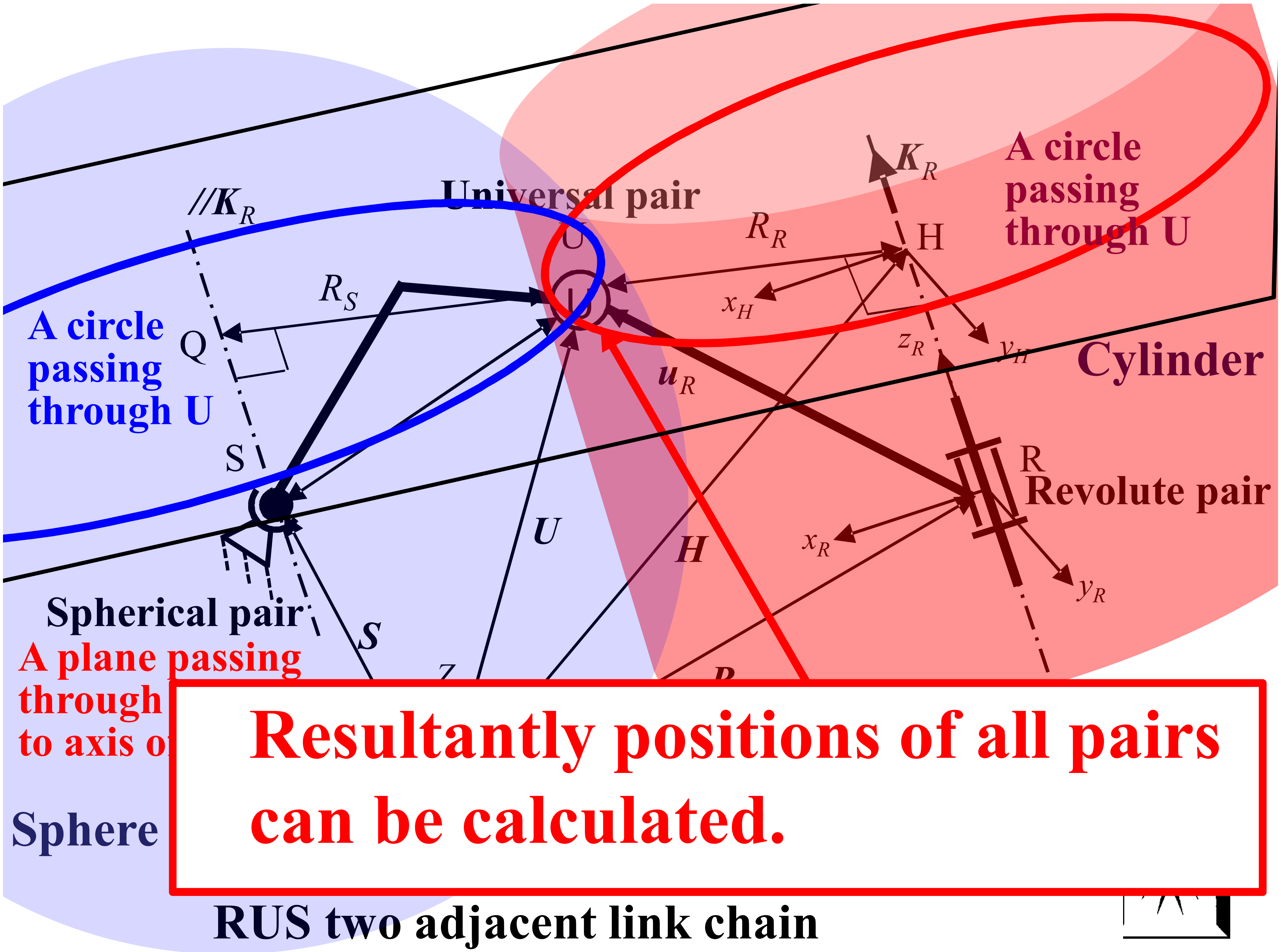
***Inverse kinematics based on RUS chain***




**6-RUS spatial parallel mechanism with 6 DOF**




# RUS two adjacent link chain

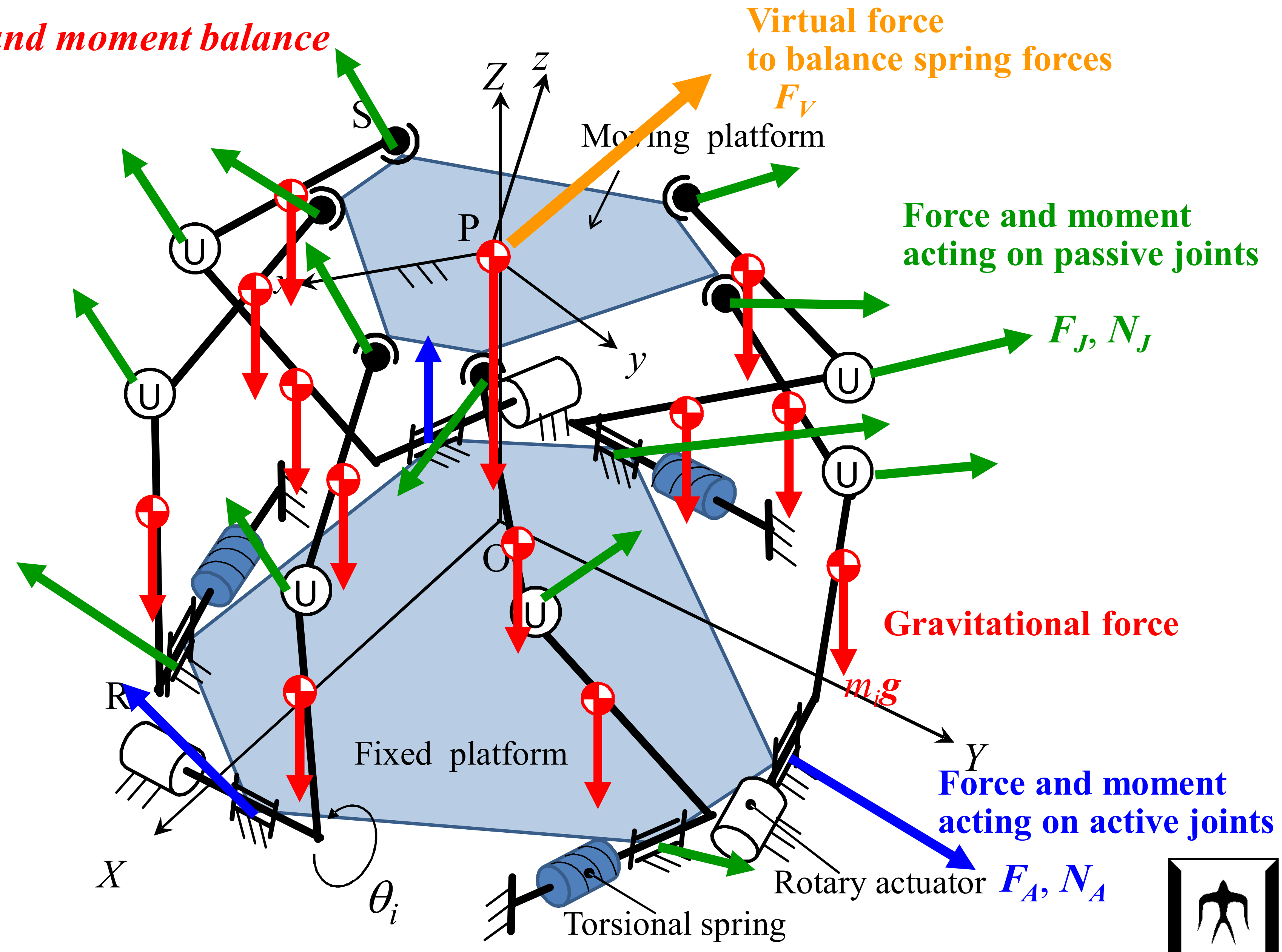



**Resultantly positions of all pairs can be calculated.**

**RUS two adjacent link chain**

# *Inverse statics of mechanism with springs*

*Force and moment balance*




**6-RUS spatial parallel mechanism constrained with springs**

## Statics equation:

**Moving plate:**

$$\sum_{i=1}^{6} \boldsymbol{F}_{JS,i} + \boldsymbol{F}_{V} + m_{P}\boldsymbol{g} = \boldsymbol{0}, \quad \sum_{i=1}^{6} (\boldsymbol{S}_{i} - \boldsymbol{G}_{P}) \times \boldsymbol{F}_{JS,i} = \boldsymbol{0}$$

**Upper links:**

$$-\boldsymbol{F}_{JS,i} + \boldsymbol{F}_{JU,i} + m_{U,i}\boldsymbol{g} = \boldsymbol{0}, \quad -(\boldsymbol{S}_{i} - \boldsymbol{G}_{U,i}) \times \boldsymbol{F}_{JS,i} + (\boldsymbol{U}_{i} - \boldsymbol{G}_{U,i}) \times \boldsymbol{F}_{JU,i} + \boldsymbol{N}_{JU,i} = \boldsymbol{0}$$

$$(i = 1 \sim 6)$$

**Lower links with actuator:**

$$-\boldsymbol{F}_{JU,i} + \boldsymbol{F}_{JR,i} + m_{L,i}\boldsymbol{g} = \boldsymbol{0},$$

$$-(\boldsymbol{U}_{i} - \boldsymbol{G}_{L,i}) \times \boldsymbol{F}_{JU,i} + (\boldsymbol{R}_{i} - \boldsymbol{G}_{L,i}) \times \boldsymbol{F}_{JR,i} - \boldsymbol{N}_{JU,i} + \boldsymbol{N}_{JR,i} + \boldsymbol{N}_{A,i} = \boldsymbol{0}$$

$$(i = 1,3,5)$$

**Lower links with spring:**

$$-\boldsymbol{F}_{JU,i} + \boldsymbol{F}_{JR,i} + m_{L,i}\boldsymbol{g} = \boldsymbol{0},$$

$$-(\boldsymbol{U}_{i} - \boldsymbol{G}_{L,i}) \times \boldsymbol{F}_{JU,i} + (\boldsymbol{R}_{i} - \boldsymbol{G}_{L,i}) \times \boldsymbol{F}_{JR,i} - \boldsymbol{N}_{JU,i} + \boldsymbol{N}_{JR,i} + \left[T_{R,i}\right]^{\mathrm{T}} \begin{bmatrix} 0 \\ 0 \\ K_i\theta_i \end{bmatrix} = \boldsymbol{0}$$

$$(i = 2,4,6)$$

**Constraints:**

$$\left[T_{R,i}\right]^{\mathrm{T}} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \bullet \boldsymbol{N}_{A,i} = 0 \quad (i = 2,4,6), \qquad \cdot\ \cdot\ \cdot\ \cdot$$

A system of 78 linear equations with respect to 78 unknowns of joint forces and moments, driving torques and virtual forces:

$$[A]\begin{bmatrix} F_{S,1,X} \\ F_{S,1,Y} \\ F_{S,1,Z} \\ \vdots \\ F_{U,1,X} \\ \vdots \\ N_{U,4,X} \\ \vdots \\ F_{V,X} \\ F_{V,Y} \\ F_{V,Z} \end{bmatrix} = [b]$$

Virtual forces:

$F_{V,X}$, $F_{V,Y}$, $F_{V,Z}$

can be calculated with Gauss-Jordan method.

