May 25, 2023

Advanced Mechanical Elements
(Lecture 7)

Kinetostatic analysis and motion control of underactuated wire-driven mechanisms

- Motion control of wire-driven underactuated mechanisms under gravitational force -

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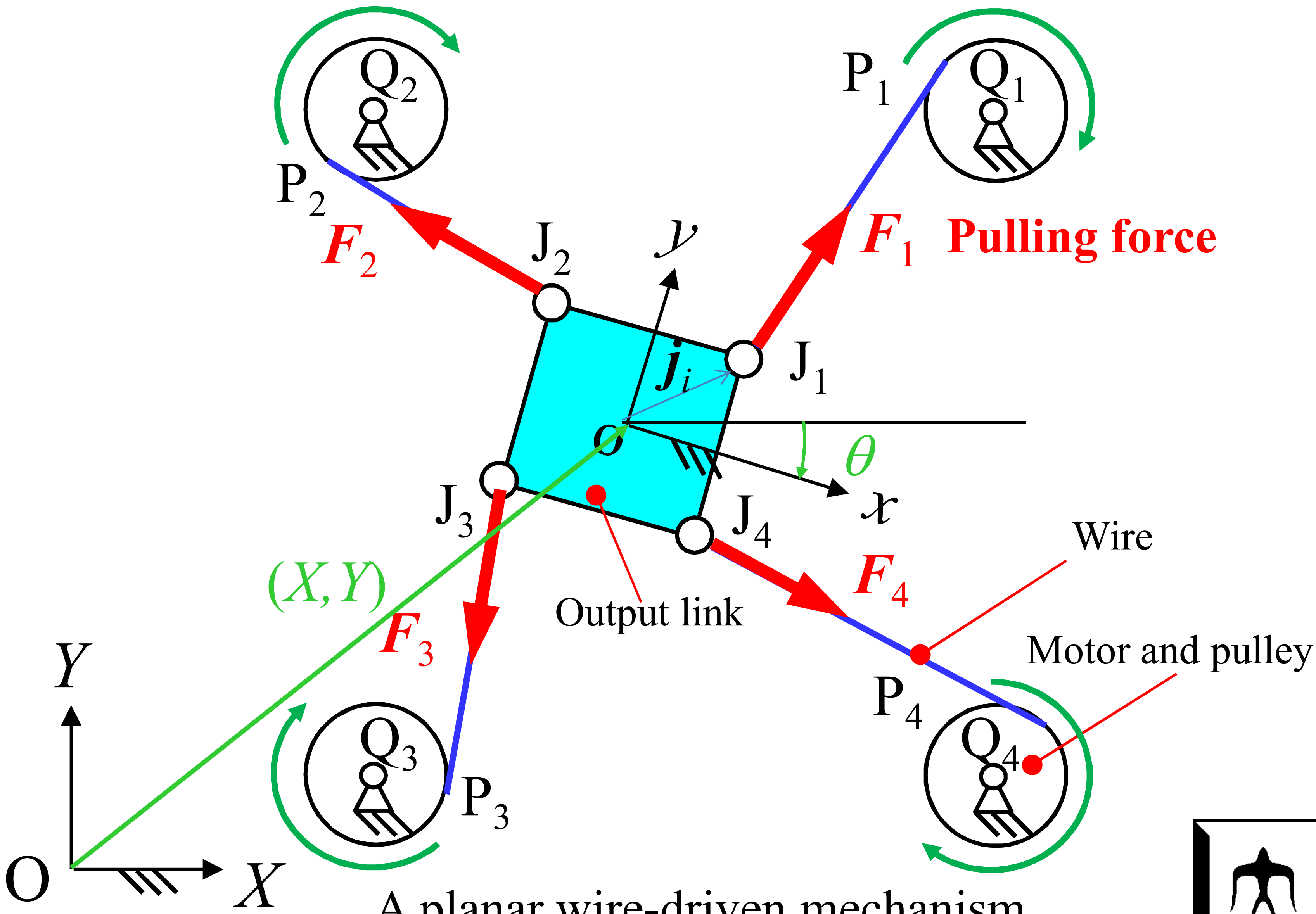
1. Conventional Wire-driven Mechanisms



A planar wire-driven mechanism

Inverse kinematics:

$$l_i = \left| \boldsymbol{P}_i - \boldsymbol{J}_i \right|$$

$$\boldsymbol{J}_i = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \boldsymbol{j}_i + \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$(\boldsymbol{P}_i - \boldsymbol{J}_i) \cdot (\boldsymbol{Q}_i - \boldsymbol{P}_i) = 0$$

Inverse statics:

$$\sum_i F_i \frac{\boldsymbol{P}_i - \boldsymbol{J}_i}{\left| \boldsymbol{P}_i - \boldsymbol{J}_i \right|} = \boldsymbol{0}$$

$$\sum_i (\boldsymbol{J}_i - \begin{bmatrix} X \\ Y \end{bmatrix}) \times F_i \frac{\boldsymbol{P}_i - \boldsymbol{J}_i}{\left| \boldsymbol{P}_i - \boldsymbol{J}_i \right|} = 0$$

$$F_i > 0$$

Wire cannot generate pushing force!

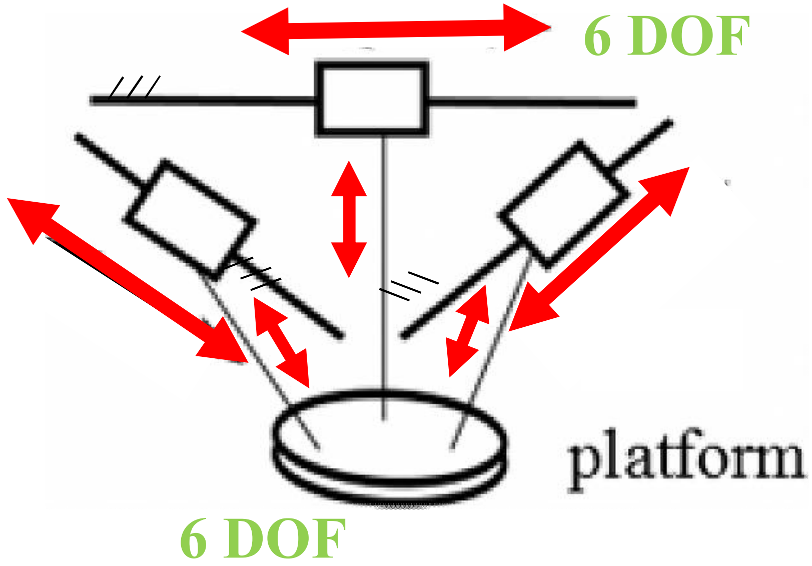
The both of kinematics and statics should be taken into account. (Wire tension should be controlled.)

Number of wires should be more than DOF because wire cannot generate pushing force.

$$\Rightarrow \left[\boldsymbol{A}_{3\times 4} \right] \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \boldsymbol{b}_{3\times 1}$$

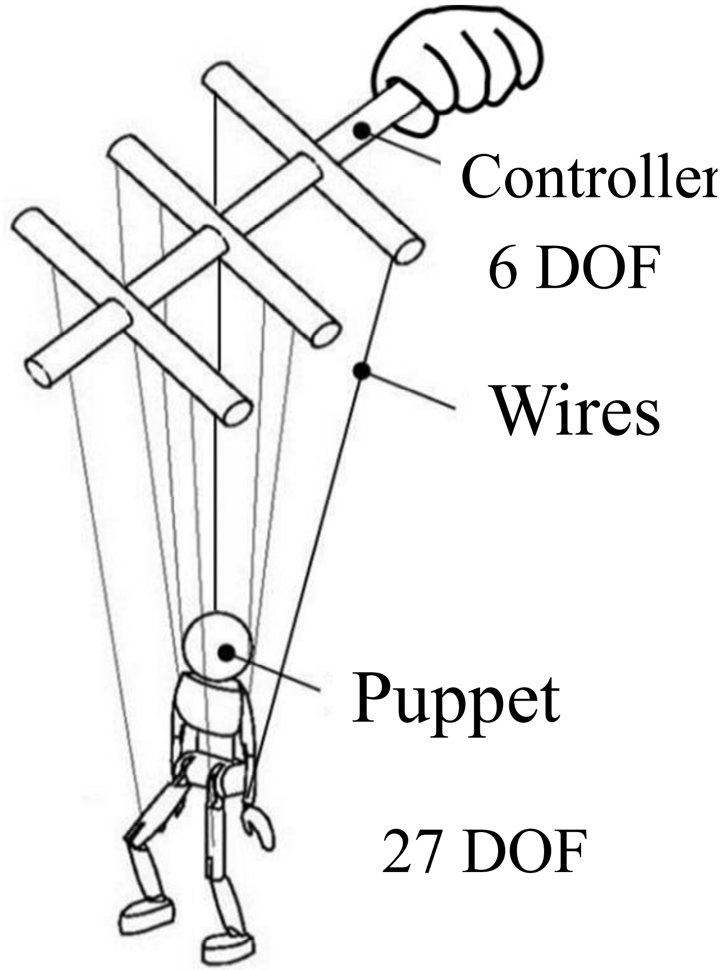
$$\Rightarrow \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \left[\boldsymbol{A}_{3\times 4} \right]^{\#} \boldsymbol{b}_{3\times 1} + (\boldsymbol{I} - \left[\boldsymbol{A}_{3\times 4} \right]^{\#} \left[\boldsymbol{A}_{3\times 4} \right]) \boldsymbol{k}$$

2. Underactuated Wire-driven Mechanisms



A general wire-driven mechanism
(6 DOF crane)

Position and posture control of moving table by adjusting wire lengths and wire winging position



Marionette as a typical mechanism

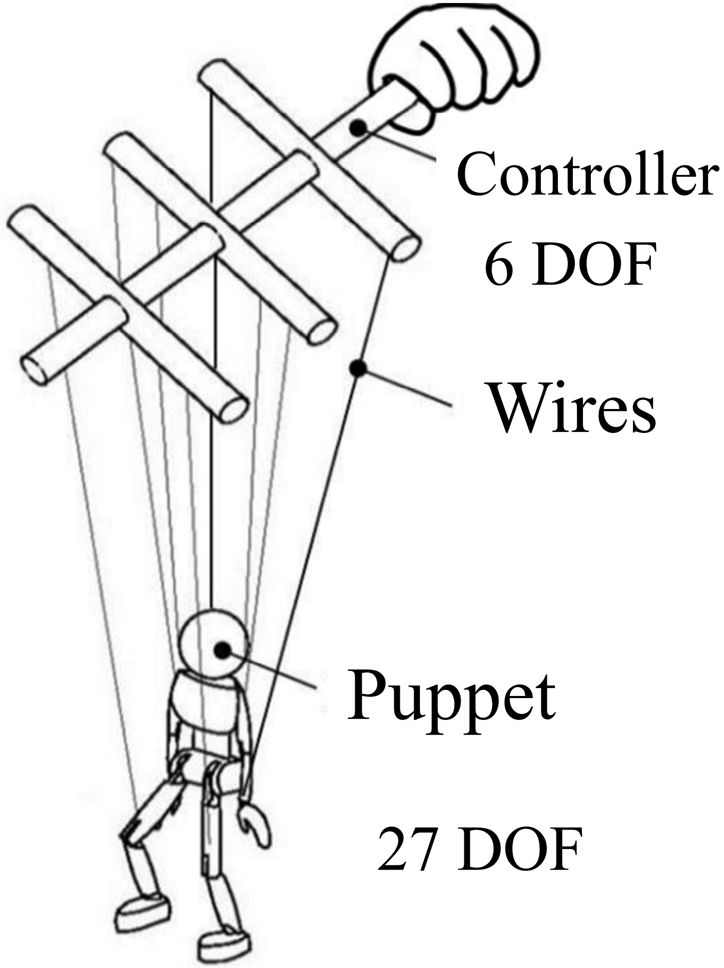
Marionette:

- An underactuated wire-driven mechanism
- Manipulated by a puppeteer with a good skill to express human or animal’s motion

It is difficult to determine the optimum motion of controller for desired motion of puppet.