## *Inverse kinetostatics analysis of 6-RUS spatial parallel mechanism with springs*

The assumed virtual forces, $F_{V,X}$, $F_{V,Y}$, $F_{V,Z}$
should be zeros.

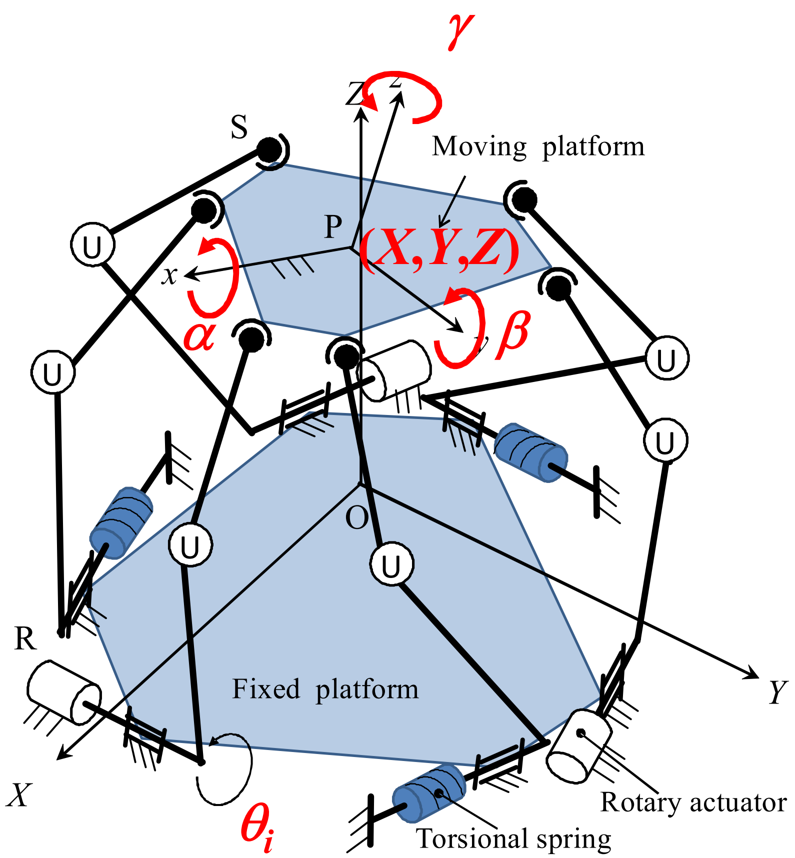
Procedure to obtain the balanced configuration
of the mechanism with springs:

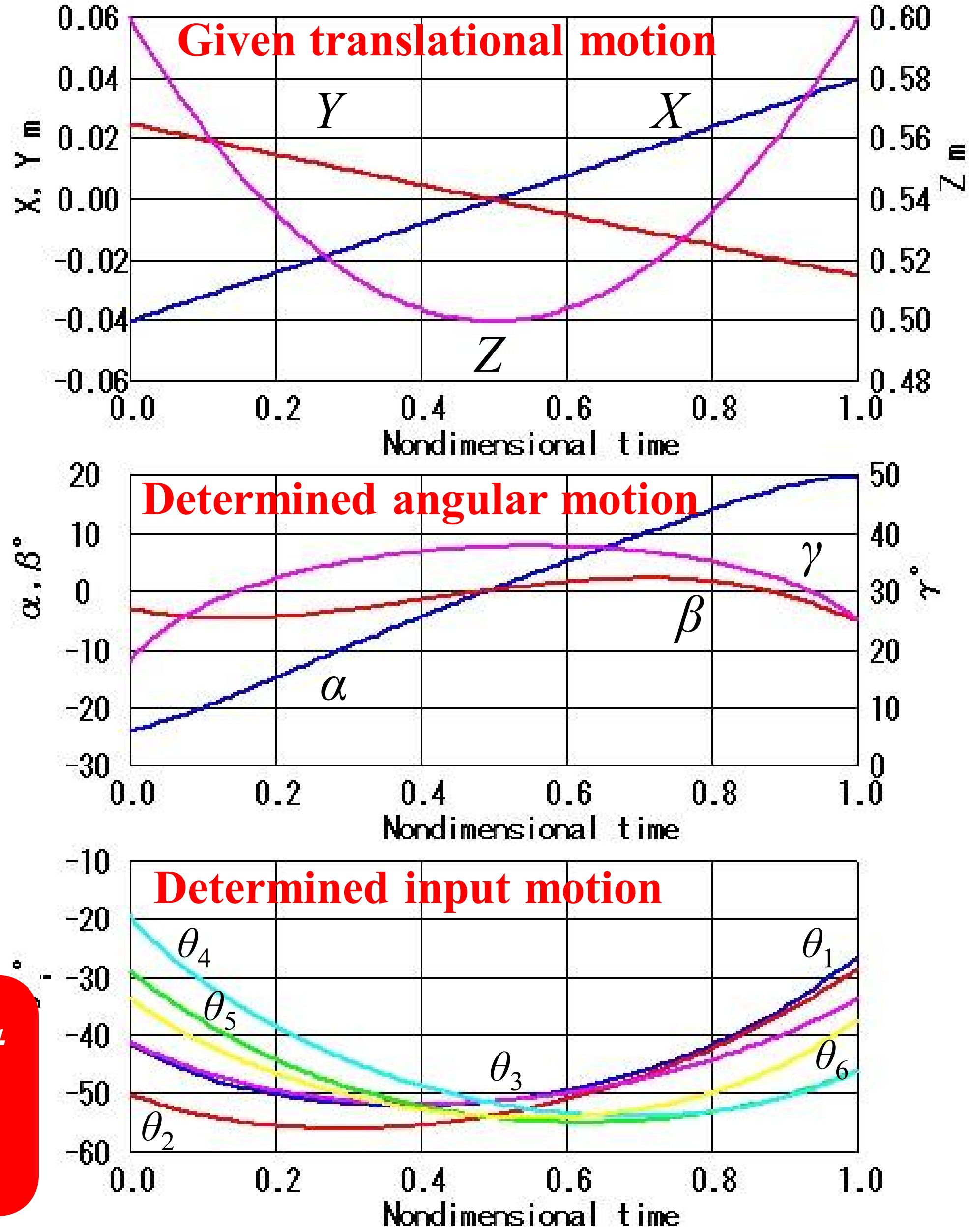
(1)Specify translational motion of moving platform, $X$, $Y$, $Z$

(2)Assume angular motion of moving platform, $\alpha$, $\beta$, $\gamma$,
as variables

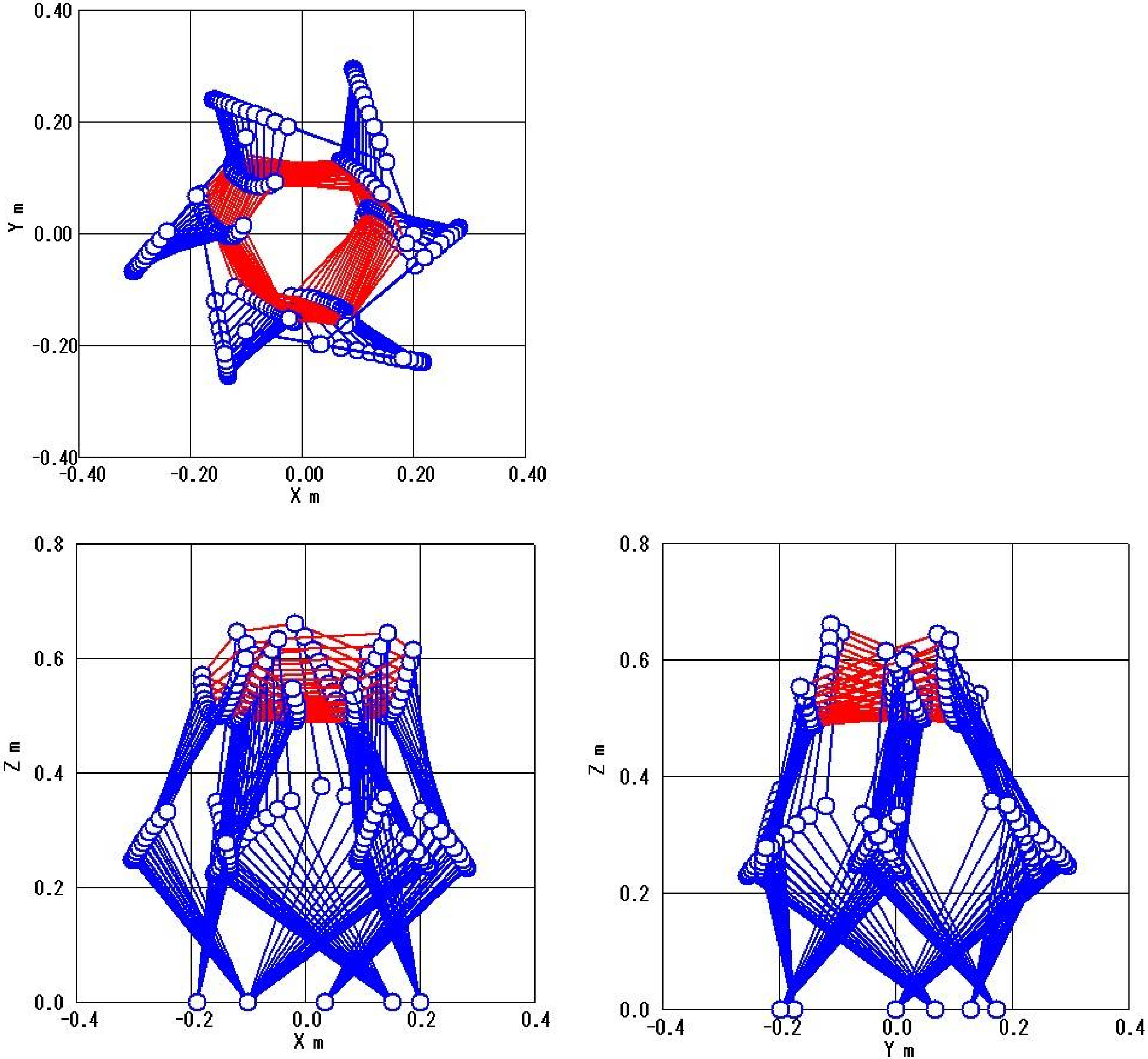
(3)Inverse statics analysis to calculate virtual forces,
$F_{V,X}$, $F_{V,Y}$, $F_{V,Z}$

# *Example of inverse kinetostatics analysis of 6-RUS spatial parallel mechanism with springs*




***Smooth displacement can be obtained.***


0.40
0.20
0.00
-0.20
-0.40
-0.40
-0.20
0.00
0.20
0.40
X m
Y m
0.8
0.6
0.4
0.2
0.0
-0.4
-0.2
0.0
0.2
0.4
X m
Z m
0.8
0.6
0.4
0.2
0.0
-0.4
-0.2
0.0
0.2
0.4
Y m
Z m

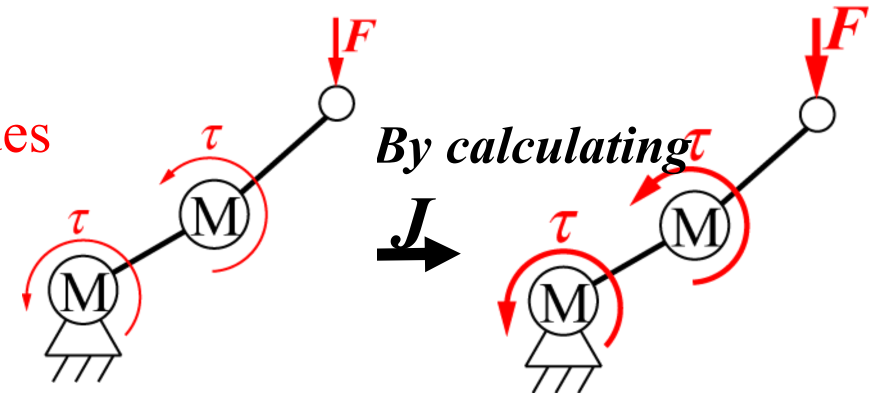
# 4. Application to Variable Stiffness Mechanisms

## 4.1 Objective

To control flexibility of robots with redundancy

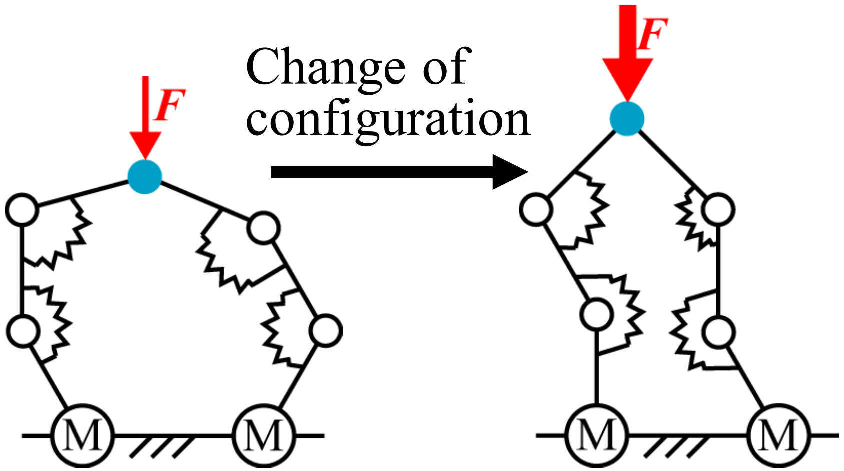
Compliance/impedance control

- To control stiffness with actuator torques
- Real time sensing and calculation

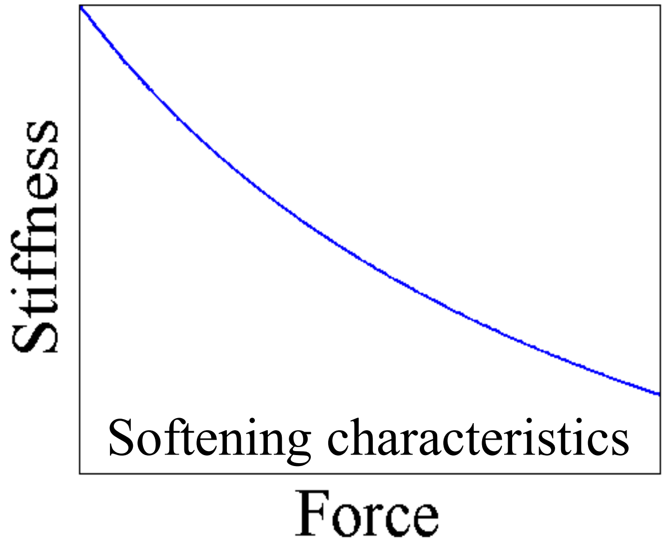


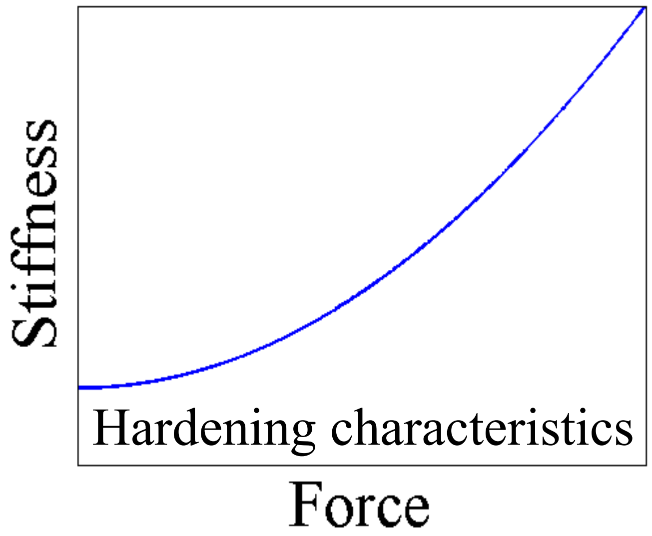

Utilizing passive elastic elements

- To control stiffness
  by configuration of mechanism
- Real time sensing is not required.



To control of nonlinear stiffness of a redundant closed-loop mechanism with elastic elements



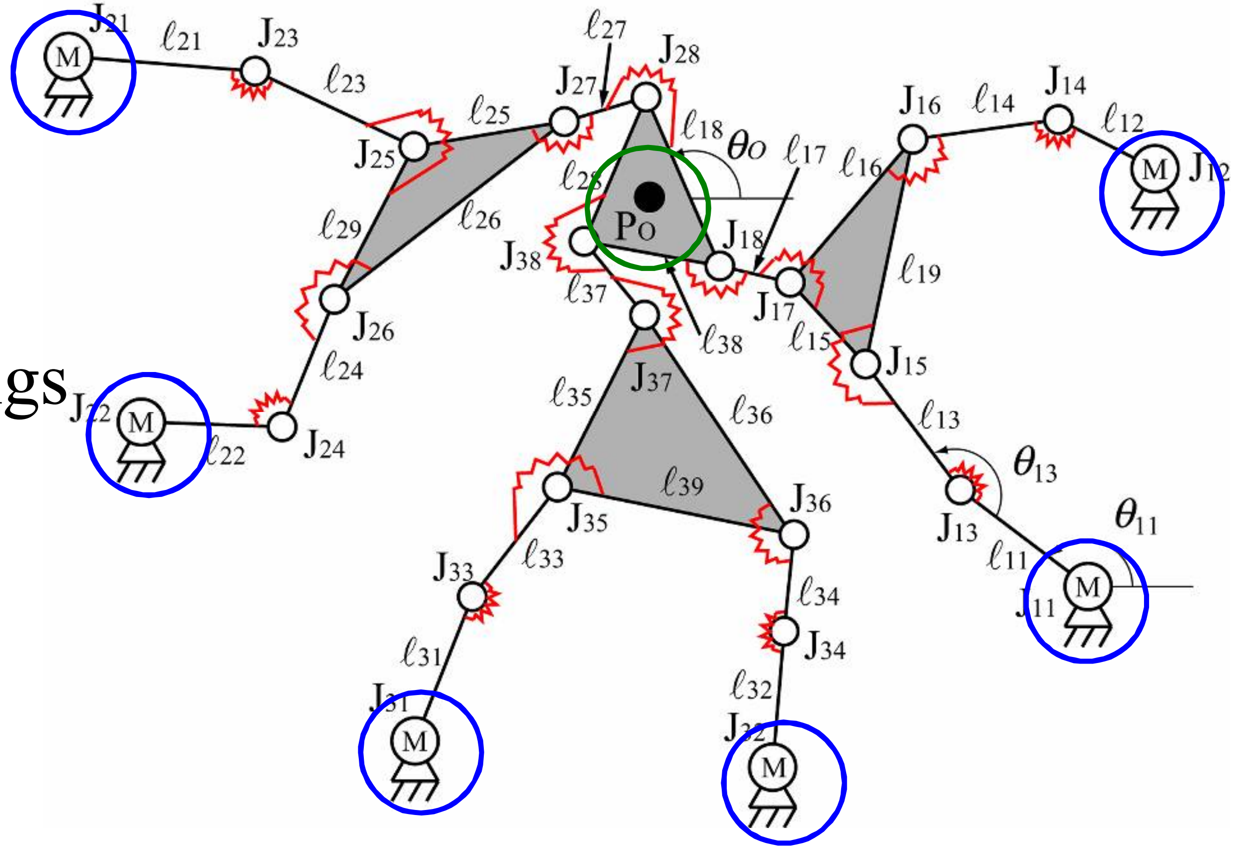




For example, control of soft or hard spring characteristics

- To analyze output stiffness with respect to actuator inputs (Forward kinetostatics analysis)
- To calculate the desired actuator inputs for the specified output stiffness (Inverse kinetostatics analysis)
- To validate the proposed method with experiments