Reasons:

- Lack of controllable DOF
- Wire-driven
- Not only kinematics but also statics due to gravitational force

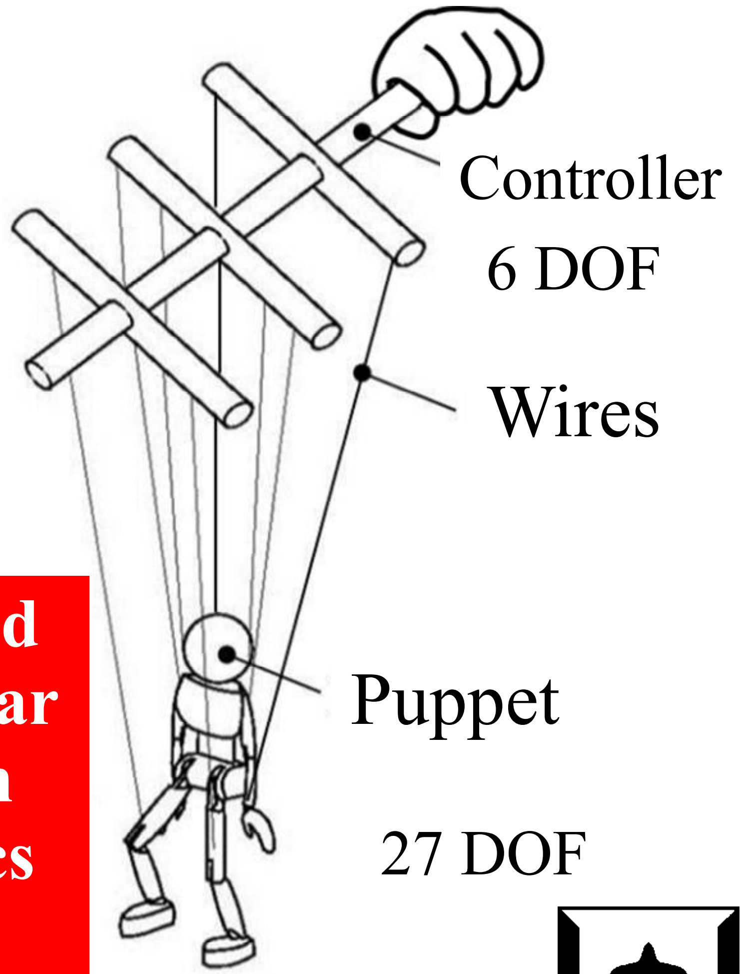


Some researchers deal with a marionette with controller with enough DOF or directly control wire length.

It is thus expected to establish the control method for underactuated marionette or the design method for optimum controller.

Objectives

To establish the general control method to generate the desired motion of planar and spatial underactuated wire-driven link mechanisms based on kinetostatics analyses



3. Kinetostatics Analysis of Underactuated Wire-driven Mechanisms

3.1 Example of planar wire-driven link chain

Wire-driven three links chain

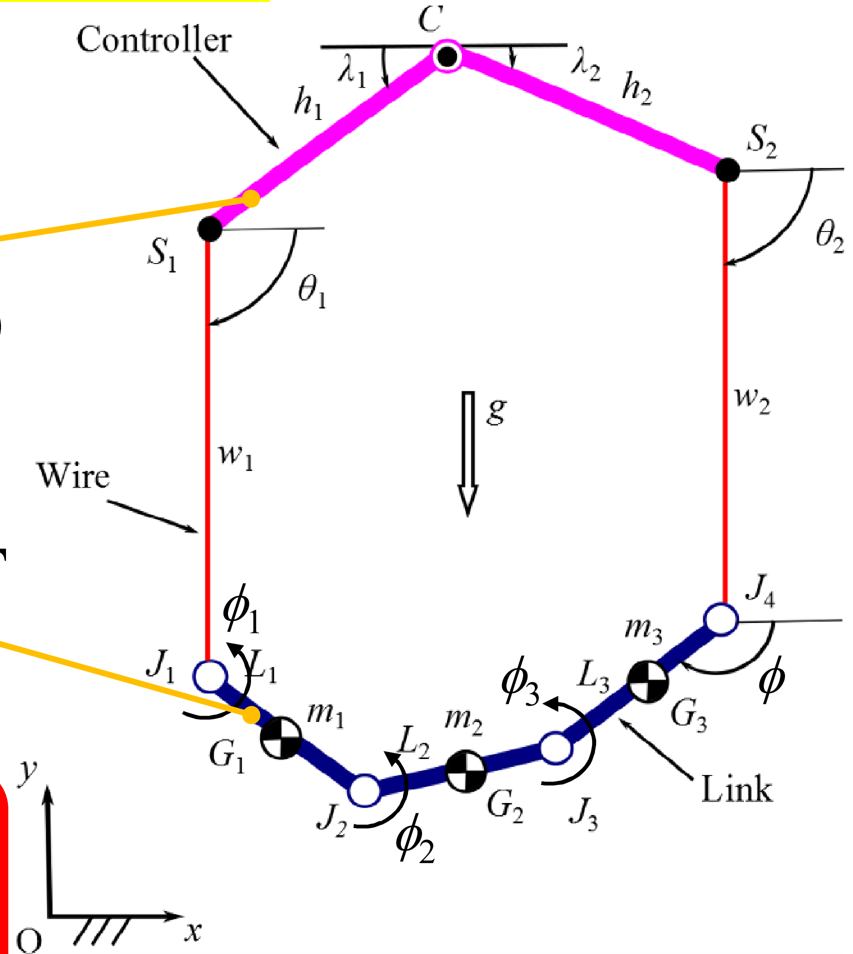
Two links controller with 4DOF

- Position of revolute joint C (2DOF)
- Posture angles λ_1, λ_2 (2DOF)

Three links output chain with 5DOF

- Position of point J_1 (2DOF)
- Posture angles ϕ_1, ϕ_2, ϕ_3 (3DOF)

The simplest underacutuated wire-driven mechanism

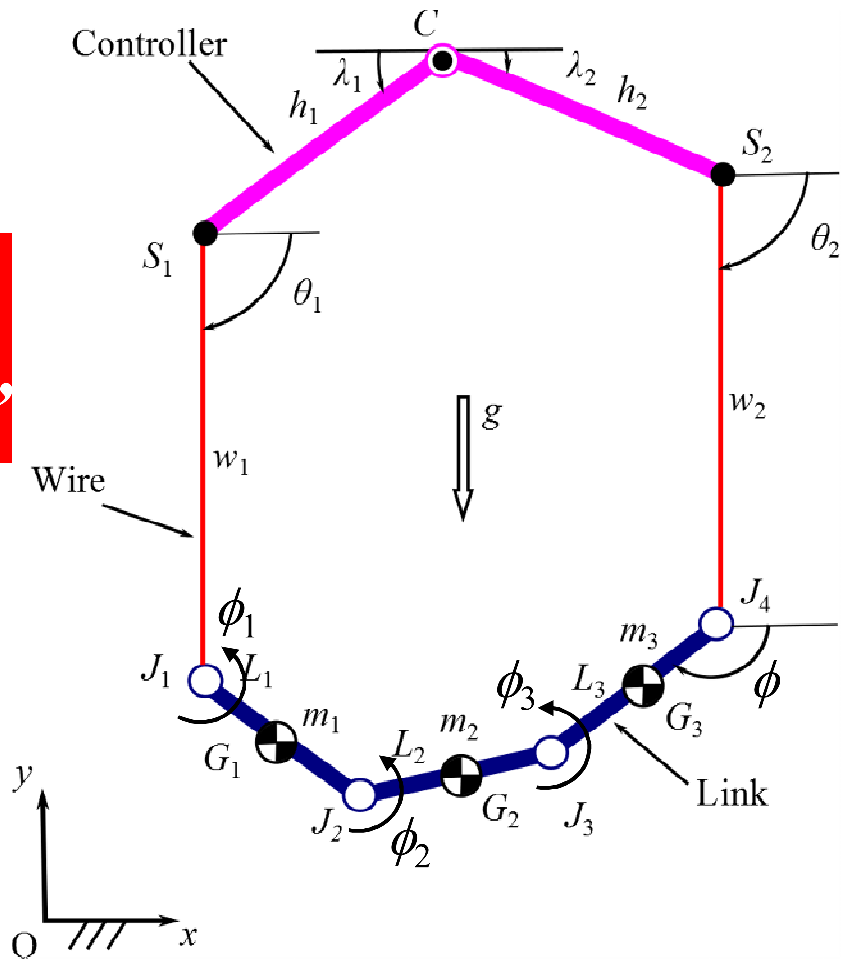


A wire-driven three links chain

3.2 Forward analysis

Configuration of controller: $\boldsymbol{C}, \lambda_1, \lambda_2$ → **Configuration of link chain:** $\boldsymbol{J}_1, \phi_1, \phi_2, \phi_3$

"To obtain configuration of output link chain, which has the minimum potential energy"



A wire-driven three links chain

Procedure:

(1)**Specify configuration of controller C, λ_1, λ_2**

(2)**Obtain wire connecting points S_1, S_2**

Wires can be assumed rigid and massless

(3)**Assume posture angles θ_1, θ_2**

(4)**Obtain wire connecting points J_1, J_4**

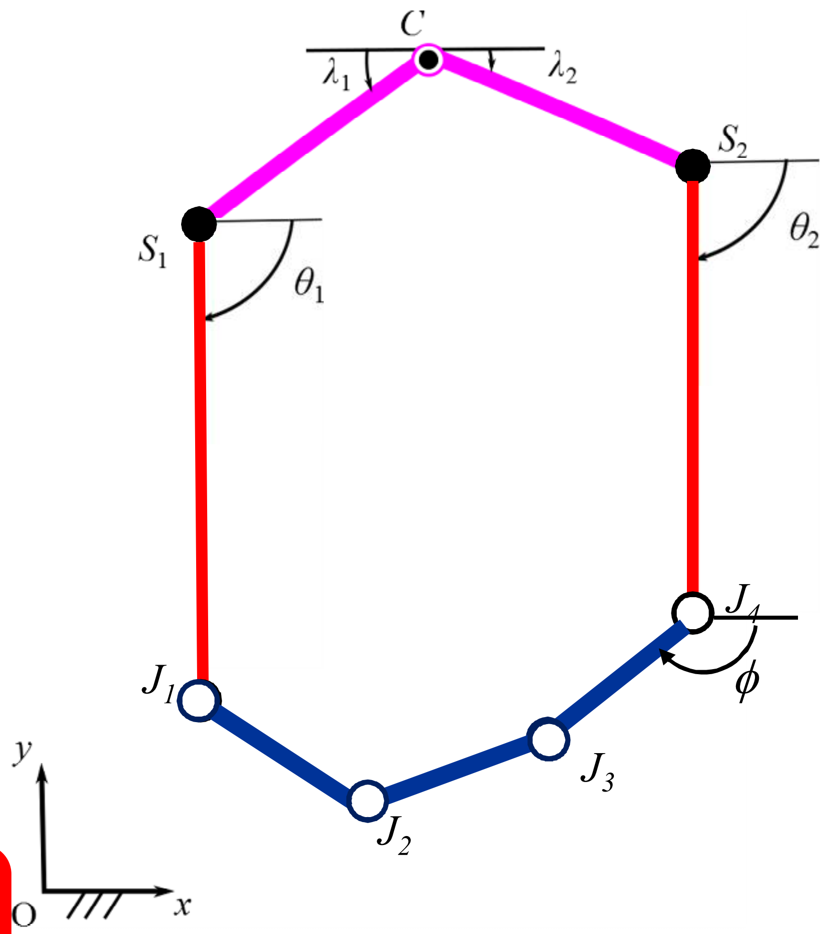
(5)**Assume link angle ϕ**

(6)**Joint J_3 can be calculated**

(7)**Joint J_2 can be calculated**

because J_2 is located at center of 2 adjacent links

Configuration of output link chain can be determined with respect to θ_1, θ_2, ϕ.



A wire-driven three links chain

(8)Calculate center of gravity of links, $G_i(\theta_1, \theta_2, \phi)$

(9)Calculate of y-coordinate of center of gravity of output link chain

Objective function:

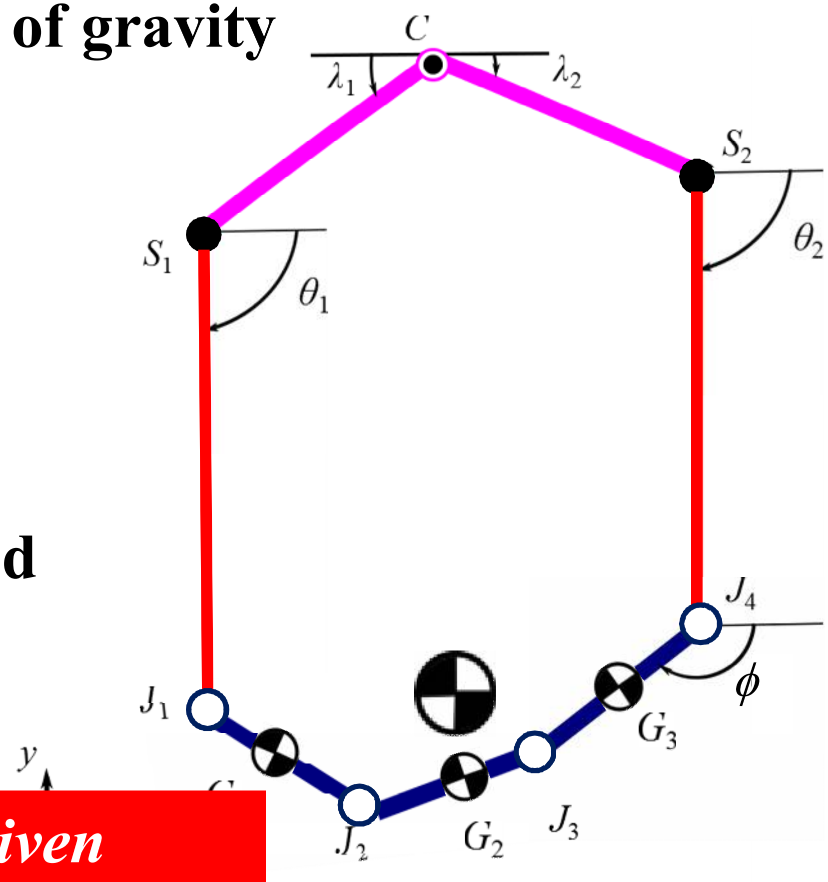
$$\Phi(\underline{\theta_1, \theta_2, \phi}) = y_G = \frac{\sum_i m_i y_{G,i}(\theta_1, \theta_2, \phi)}{\sum_i m_i}$$

Design variables

Optimization with the gradient method

=Minimization of potential energy

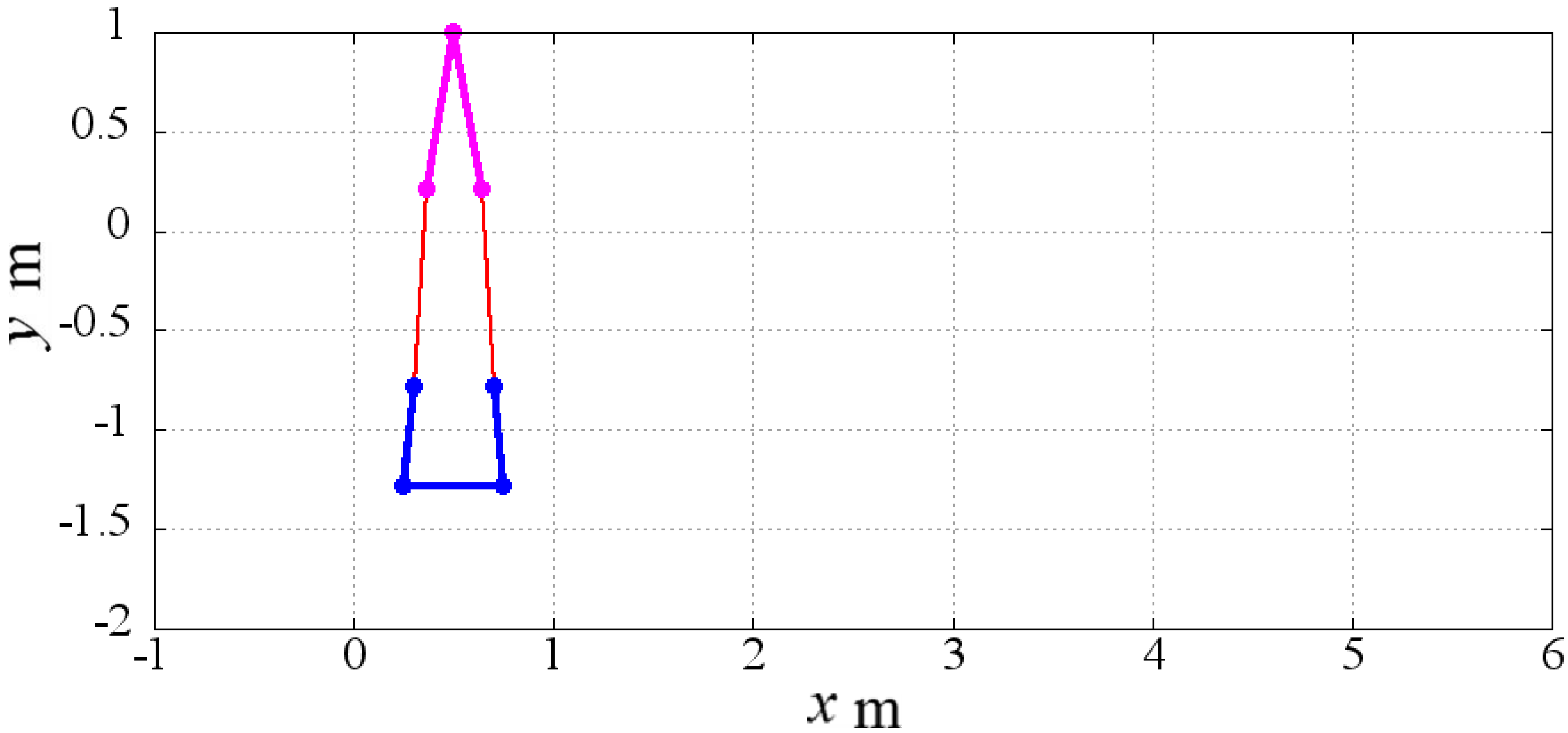
Configuration of underactuated wire-driven mechanism can be determined with the obtained θ_1, θ_2, ϕ



A wire-driven three links chain

Example of forward analysis

In case where C moves along a straight line and control links becomes open



Motion of output link chain can be calculated.

3.3 Inverse analysis

Configuration of link chain: $\boldsymbol{J}_1, \phi_1, \phi_2, \phi_3$ → **Configuration of controller:** $\boldsymbol{C}, \lambda_1, \lambda_2$

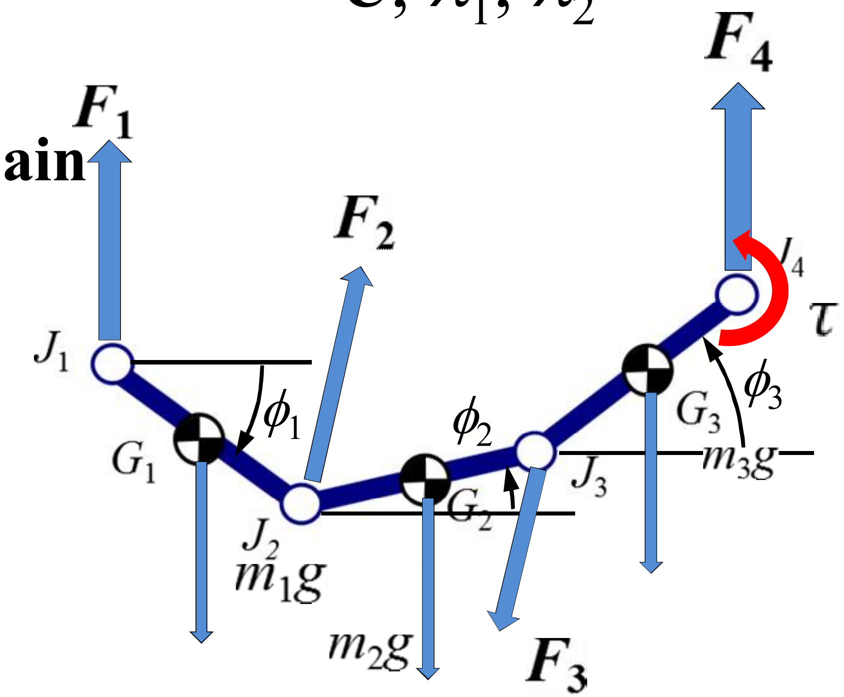
Procedure:

(1)Specify configuration of link chain

$\boldsymbol{J}_1, \phi_1, \phi_2, \phi_3$

Force balance

(2)Assume joint forces $\boldsymbol{F}_i$, gravitational forces $m_i\boldsymbol{g}$, virtual torque τ



(3)Solve static equations

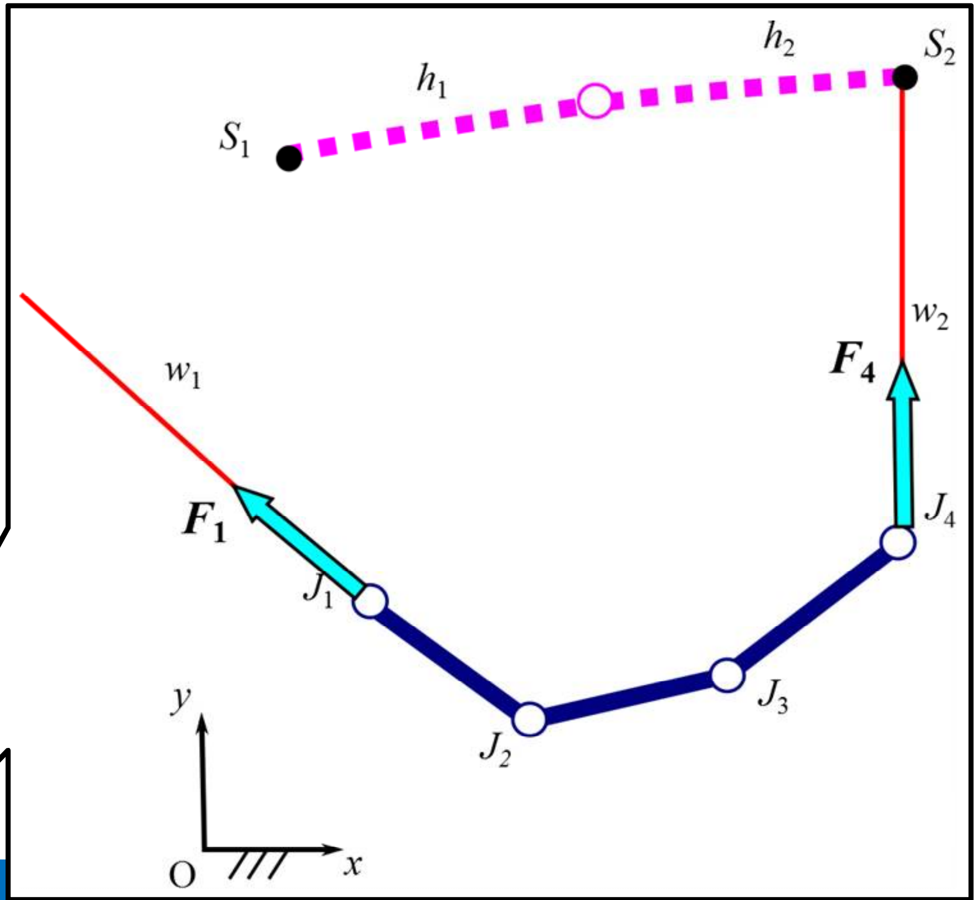
$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ y_1 - y_{G,1} & x_{G,1} - x_1 & y_2 - y_{G,1} & x_{G,1} - x_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & y_{G,2} - y_2 & x_2 - x_{G,2} & y_3 - y_{G,2} & x_{G,2} - x_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & y_{G,3} - y_3 & x_3 - x_{G,3} & y_4 - y_{G,3} & x_{G,3} - x_4 & 1 \end{bmatrix} \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \\ \tau \end{bmatrix} = \begin{bmatrix} 0 \\ m_1 g \\ 0 \\ 0 \\ m_2 g \\ 0 \\ 0 \\ m_3 g \\ 0 \end{bmatrix}$$

Wire connecting points S^*_1, S^*_2 should be located in direction of forces F_1, F_4

(4) S^*_1, S^*_2 can be calculated as

$$S_1^* = \frac{w_1}{|F_1|} F_1 + J_1, \quad S_2^* = \frac{w_2}{|F_4|} F_4 + J_4$$