# 4.2 Target mechanism

- A planar closed-loop mechanism with 9DOF and 6 actuators
  - 6 active pairs
  - 18 passive pairs
  - 19 moving links




- Torsional coil springs at all passive pairs

- Output point on on a moving table

- 3 non-actuated DOF

# 4.3 Configuration determining parameter

Actuator inputs: $\Theta$ → Force balanced configuration

3 non-actuated DOF

↓

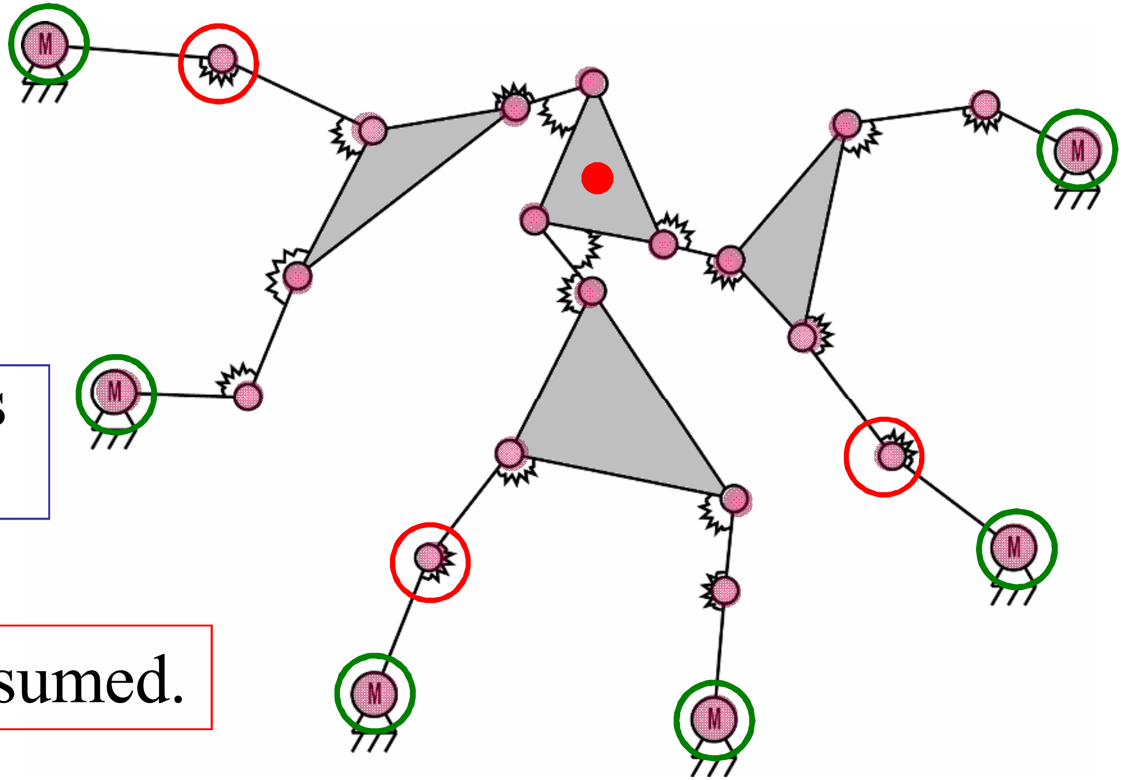
3 C.D.P: $\Phi$

↓

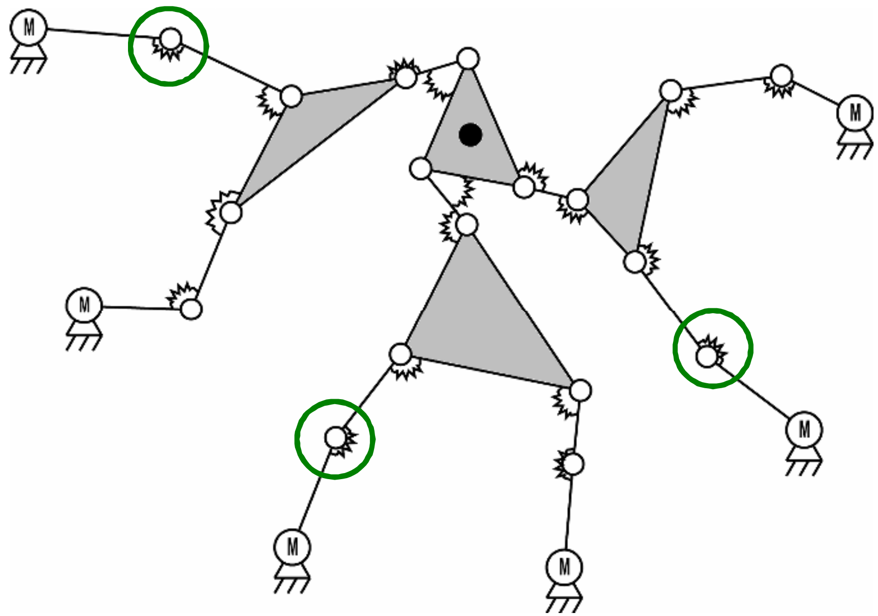
Closed-loop kinematics calculation

↓

Configuration can be assumed.



# 4.4 Force balance equation with virtual forces



Joint forces $\boldsymbol{F}_J$ and driving forces $\boldsymbol{F}_A$ are totally 54.

$\boxed{\text{3 virtual torques: } \boldsymbol{F}_I}$

19 moving links

↓

$\boxed{\text{57 force balance equations}}$

$$[A(\boldsymbol{\Theta},\boldsymbol{\Phi})]\begin{bmatrix}\boldsymbol{F}_J\\ \boldsymbol{F}_A\\ \boldsymbol{F}_I\end{bmatrix}=[b(\boldsymbol{\Theta},\boldsymbol{\Phi},\boldsymbol{F}_E)]$$

Essentially $\boldsymbol{F}_I = \boldsymbol{0}$

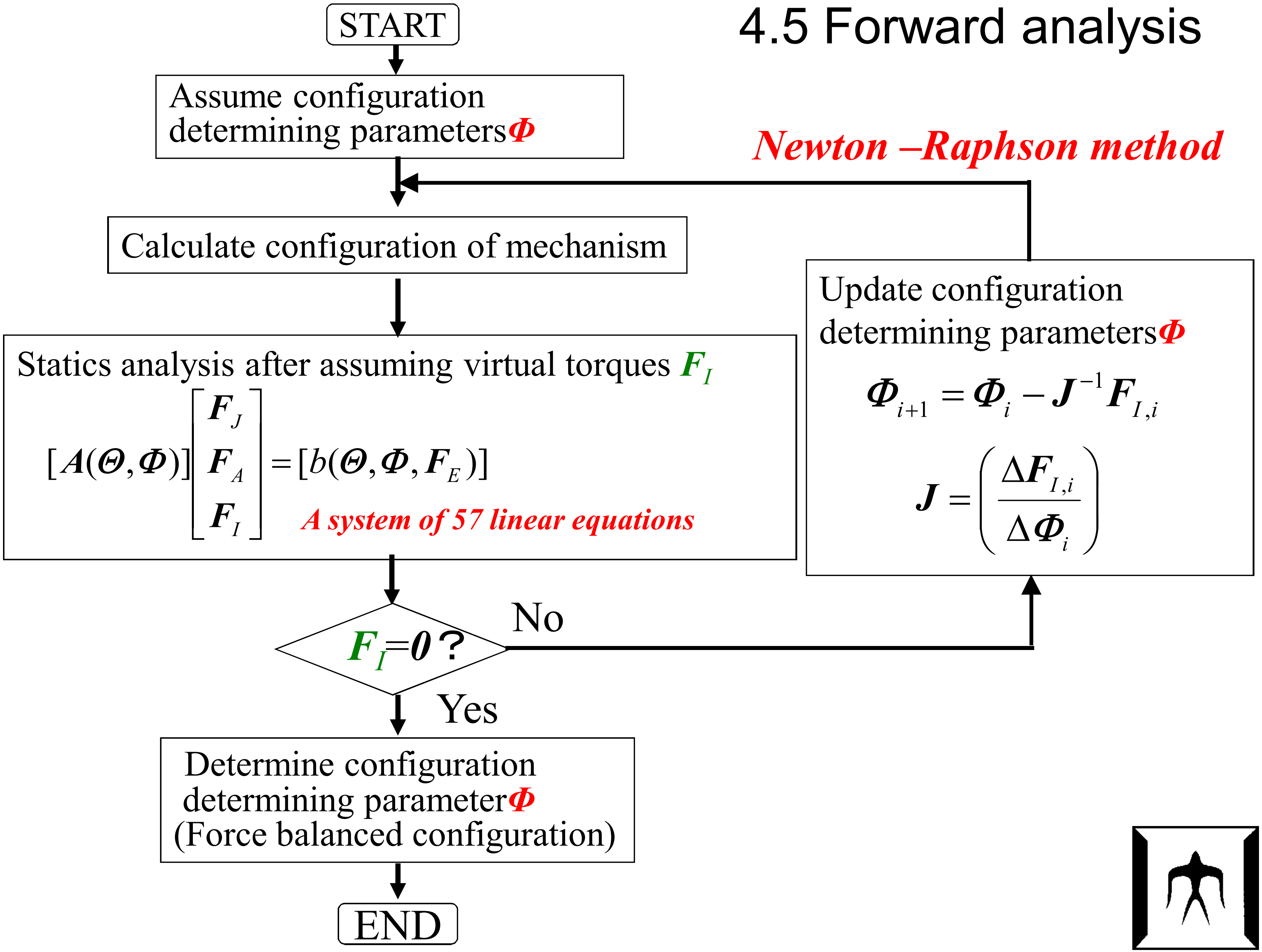
C.D.P. should be modified.

$$\boldsymbol{\Phi}_{i+1} = \boldsymbol{\Phi}_i - \boldsymbol{J}^{-1}\boldsymbol{F}_{I,i}$$

$$\boldsymbol{J} = \left(\frac{\Delta \boldsymbol{F}_{I,i}}{\Delta \boldsymbol{\Phi}_i}\right)$$

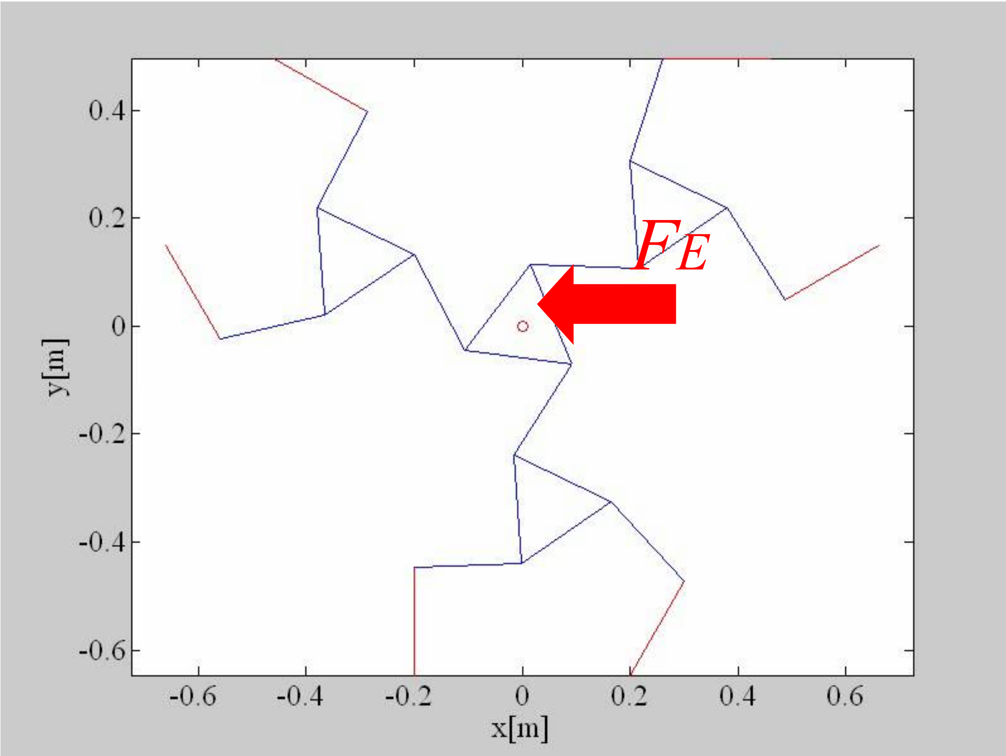
***Force balanced configuration $\boldsymbol{\Phi}$ is obtained when $\boldsymbol{F}_I \doteqdot 0$***