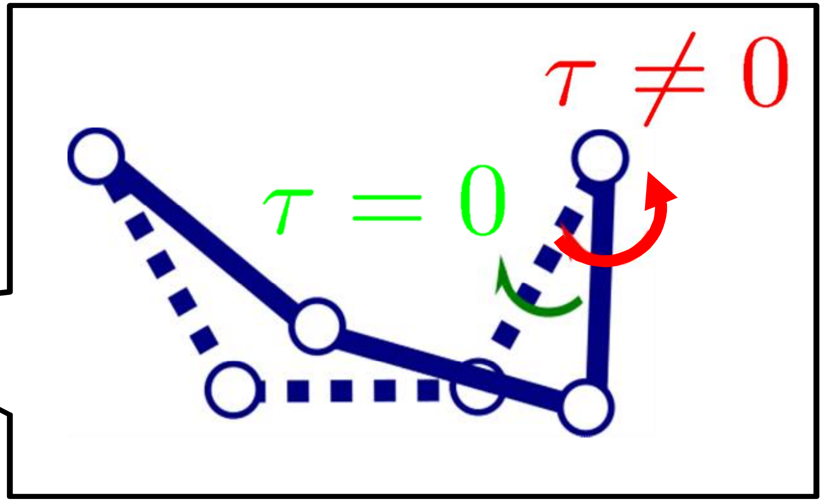
(5) Configuration of control link C, λ_1, λ_2, can be calculated with S^*_1, S^*_2



However, there exists impossible Configuration:

1)Control link cannot reach S^*_1, S^*_2

2)Virtual torque is not equal to zero



(6)Modify configuration of output link

Modifying with optimization

Objective function :

$$\Phi(\psi^*) = p_1 \sum_i \left| \boldsymbol{S}_i^* - \boldsymbol{S}_i \right|^2 + p_2 \sum_j \tau_j^2$$

Design variable

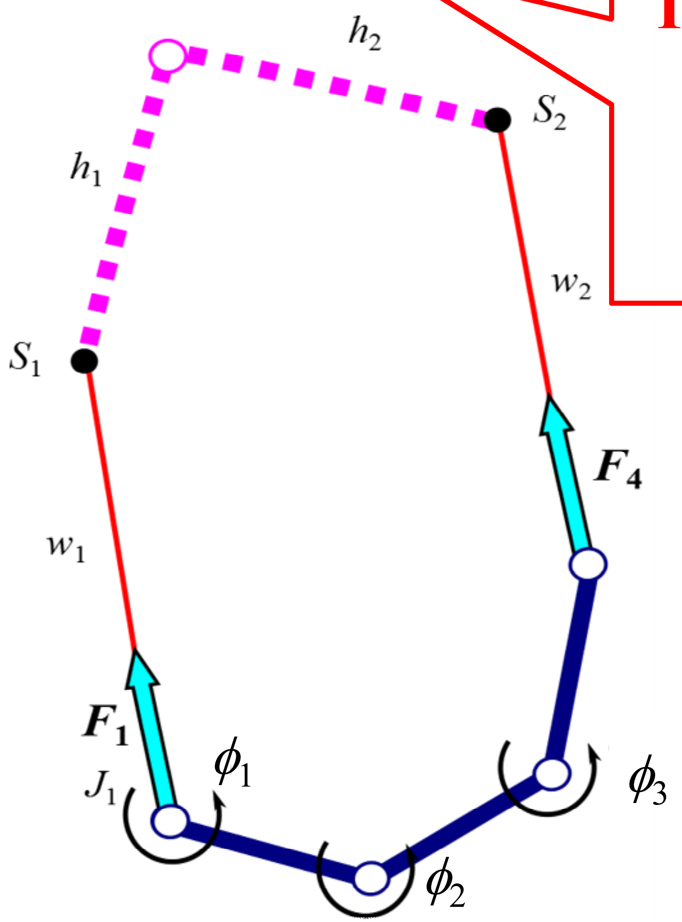
To make controller reach to wire connecting points

To make virtual torques zero

To make modification small

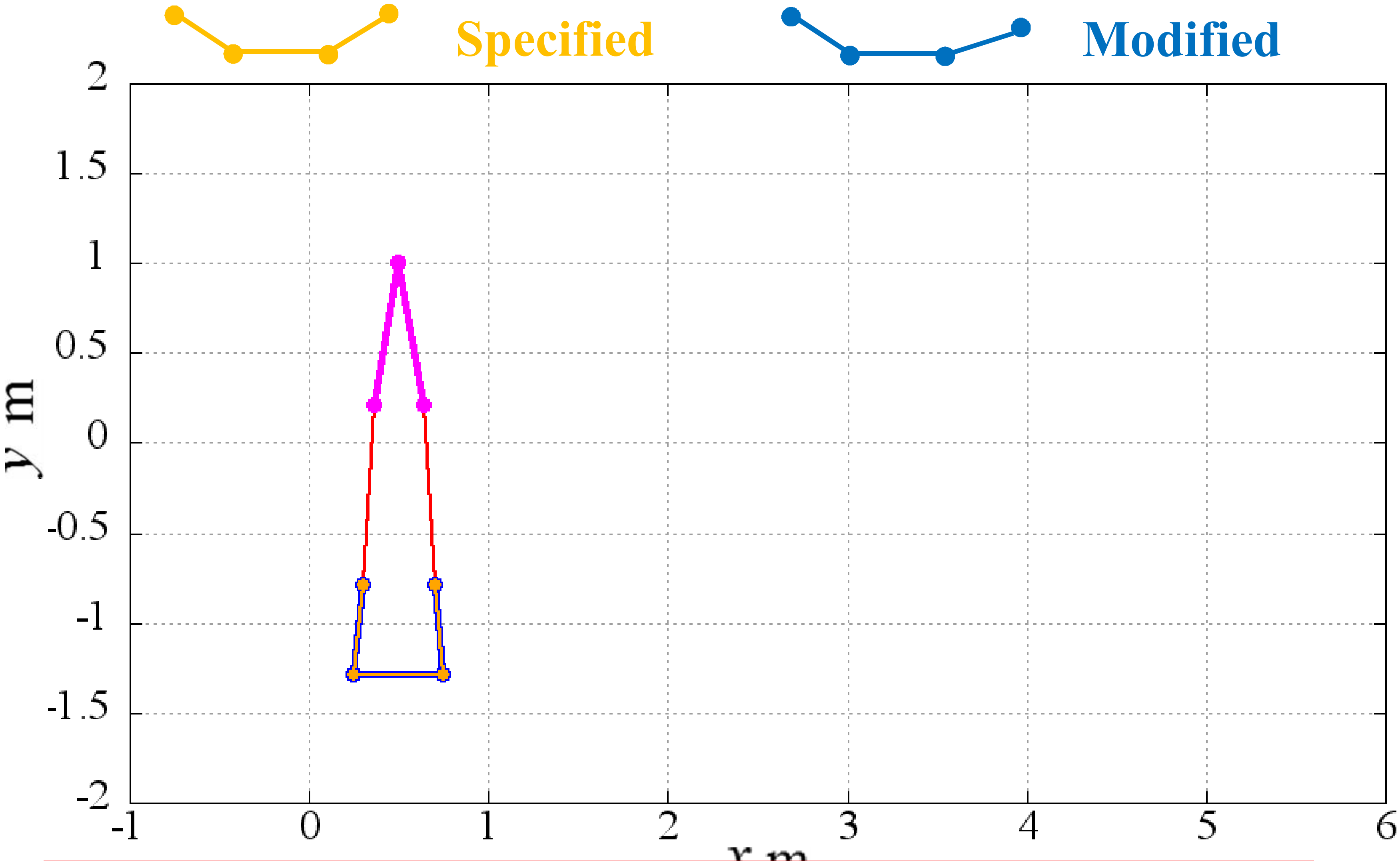
Set the link angle which has maximum sensitivity

$\max \left(\dfrac{\partial \Phi}{\partial \phi_i} \right)$ **as design variable** $\phi_i \rightarrow \psi^*$

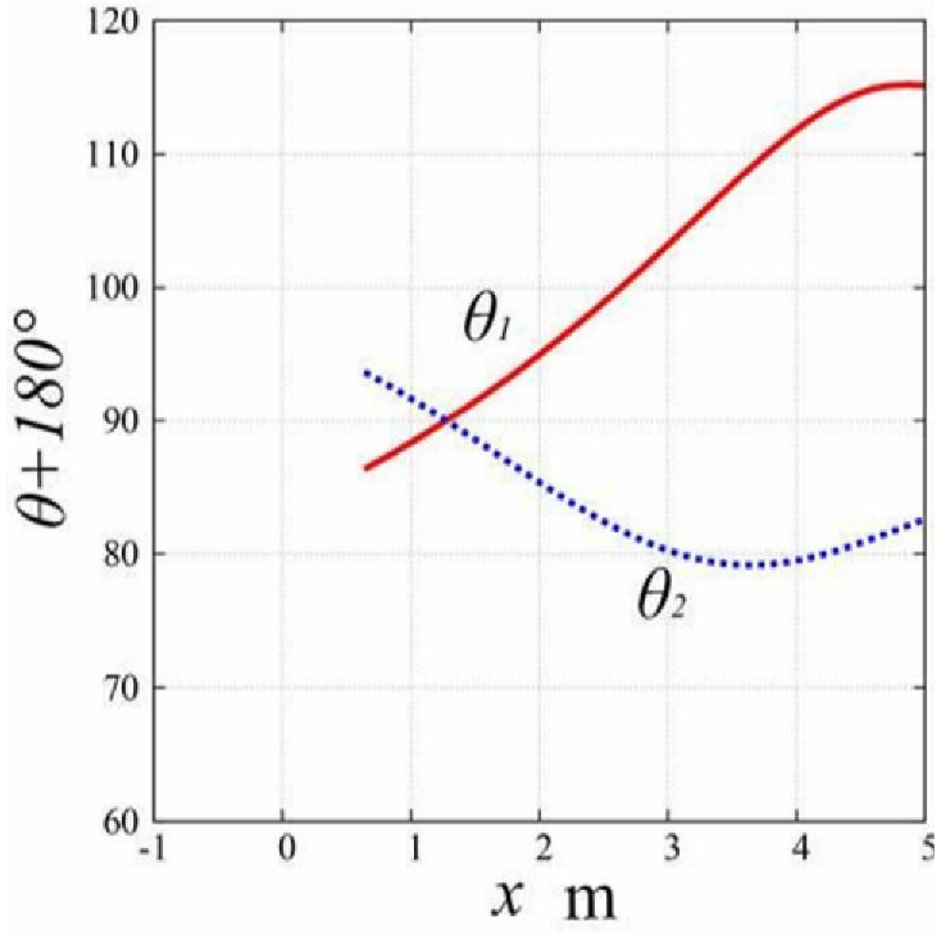


Possible configuration of output link chain can be obtained with the gradient method

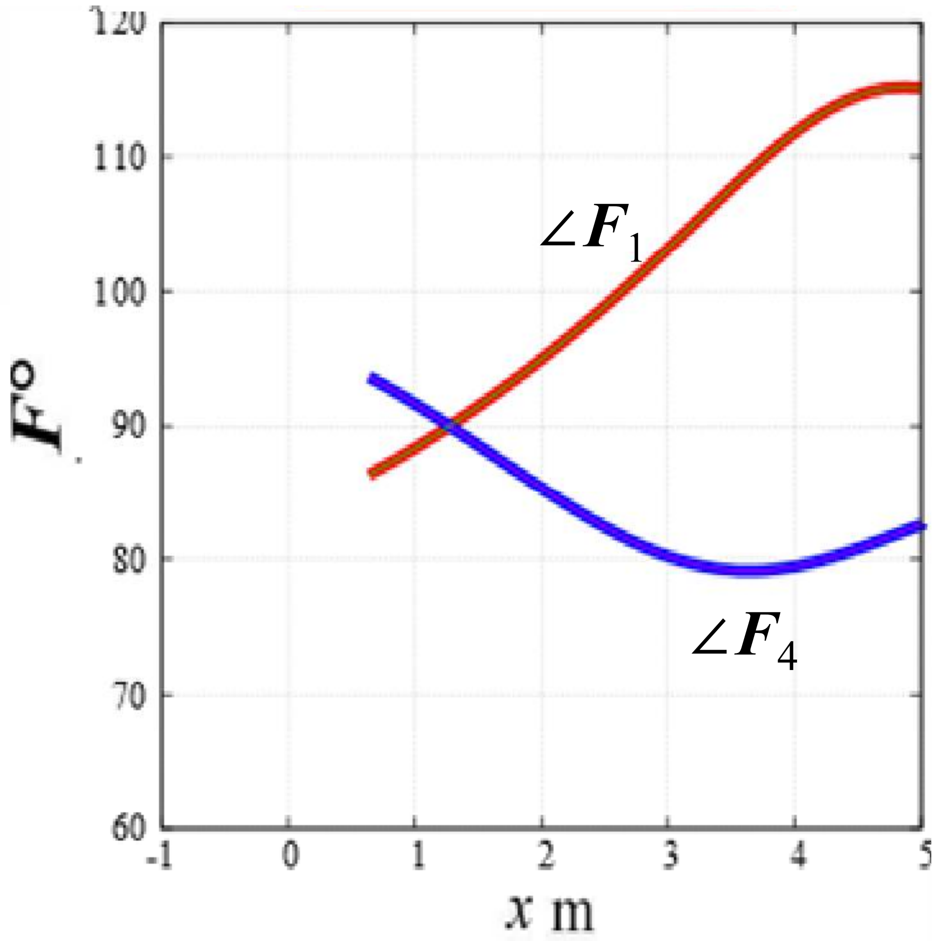
Example of inverse analysis



Expected configuration of output link chain can be calculated with inverse analysis.