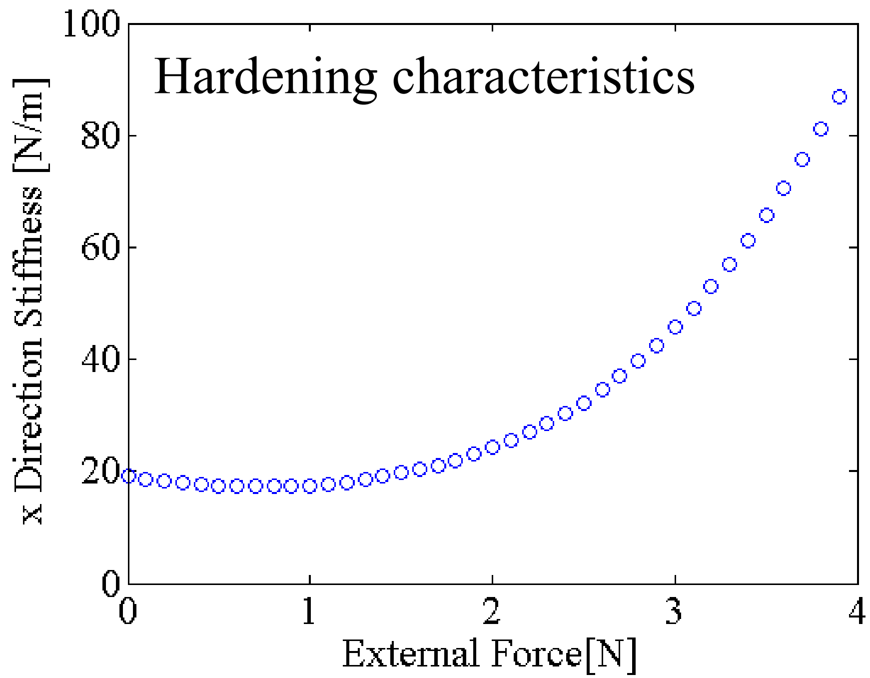
# 4.5 Forward analysis



# Analysis of nonlinear stiffness

## − sequential kinetostatics analysis with external force



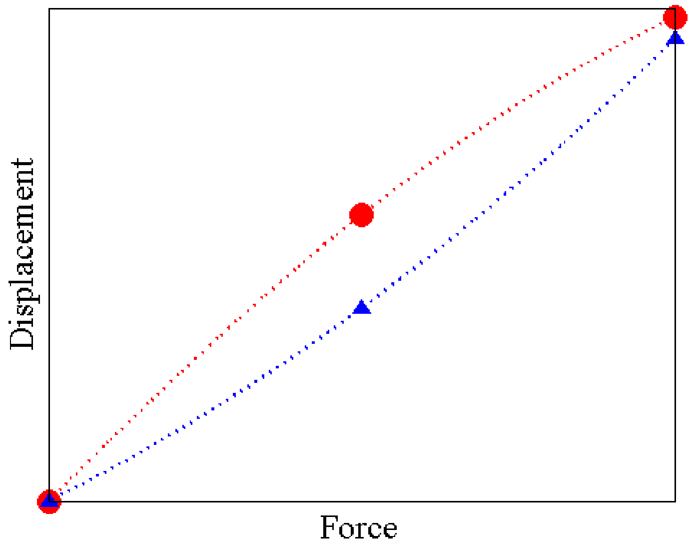

***Output displacement and nonlinear stiffness of output point can be obtained.***

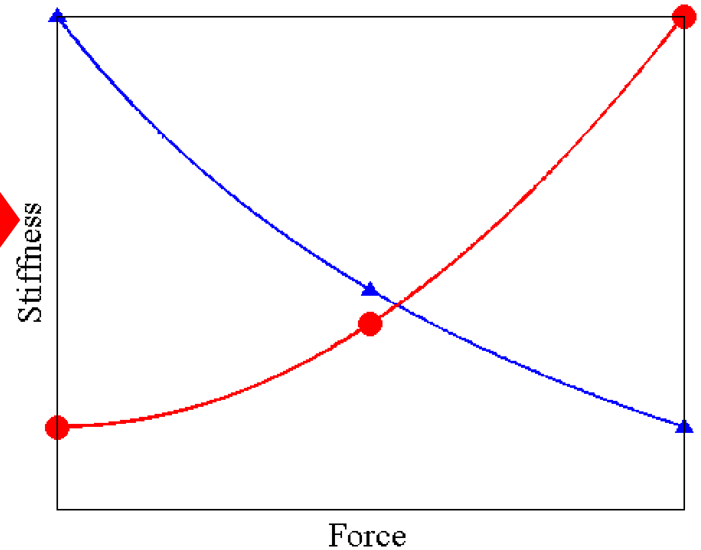
# 4.6 Inverse analysis

***Specification of nonlinear stiffness with precision points***

**Nonlinear stiffness** → **Actuator inputs**

**3 sets of forces and displacements of output point are specified**

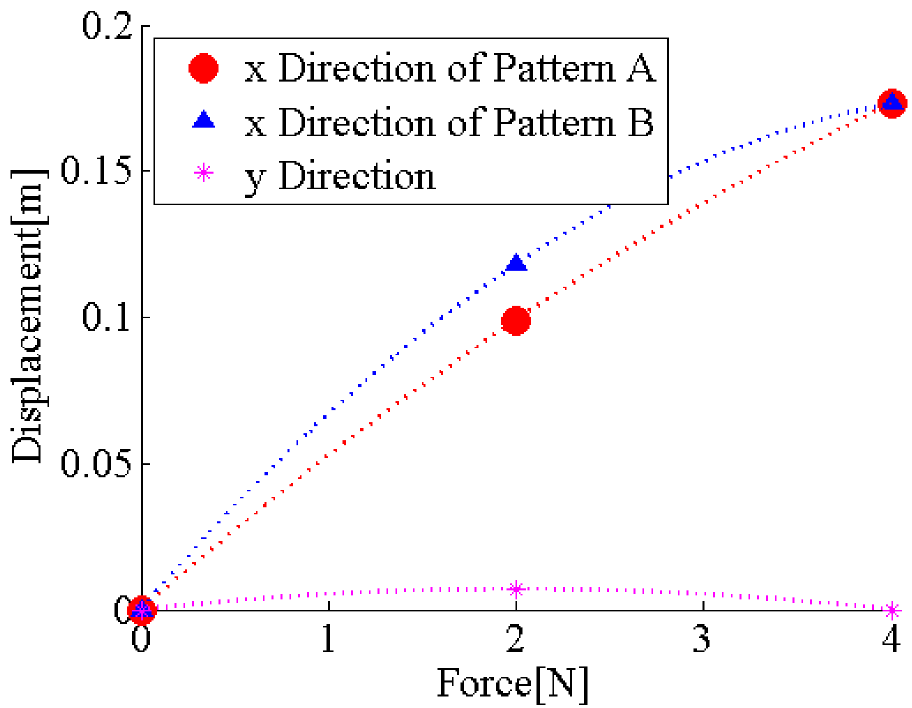


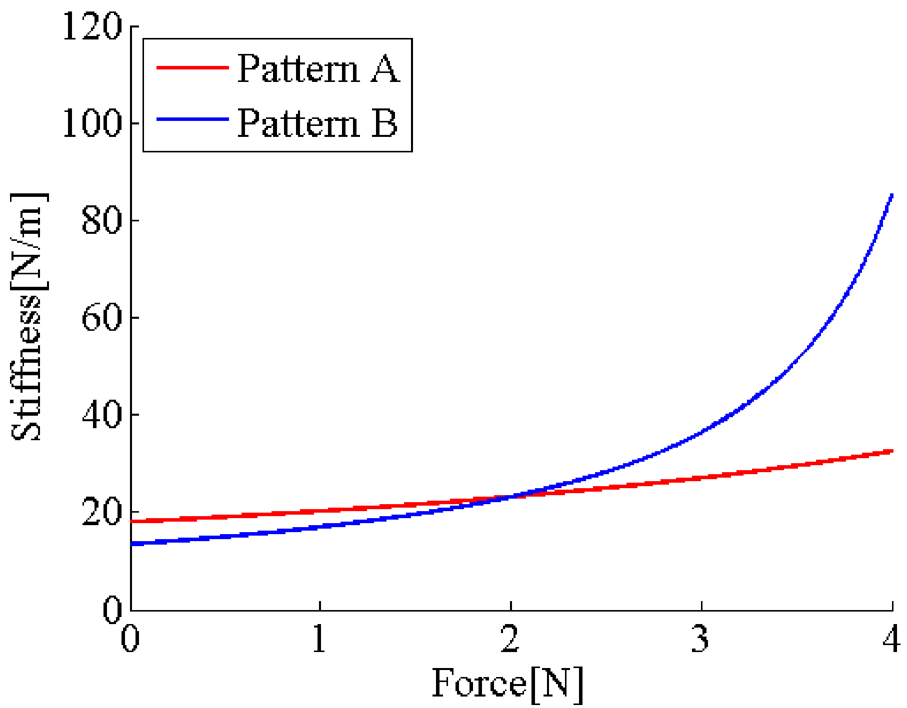




***3 statuses are simultaneously analyzed.***

# *Examples of analysis*

## To control nonlinear stiffness for 2 patterns

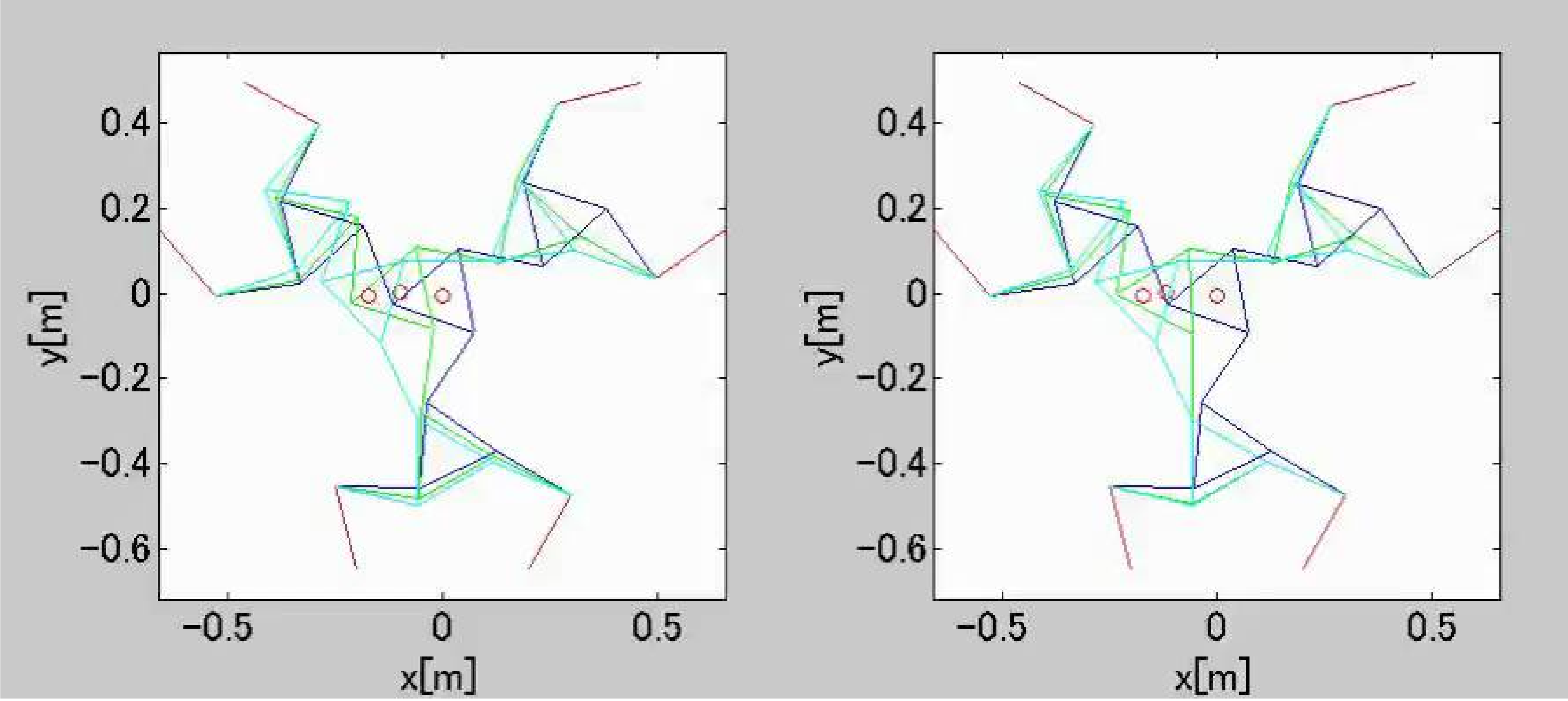


Stiffness pattern A

Force balanced configuration

Stiffness pattern B

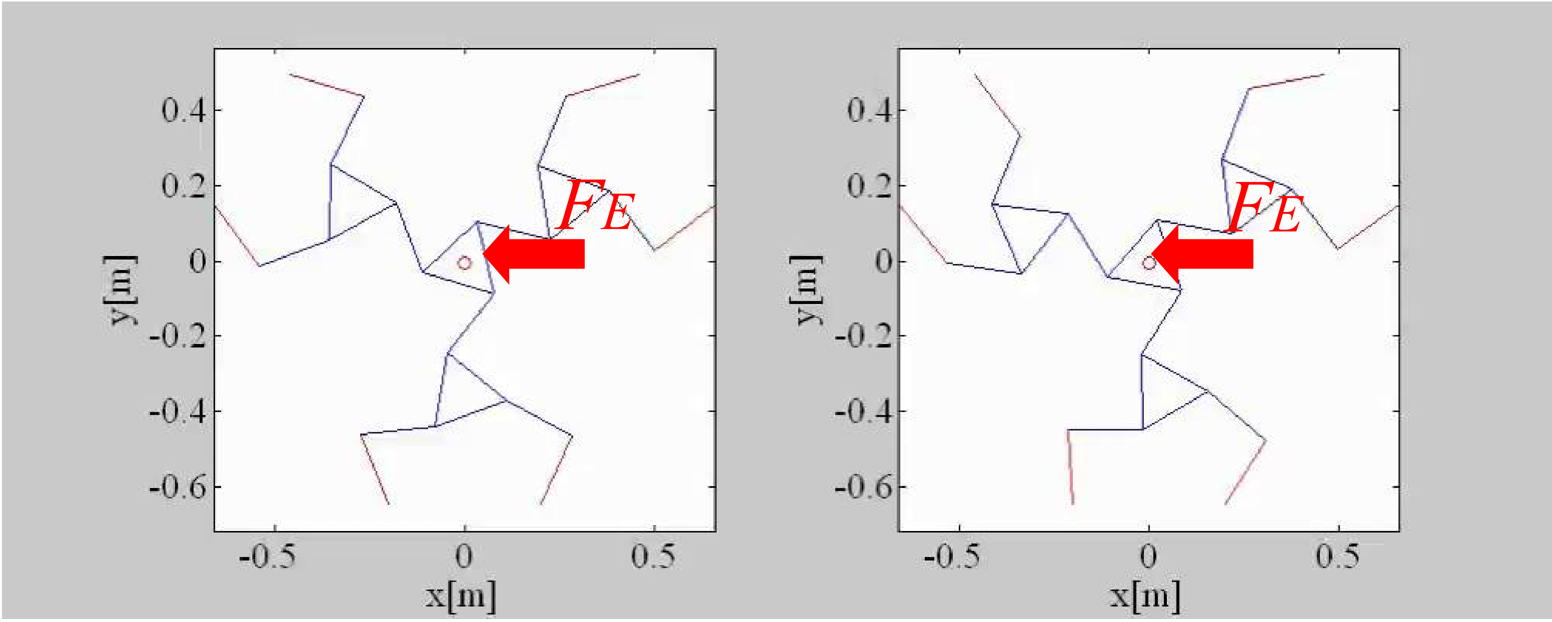


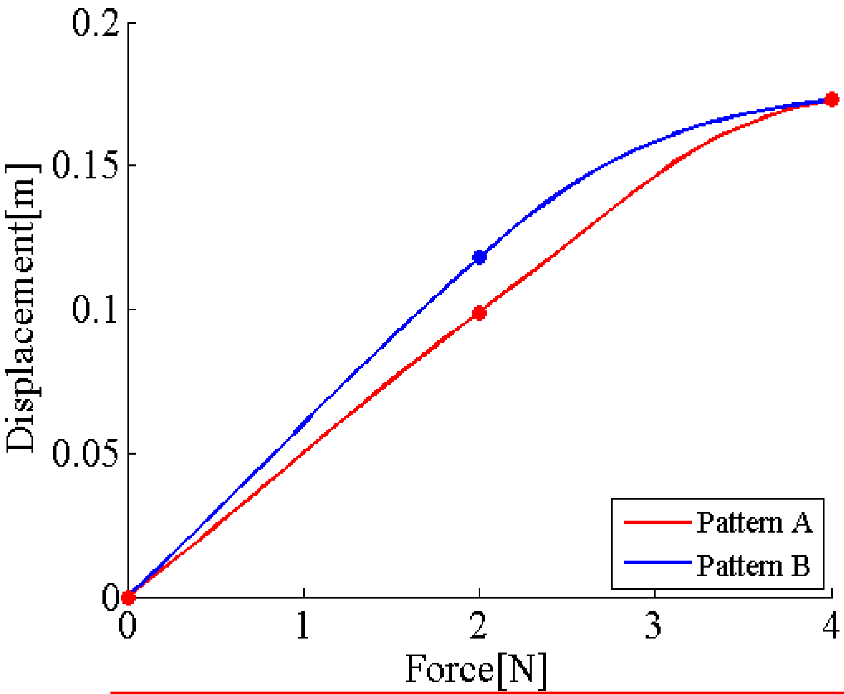

***By assuming 3 sets of displacement and forces, actuator inputs to generate nonlinear stiffness can be obtained.***

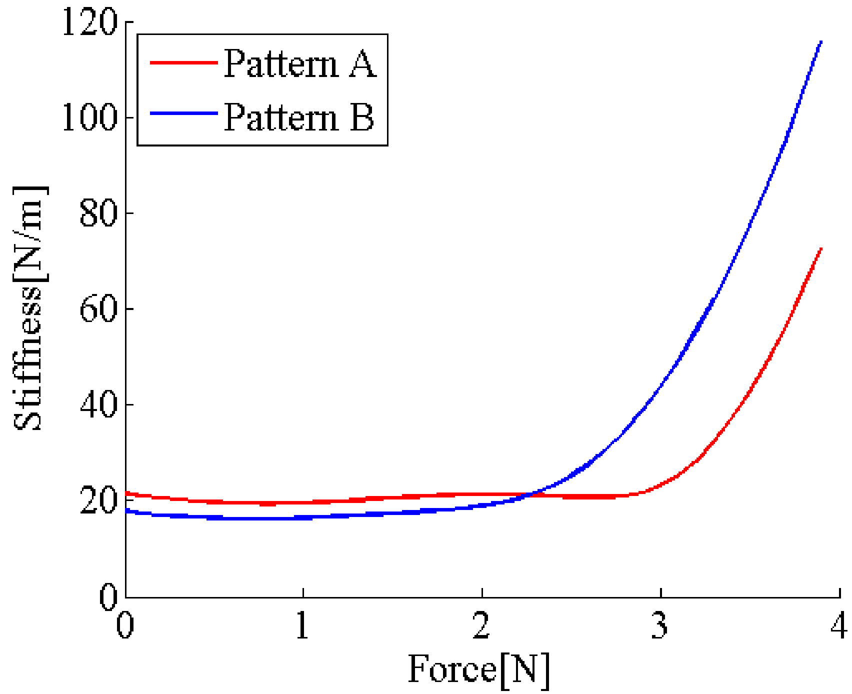
$$\boldsymbol{P}_{O,A} = \begin{bmatrix} 0.000 & -0.099 & -0.173 \\ -0.005 & 0.002 & -0.005 \end{bmatrix} \quad \boldsymbol{P}_{O,B} = \begin{bmatrix} 0.000 & -0.118 & -0.173 \\ -0.005 & 0.002 & -0.005 \end{bmatrix}$$

$F_E$=0[N]

$F_E$=4[N]



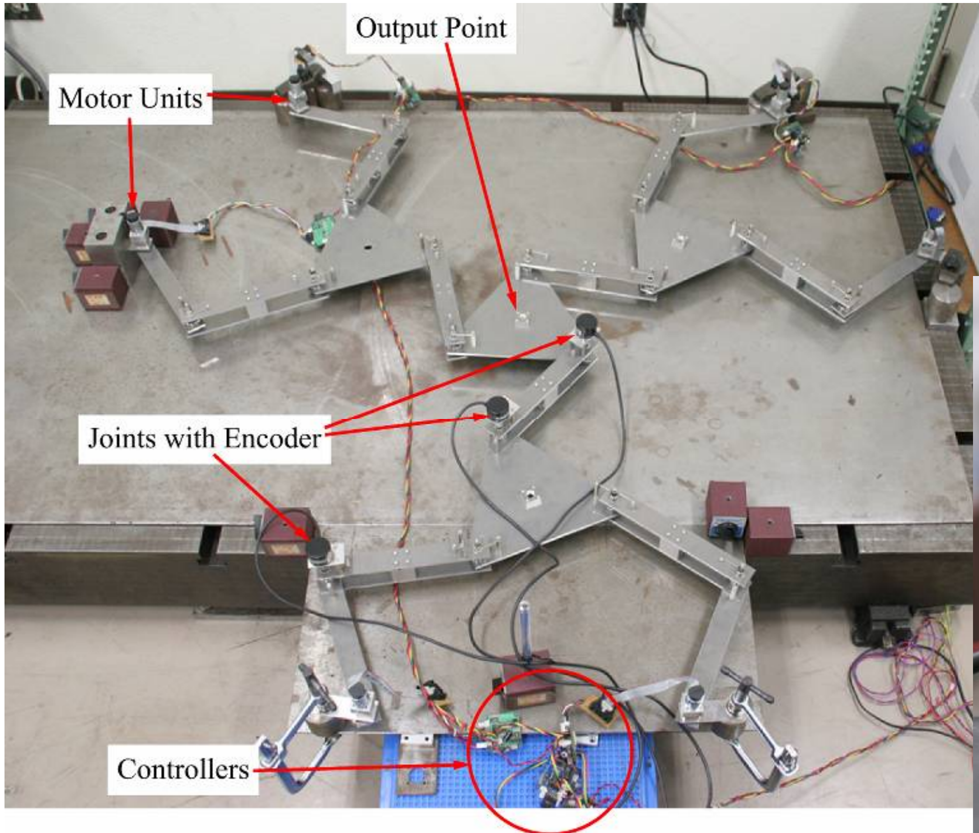

***Both of displacement and flexibility can be controlled.***

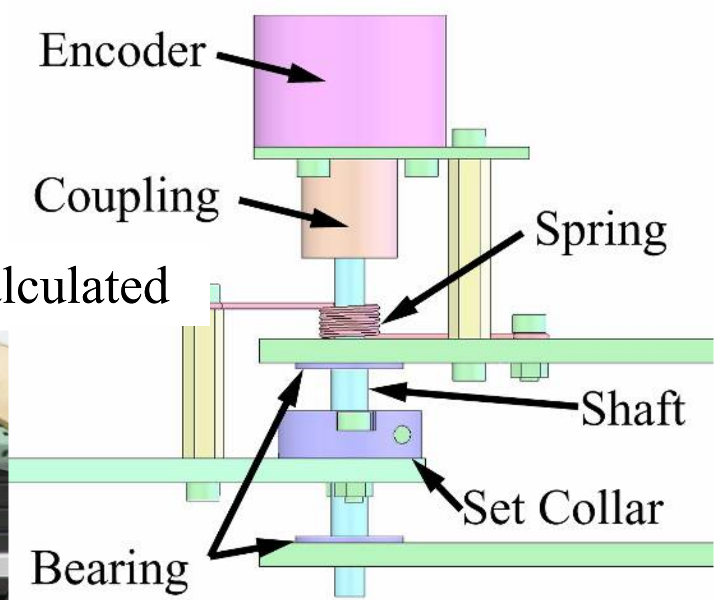
# 4.7 Experimental validation

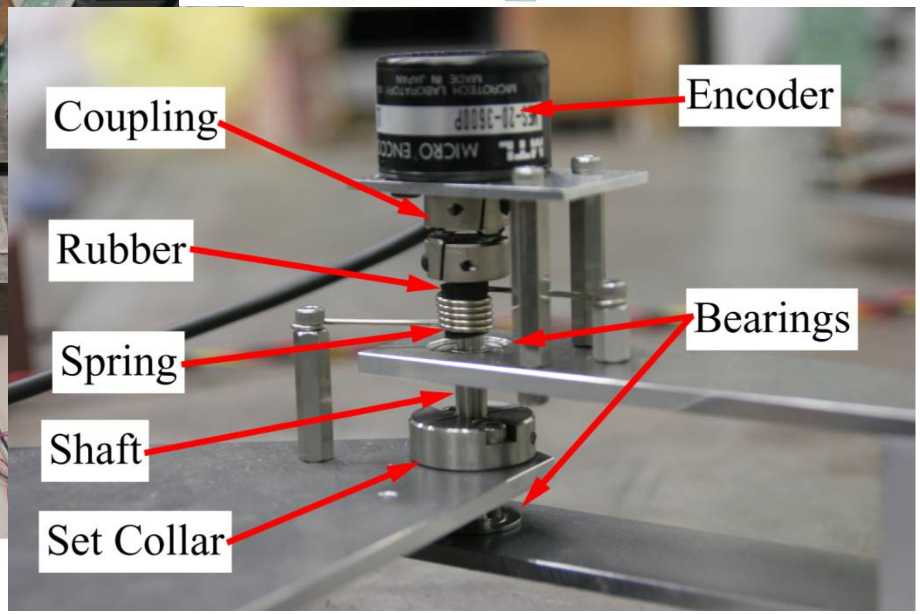
3 rotary encoders are mounted on passive joints