Wire angles



Force angles

Directions of wire agree with those of forces → Configuration can be correctly modified

Let's have a 10 minutes break here!

If possible, would you please answer to 2021 1Q Course Survey of Study Effectiveness for this course 'Advanced Mechanical Elements' now?

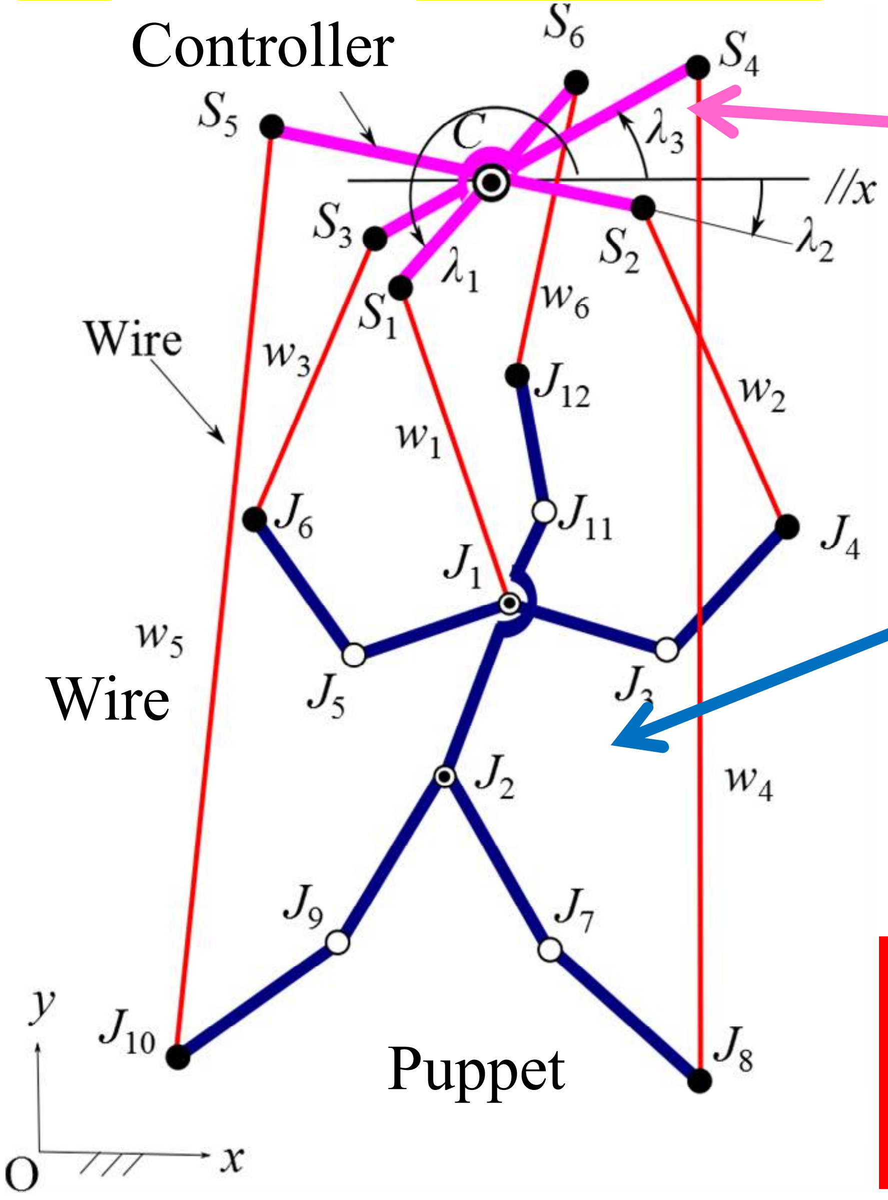
The web-site to answer the survey is as follows:

https://www.ks-fdcenter.net/fmane_titech/Ans?ms=t&id=titech&cd=tNCE234B

The deadline to answer is June 10, Thursday.

4. Motion Planning of Human Type Marionettes

4.1 A planar marionette



A controller with 5 DOF:

- Position of triple revolute joint : C (2DOF)
- Posture angles: $\lambda_1, \lambda_2, \lambda_3$ (3DOF)

A puppet with 12 DOF:

- Left hand and right foot are connected with a control bar.
- Right hand and left foot are connected with a control bar.

It is just an underactuated mechanism however does not require virtual torque.

4.2 Forward analysis of marionette

Configuration of controller: → **Configuration of puppet:**

$\boldsymbol{C}, \lambda_1, \lambda_2, \lambda_3$ → $\boldsymbol{J}_1, \dots, \boldsymbol{J}_{12}$

Same procedure as three-link chain :

(1)Specify configuration of controller: $C, \lambda_1, \lambda_2, \lambda_3$

(2)Obtain wire connecting points: $\boldsymbol{S}_1, \dots, \boldsymbol{S}_6$

(3)Assume posture angles of massless wires, $\theta_1, \dots, \theta_6$

(4)Obtain wire connecting points: $\boldsymbol{J}_1, \boldsymbol{J}_4, \boldsymbol{J}_6, \boldsymbol{J}_8, \boldsymbol{J}_{10}, \boldsymbol{J}$

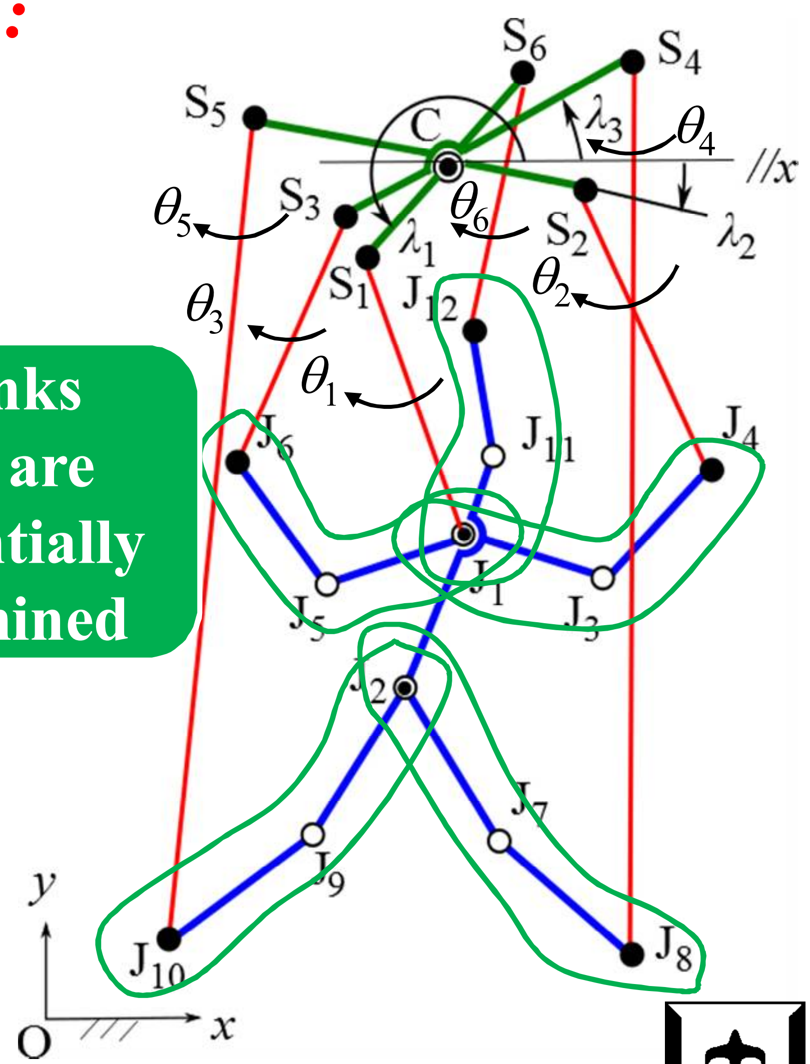
(5)Obtain configuration of puppet based on kinematics

(6)Obtain center of gravity of links

(7)Solve the optimum wire angles which minimize potential energy of whole puppet

(8)Forward analysis is completed

Two links chains are sequentially determined



4.3 Inverse analysis of marionette

Configuration of puppet: $\boldsymbol{J}_1, \ldots, \boldsymbol{J}_{12}$ → **Configuration of controller:** $\boldsymbol{C}, \lambda_1, \lambda_2, \lambda_3$

Same procedure as three-link chain :

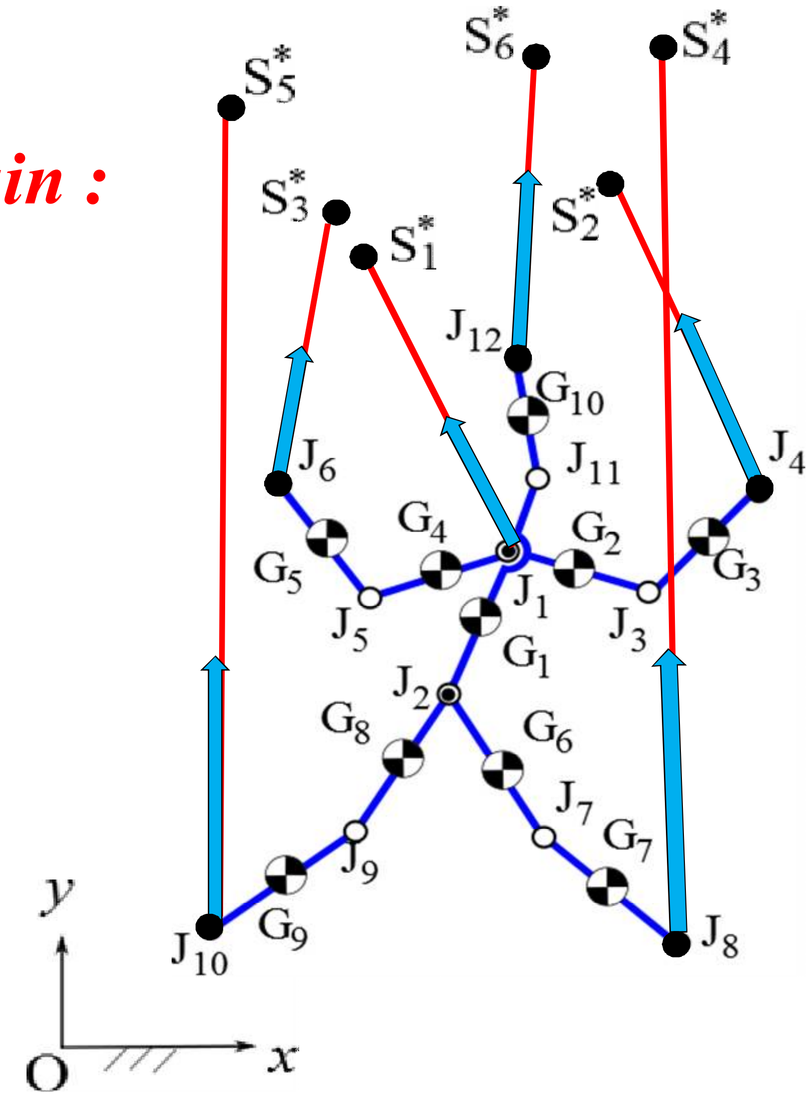
(1)Specify configuration of puppet $\boldsymbol{J}_1, \ldots, \boldsymbol{J}_{12}$

(2)Inverse statics analysis

"30 variables"

(3)Determine direction of wires

(4)Obtain wire connecting points on a controller: $\boldsymbol{S}_1^*, .., \boldsymbol{S}_6^*$



(5)Determine configuration of a controller

"Generally no solution due to lack of DOF"

(6)Optimize the rotating angle of link chain in the puppet to satisfy the constraint

Design variables:

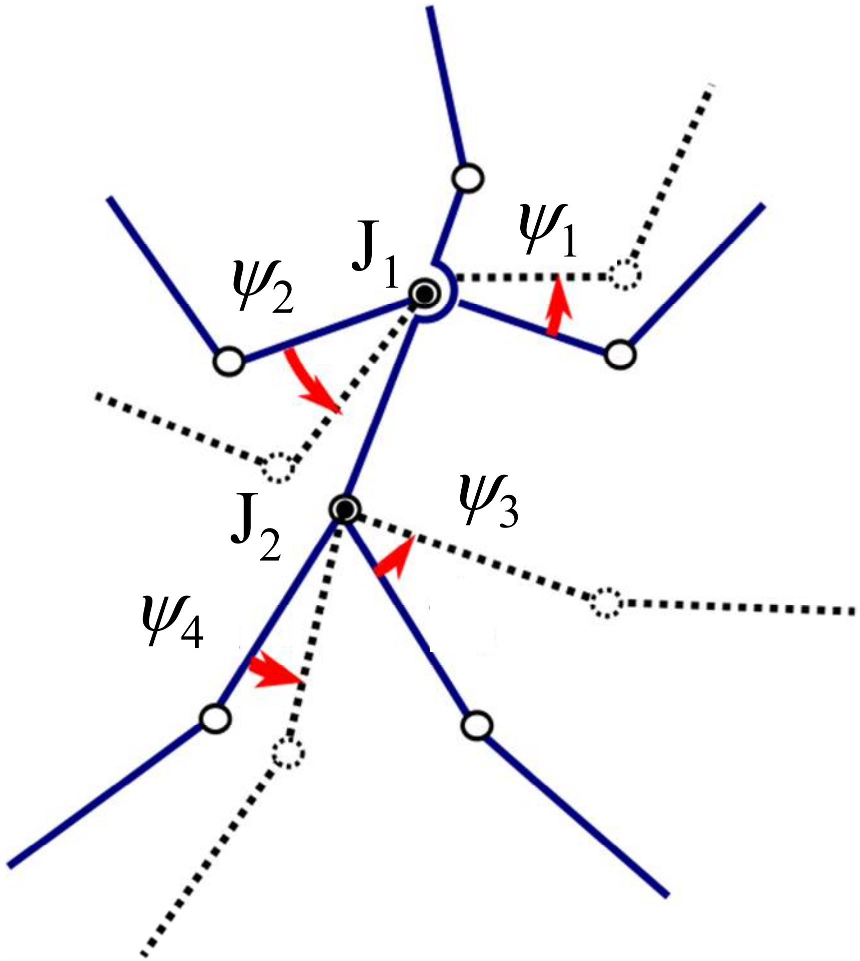
Rotation angles of link chains about J_1, J_2 : ψ_i (i=1,2,3,4)

Objective function:

Summation of distances between S_k^* and S_k:

$$\Phi_I(\psi_1,\psi_2,\psi_3,\psi_4) = \sum_k \left| \boldsymbol{S}_k^* - \boldsymbol{S}_k \right|$$

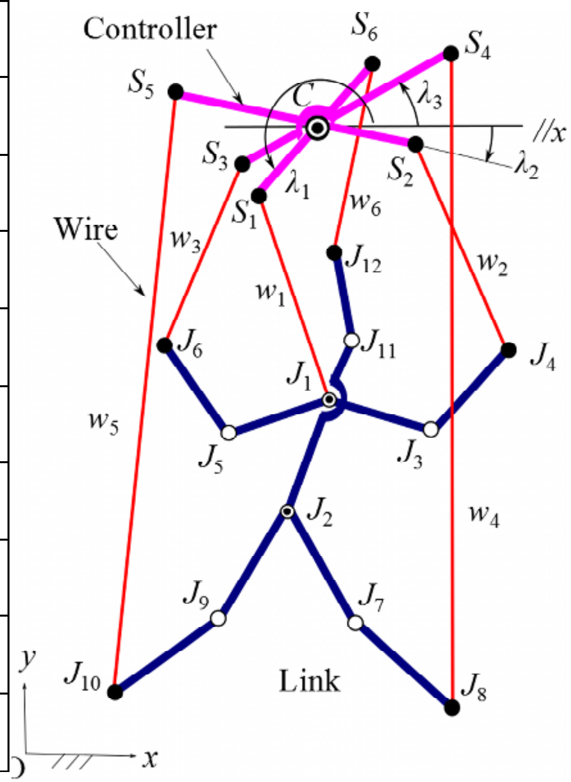
It should be zero.

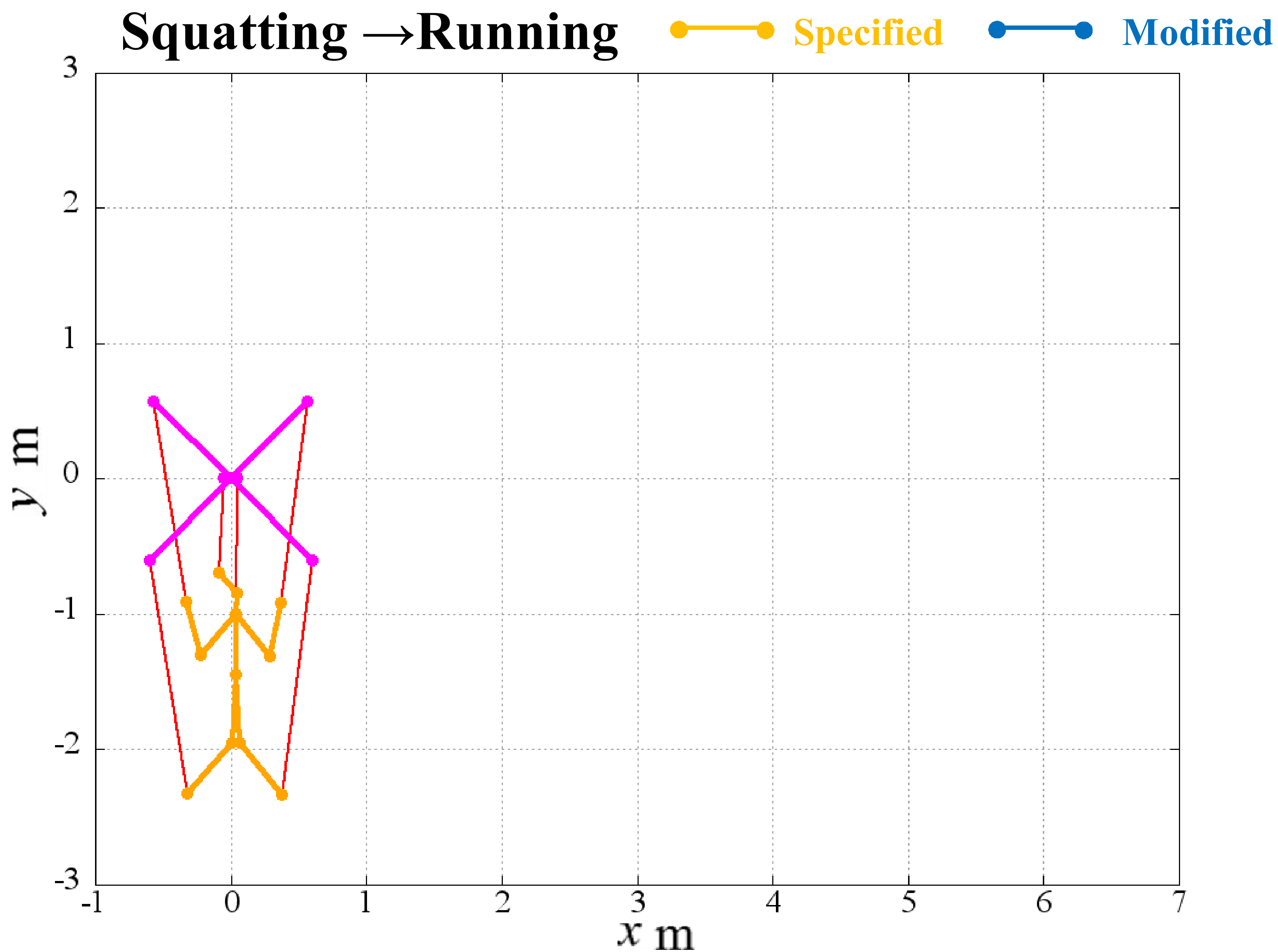


Example of inverse analysis of marionette

Specifications of planar marionette

J_1J_2	0.45 m	J_1J_{11}	0.60 m	J_1J_3	0.40 m
J_3J_4	0.40 m	J_1J_5	0.40 m	J_5J_6	0.40 m
J_2J_7	0.50 m	J_7J_8	0.50 m	J_2J_9	0.50 m
J_9J_{10}	0.50 m	$J_{11}J_{12}$	0.20 m	CS_1	0.05 m
CS_2	0.80 m	CS_3	0.80 m	CS_4	0.85m
CS_5	0.85m	CS_6	0.05m	w_1	1.00 m
w_2	1.50 m	w_3	1.50 m	w_4	1.75 m
w_5	1.75 m	w_6	0.70 m	m_1	0.60 kg
m_2	0.40 kg	m_3	0.40 kg	m_4	0.40 kg
m_5	0.40 kg	m_6	0.50 kg	m_7	0.50 kg
m_8	0.50 kg	m_9	0.50 kg	m_{10}	0.20 kg





An example of inverse analysis of a planar human type marionette

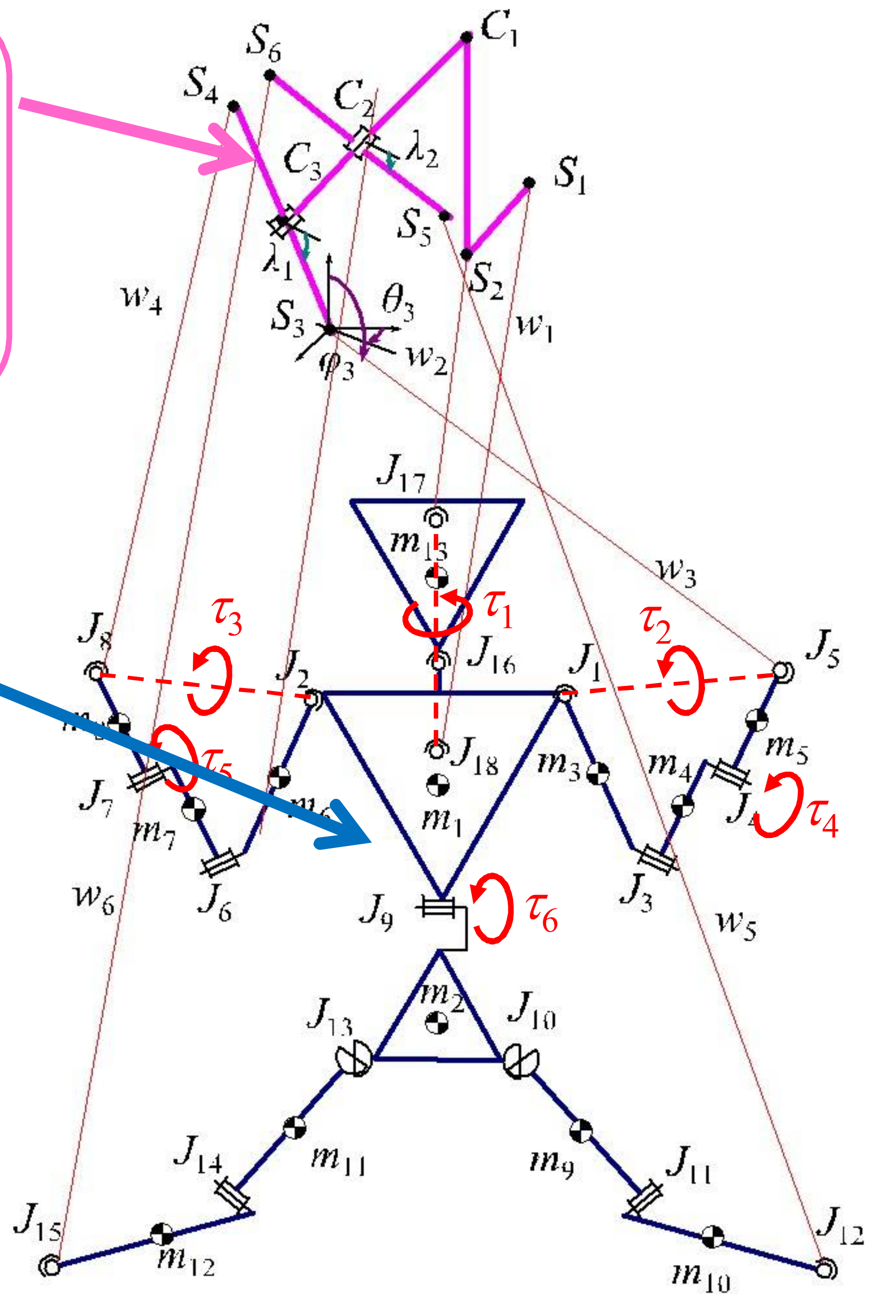
4.2 A spatial marionette

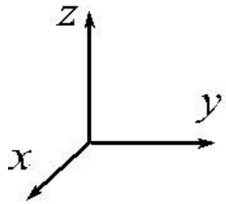
A controller with 5 DOF:

- Position of a link, C_1 (3DOF)
- Posture angles:
  λ_1, λ_2 (2DOF)

A puppet with 25 DOF:

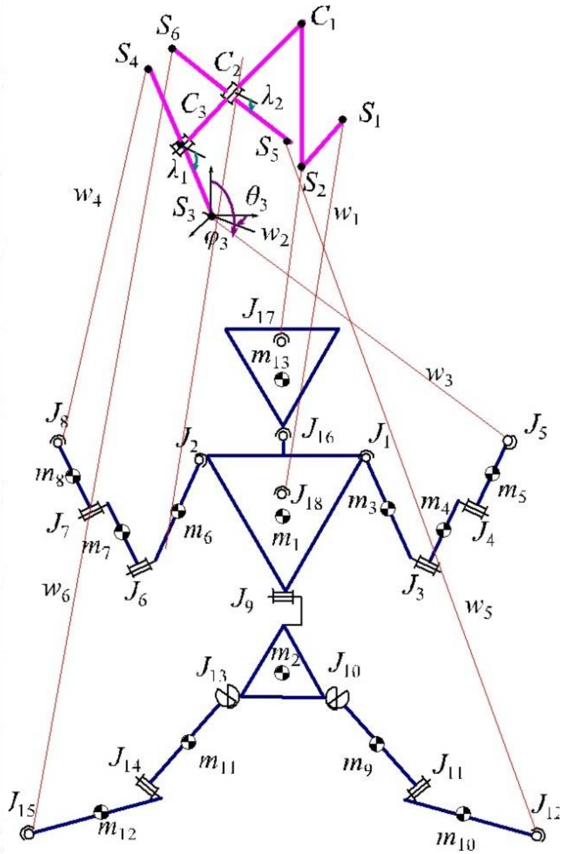
- With revolute, universal and spherical joints
- **6 virtual torques are assumed.**

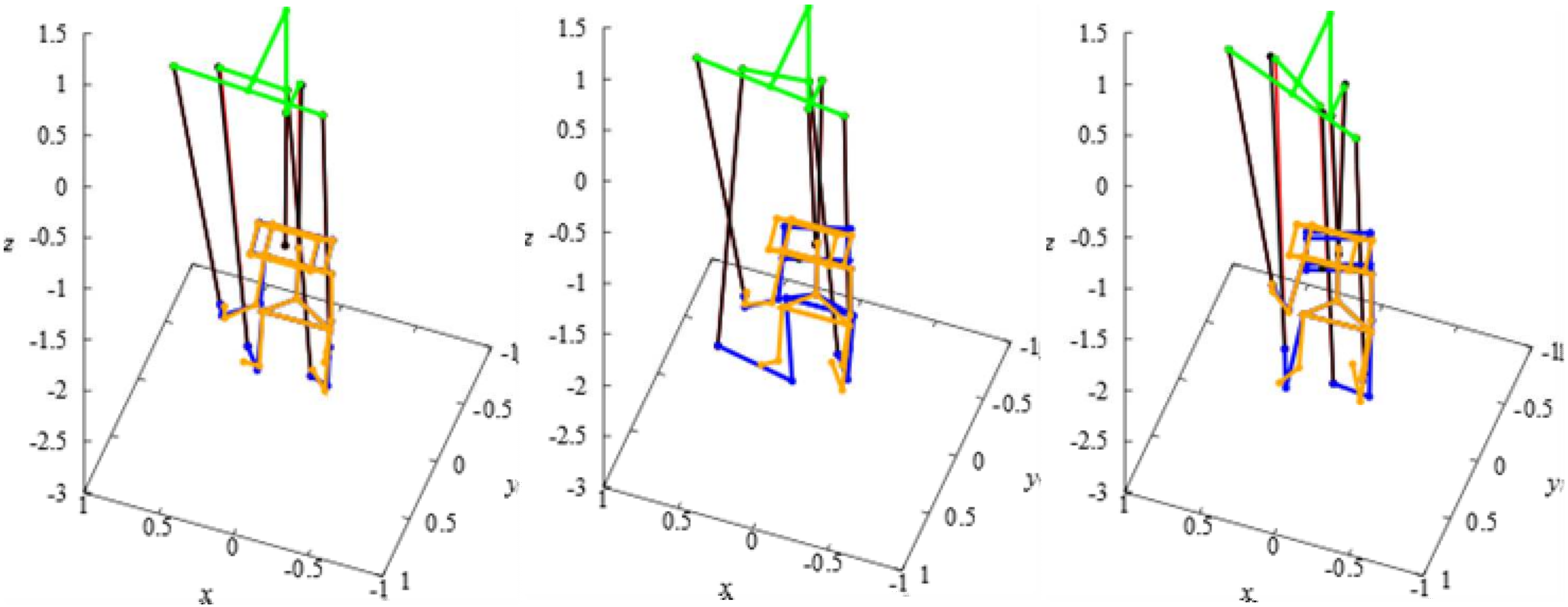




Specifications of spatial marionette

J_1J_2	0.090 m	$J_{16}J_{17}$	0.080 m	$J_{11}J_{12}$	0.100 m
J_3J_4	0.080 m	$J_{16}J_{18}$	0.030 m	$J_{18}J_9$	0.060 m
J_7J_8	0.020 m	J_4J_5	0.020 m	J_2J_6	0.080 m
$J_{10}J_{11}$	0.100 m	J_9J_{10}	0.050 m	J_9J_{13}	0.050 m
$J_{13}J_{14}$	0.100 m	J_1J_3	0.080 m	J_6J_7	0.080 m
$J_{10}J_{13}$	0.090 m	$J_{14}J_{15}$	0.100 m	S_1S_2	0.050 m
C_1S_2	0.200 m	C_1C_2	0.120 m	C_2C_5	0.100 m
C_2C_6	0.100 m	C_2C_3	0.020 m	C_3S_3	0.050 m
C_3S_4	0.050 m	w_1	0.340 m	w_2	0.250 m
w_3	0.580 m	w_4	0.580 m	w_5	0.660 m
w_6	0.660 m	m_1- m_{13}		0.10 kg		



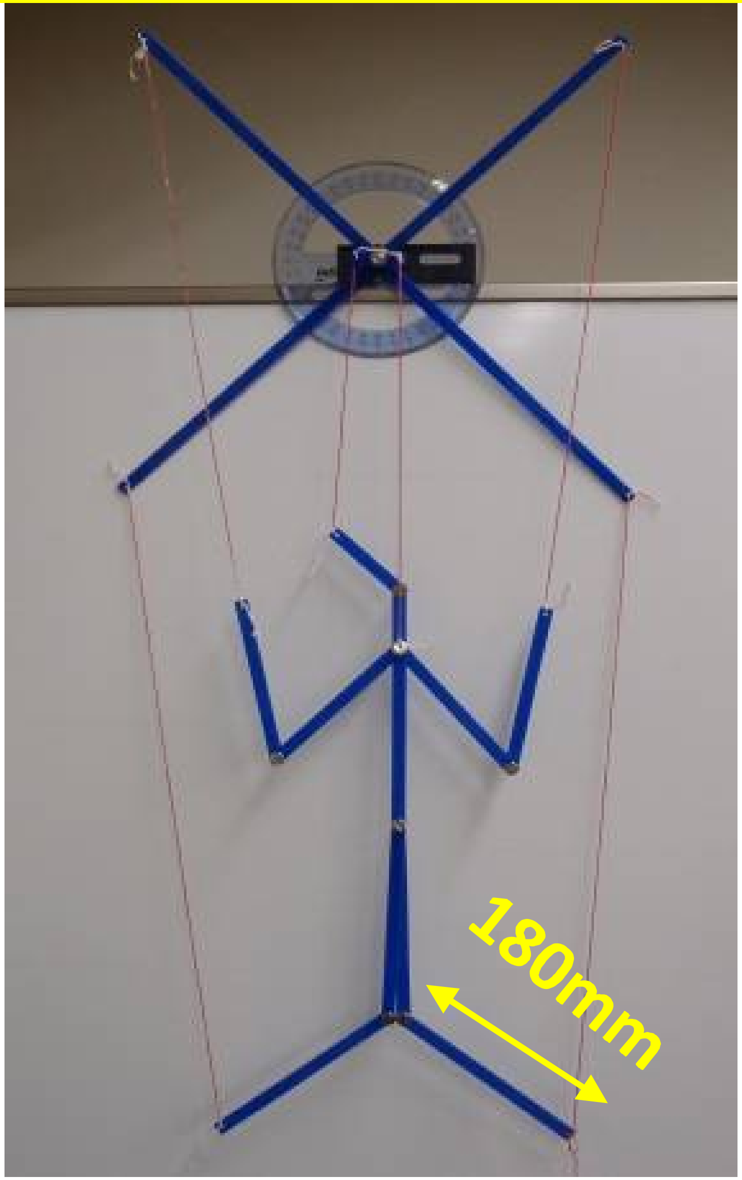


Example of inverse analysis of spatial human type marionette

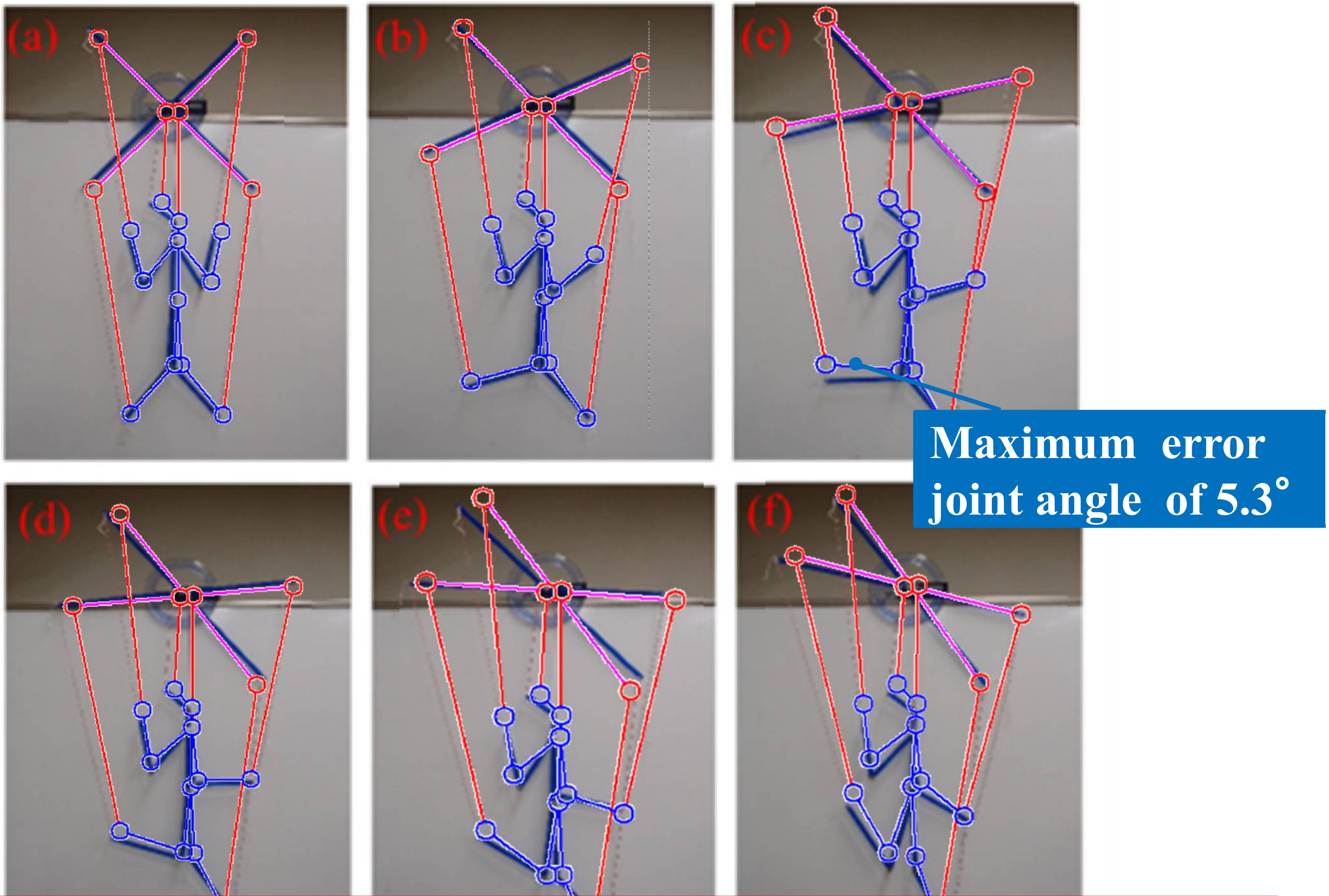
Directions of wire agree with those of forces ➡ Configuration can be correctly modified

5. Experiments

5.1 A planar marionette



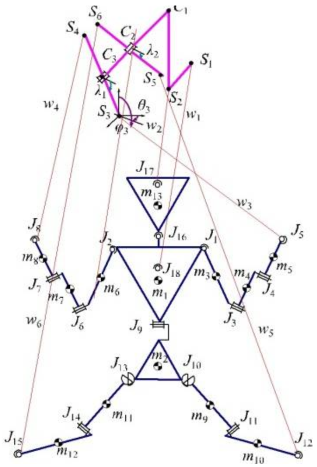
- Links are made of acrylic plate with 4mm in thickness.
- Revolute joints are metal shaft with screws at both ends and two bolts as flange.
- Wires are cotton thread.