↓

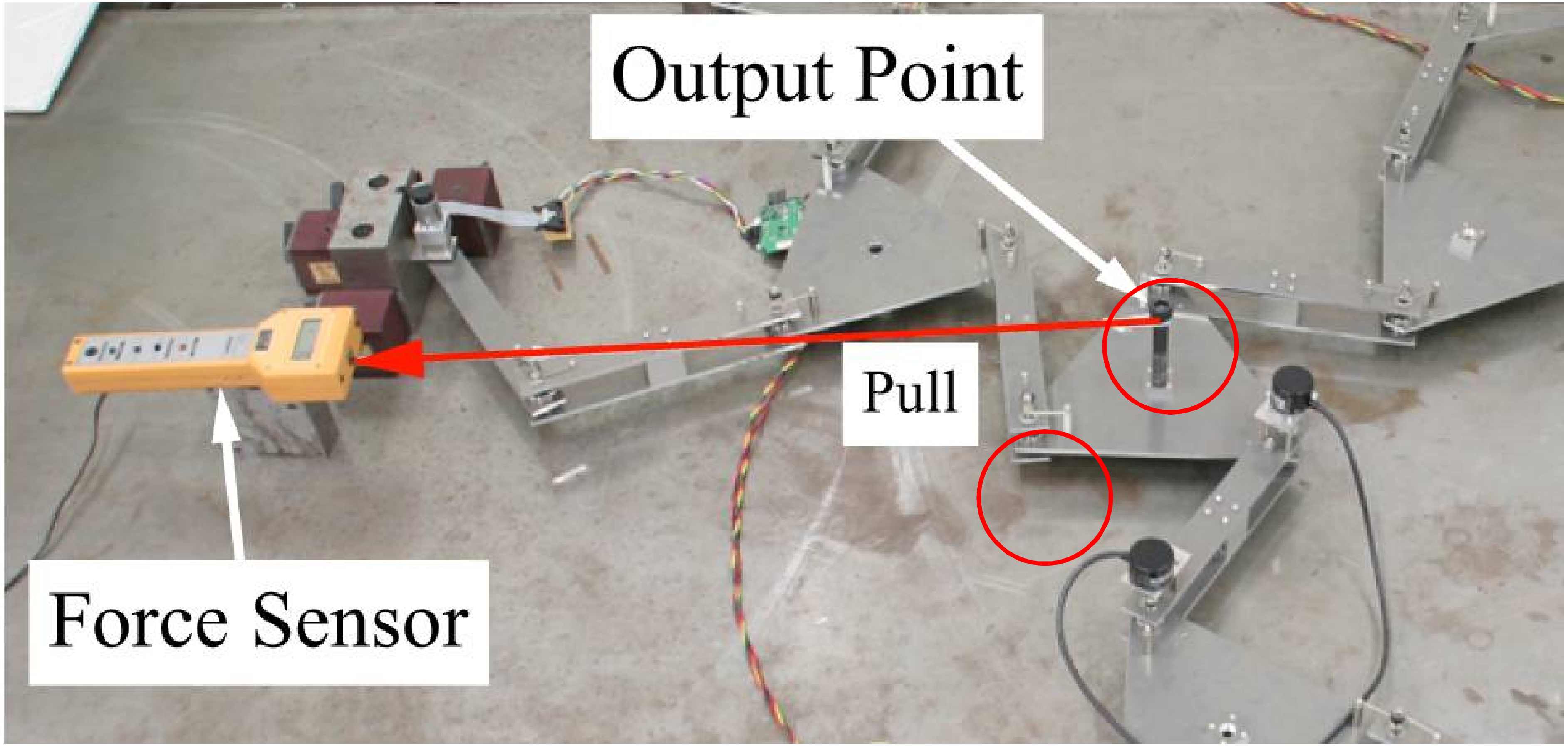
All joint positions can be calculated



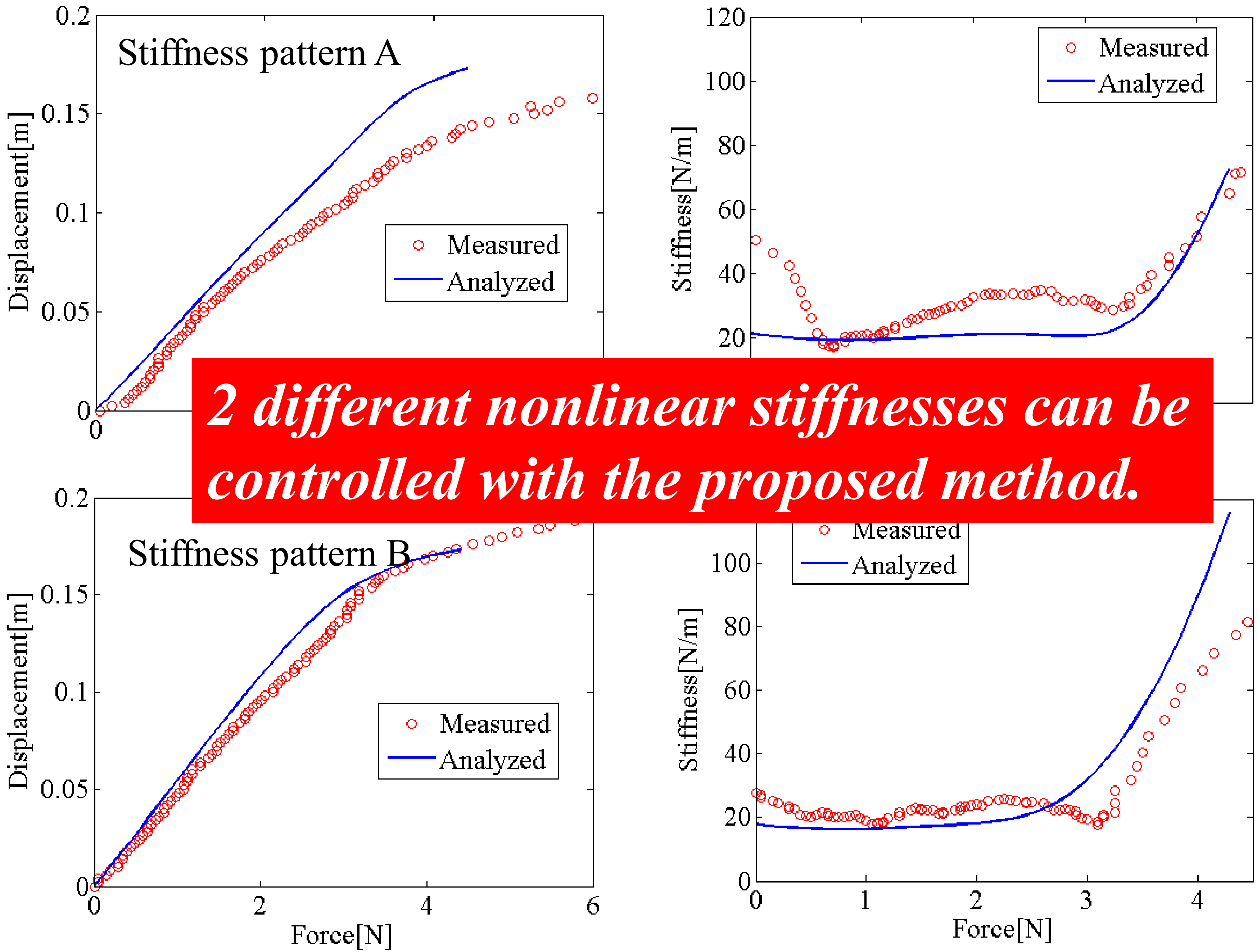



# Measurement of elastic displacement




*By applying external force, displacement and force are measured with rotary encoders and force sensor.*




***2 different nonlinear stiffnesses can be controlled with the proposed method.***

# 5. Concluding remarks

**Aiming to control underactuated link mechanisms constrained with elastic elements, forward/inverse kinetostatic analyses are established.**

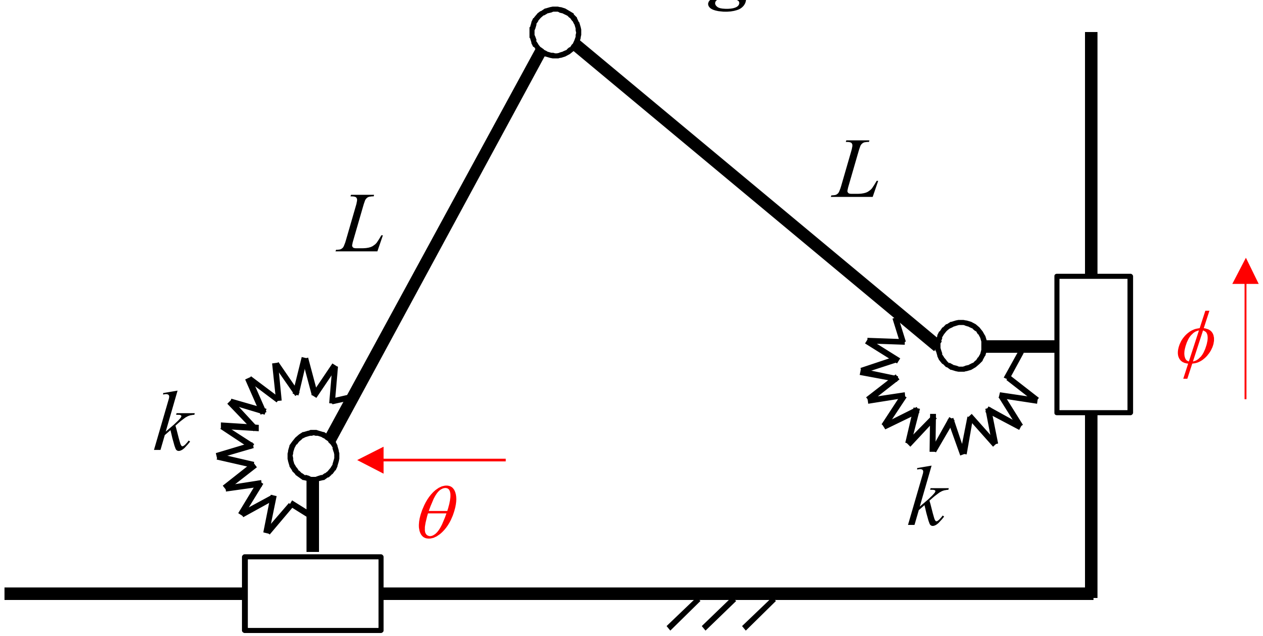
**(1)Inverse kinetostatic analysis based on statics analysis taking account of virtual torque and solution of nonlinear equations with respect to unknown joint variables is proposed.**

**(2)A planar 5-bar mechanism with 2 DOF and 1 actuator and a spatial 6RUS parallel manipulator with 6 DOF and 3 actuators are analyzed.**

**(3)Output stiffness of redundant parallel underactuated link mechanism constrained with elastic elements can be controlled by changing mechanism configuration while keeping the desired output displacement.**

# Homework 6

**Derive the statics equation of the following underactuated mechanism with input motion $\theta$ (active) and output motion $\phi$ ,(passive), which is constrained with two torsional springs of torsional stiffness $k$ and natural angle $\pi$.**




**The result will be summarized in A4 size PDF with less than 4 pages and sent to Prof. Iwatsuki via T2SCHOLAR by May 25, 2023.**