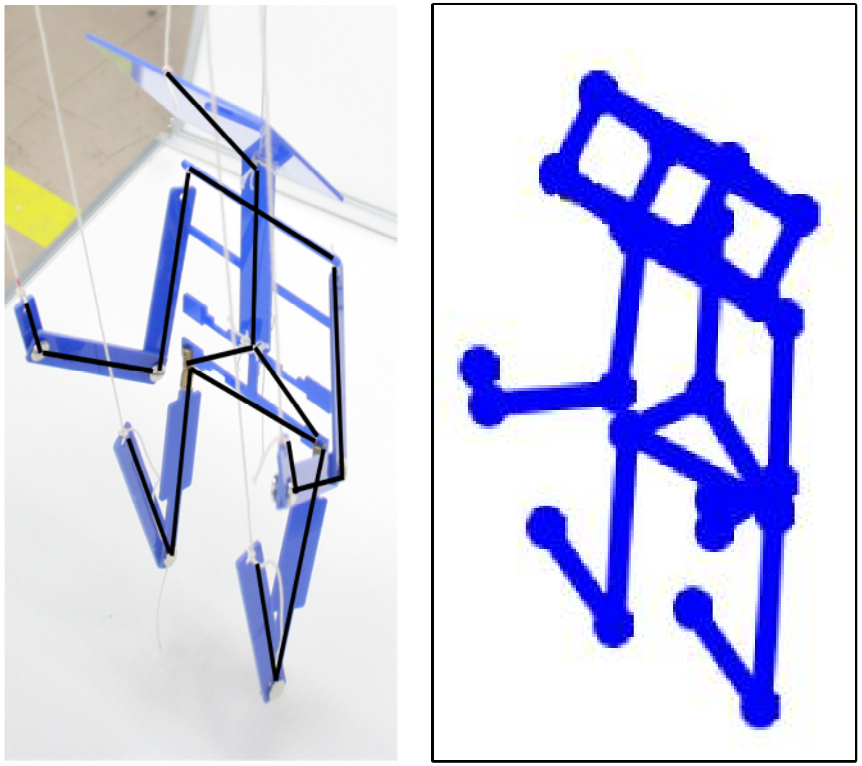


The prototype marionette can generate the desired configurations calculated with the proposed inverse analysis

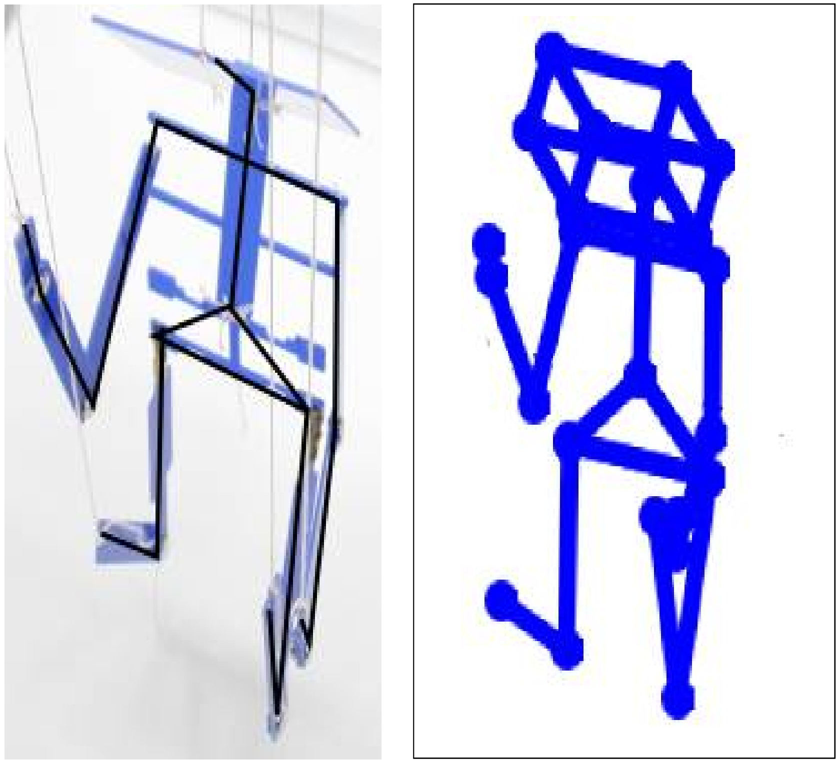
5.2 A spatial marionette

It is confirmed that the proposed method is effective and useful.





(a)Configuration 1



(b)Configuration 2

6. Concluding remarks

Aiming to establish the general method to control underactuated wire-driven mechanisms, kinetostaics analyses are proposed and examined.

(1)The configuration of link chain hung with several wires can be calculated with the optimization to minimize the vertical position of center of gravity of the chain.

(2)The configuration of controller can be calculated with wire directions based on the inverse analysis and the optimization of the modifying angles of link chains.

(3)Motion planning of planar and spatial human type marionettes can be achieved with the proposed inverse kinetostatics.

(4)The proposed method was experimentally validated with prototypes of the human type marionettes composed of links of acrylic bars and cotton threads.

Concluding remarks for whole lecture

Through this lecture, you are expected to be able to:

(1) Explain mobility of mechanism and relation between input/output motion of mechanism

(2) Analyze displacement, velocity and acceleration of planar/spatial closed-loop link mechanism with the systematic kinematic analysis method

(3) Analyze the dynamics of planar/spatial closed-loop link mechanism utilizing the systematic kinematic analysis method

(4) Explain the optimum motion control of redundant link mechanisms

(5) Explain motion control of underactuated mechanism with elastic elements

Important issues explained in this lecture are as follows:

(1)Kinematic analyses of planar/spatial link mechanism with the systematic kinematic analysis method

"Displacement, velocity and acceleration analyses of planar/spatial closed-loop link mechanism can be easily achieved."

(2)Dynamic analyses of planar/spatial link mechanism

"Driving forces and joint forces can be analyzed using the systematic kinematic analysis."

(3)Optimum motion control of redundant link mechanisms

"Dexterity can be maximized by utilizing redundancy."

(4)Motion control of underactuated link mechanisms

"Underactuated mechanisms constrained with elastic elements can be controlled by taking account of kinematics and statics."

(5)Motion control of wire-driven underactuated link mechanisims

"Marionette can be theoretically controlled."

Subject of final report

Calculate the desired input motions for the specified target output motion, (*X*,*Y*), of the planar 6-bar one-loop manipulator shown in the next page under the following conditions.

(1)You can locate 2 or 3 rotary actuators at revolute pairs.

Note: ・If you locate 2 actuators, the mechanism will become an underactuated mechanism. Therefore you have to constrain the mechanism with some elastic elements.

If you locate 3 actuators, the mechanism will become a redundant mechanism. Therefore you have to achieve the optimum inverse kinematics with a certain objective function.

(2)You can determine mechanical parameters and inertial parameters arbitrarily.

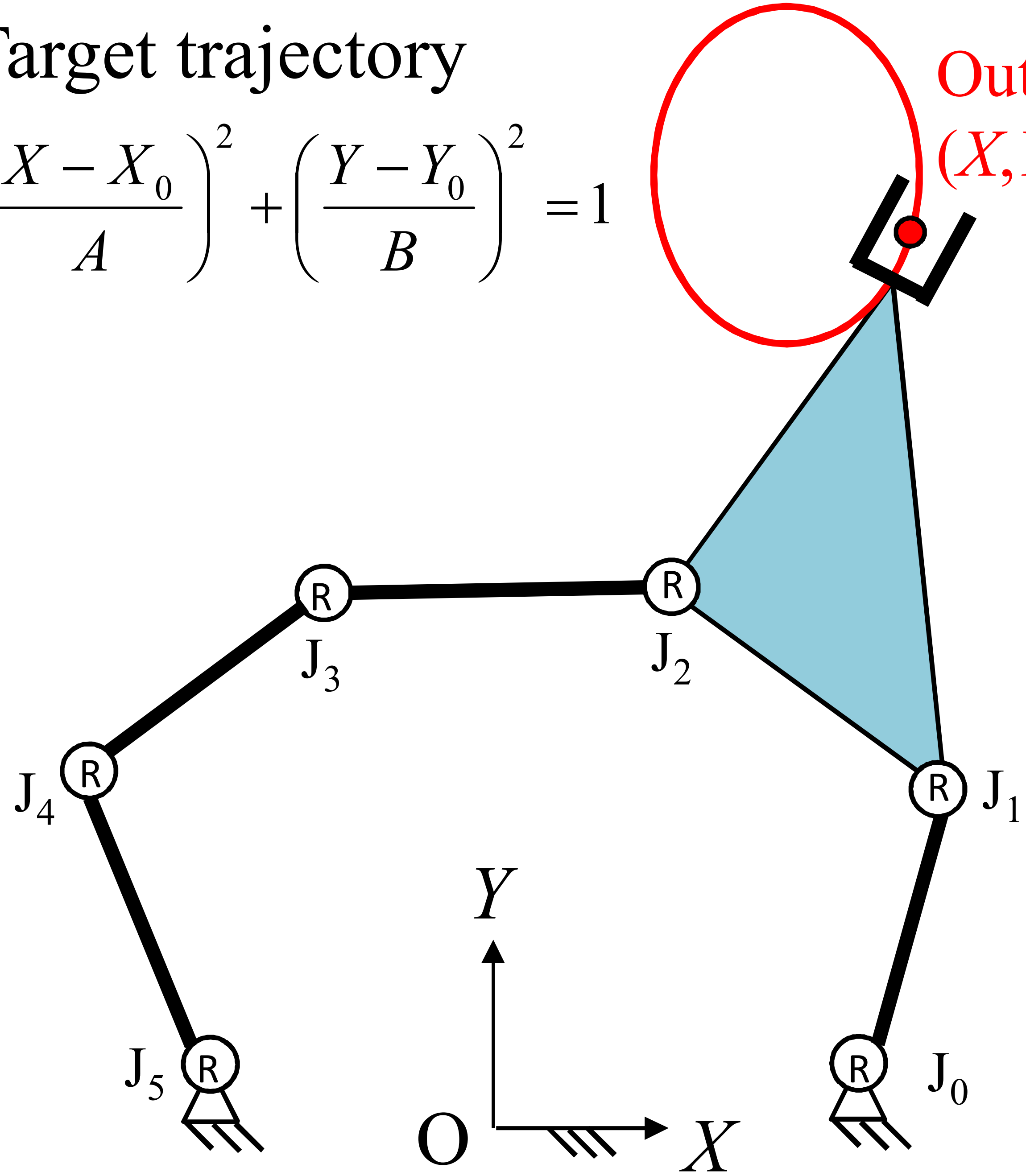
(3)The target trajectory of the output point, P, should be an ellipsoid. The output point should be driven at uniform speed or according to double dwelling function such as 5th power function. You can determine its parameters A, B, X_0 and Y_0 and their time history.

(4)You will show time histories of output/input motions.

The report will be summarized in A4 size PDF with less than 10 pages and sent to Prof. Iwatsuki via T2SCHOLA by June 9, 2023.

Target trajectory

$$\left(\frac{X - X_0}{A}\right)^2 + \left(\frac{Y - Y_0}{B}\right)^2 = 1$$



Note:
It will be a stupid solution if you will locate 3 actuators at J_0, J_1, J_2, because the mechanism becomes a serial 2R manipulator with non-necessary three links.

A planar 6-bar one-loop manipulator